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CONTINUOUS BEAMS OF ALUMINUM ALLOY TUBULAR CROSS-SECTION - PART

II: PARAMETRIC STUDY AND DESIGN

by

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ABSTRACT: Aluminum alloys are employed in a range of structural engineering applications, supported by many international design standards, but plastic design methods are generally not permitted. In the second part of this study, an extensive numerical parametric study is conducted using *ABAQUS 6.10-1* (2010) to assess the effect of key parameters, such as cross-section slenderness, cross-section aspect ratio and moment gradient on the strength, strain hardening and moment redistribution behavior of aluminum alloy continuous beams. The key input parameters and findings are reported herein. Based on both the experimental and parametric numerical results, the design provisions of the American (AA, 2010), Australian/New Zealand (AS/NZS, 1997) and European (EC9, 2007) specifications as well as the traditional plastic design method, the plastic hinge method (EC9, 2007) and the continuous strength method (CSM) for indeterminate structures, the scope of which is extended in the present study, have been evaluated. The design strengths predicted by the three specifications were found to be rather conservative, while the predications of the latter three methods are more precise and consistent. The results reveal that strain hardening at the cross-sectional level and moment redistribution at the global system level have significant influence on the performance of stocky (plastic and compact

sections) aluminum alloy members, which should be accounted for in design. Following reliability analysis, proposals are made for revised design provisions.

KEY WORDS: Aluminum alloys; Continuous beams; Continuous strength method; Indeterminate structures; Parametric study; Plastic design; Reliability analysis; Square and rectangular hollow sections; Structural design; Tubular sections.

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INTRODUCTION

The behavior and design of indeterminate aluminum alloy structures is explored in the present paper, with particular emphasis on plastic moment redistribution and strain hardening. Early studies into the inelastic behavior of aluminum alloy structures were carried out at the end of the 1970s and in the early 1980s (De Martino and Faella, 1978; De Luca, 1982). Studies of moment redistribution in indeterminate structures composed of nonlinear materials include that of Manganiello et al. (2006), who investigated the required ductility for moment redistribution in aluminum alloy continuous beams by numerical means and verified the rotation capacity requirements in EC9 (2007), and that of Theofanous et al., (in press), where significant moment redistribution in stainless steel continuous beams was observed. In the latter paper, the importance of strain hardening was also highlighted. Similar observations have been made by others, such as Kemp (1986), who reviewed previous studies into the influence of strain hardening on the

capacity of steel beams, and Byfield and Nethercot (1998) and Kemp et al. (2002), who demonstrated experimentally the benefits of strain hardening in stocky (plastic and compact) steel sections. In recent years, further studies (Byfield et al., 2005; Gardner, 2008; Gardner et al., 2011; Afshan and Gardner, 2013 and Su et al., 2014a, b) have recognized the influence of strain hardening on the ultimate strength of metallic structural elements.

For the design of aluminum alloy structural members, there are a number of established international design specifications, including the Aluminum Design Manual (AA) (2010), the Australian/New Zealand Standard (AS/NZS) (1997) and Eurocode 9 (EC9) (2007). EC9 provides the plastic hinge method in Annex H for Class 1 continuous beams as an alternative approach to that given in the main body of the code; both moment redistribution and strain hardening are considered. De Matteis et al. (2001) highlighted that EC9 was the first code to allow full inelastic analysis of aluminum alloy structures. However, plastic design is currently not available in the American and Australian/New Zealand Standards. A number of previous studies on aluminum alloy flexural members (Moen et al., 1999; Zhu and Young, 2006, 2009; Kim and Peköz, 2010; Su et al., 2013b) reported that although the existing design specifications are widely used by structural engineers, they are not fully efficient without considering strain hardening. To improve design efficiency, a deformation-based approach, the continuous strength method (CSM) for aluminum alloy continuous beams, is proposed in the present study and verified by comparison with test and numerical results. The CSM allows for moment redistribution as well as a systematic exploitation of strain hardening.

The aims of this investigation are to generate parametric numerical results for aluminum alloy continuous beams based on the finite element (FE) models validated in the companion paper and to evaluate the accuracy of different design methods. The considered design methods are those set out in the American (AA, 2010), Australian/New Zealand (AS/NZS, 1997) and European (EC9, 2007) specifications together with the traditional plastic design method, the EC9

Annex H plastic hinge method (EC9, 2007) and the continuous strength method (CSM) for indeterminate structures.

NUMERICAL PARAMETRIC STUDY

In this section, the validated FE model is used to conduct a parametric study by a FE package ABAQUS version 6.10-1 (2010), aiming to develop a comprehensive understanding of the inelastic behavior of aluminum alloy continuous beams. The parametric study was carried out to expand the available structural performance data over a wider cross-section slenderness range and to investigate the effect of key factors, such as the cross-section aspect ratio and the moment gradient, on the performance of aluminum alloy continuous beams. Cross-sections with outer wall dimensions up to 180 mm and the thickness varying between 3.5 mm and 12.0 mm were modeled. Thus, aspect ratios from 0.33 to 3.40 and a wide range of plate slenderness (b/t ratios: 4.25-55.14), covering the four cross-section classes, were considered. The overall beam lengths were 1690 mm, 2490 mm and 3690 mm for small (width 50 mm $\times$ height 130 mm, width 130 mm $\times$ height 50 mm), medium (width 140 mm $\times$ height 100 mm) and large (width 180 mm $\times$ height 180 mm, width 160 mm $\times$ height 200 mm) cross-sections, respectively. A total of 20 different cross-section dimensions were considered, together with two aluminum alloys and three loading configurations. The material stress-strain curves obtained from the tensile coupon tests carried out on tube H64 $\times$ 64 $\times$ 3.0 and +N95 $\times$ 50 $\times$ 10.5 (a tube reported in Su et al.(2013), with the yield stress $f_y = 109.5$ MPa and the ultimate stress $f_u = 177.4$ MPa) were used to define material properties for high strength and normal strength aluminum alloys in the parametric study, respectively. The local geometric imperfections were assumed to be of the form of the lowest appropriate elastic buckling mode shape with an amplitude of 0.2 mm, as measured in the experimental program.

In order to build up a large structural performance database of aluminum alloy continuous beams and thereafter to assess the different design approaches, a total of 120 numerical results

have been generated herein. The FE models also enable careful examination of the inelastic behavior and moment redistribution in the considered indeterminate structural systems. The newly generated numerical results are presented in Table 1, with the same labeling system as described in the companion paper. In the numerical models, the ultimate loads were determined when either a plastic collapse mechanism formed or the material fracture strain ε_f was reached on the tension flange, whichever occurred first.

CONTINUOUS STRENGTH METHOD (CSM) FOR INDETERMINATE ALUMINUM ALLOY STRUCTURES

In this section, the continuous strength method (CSM) is introduced (Gardner, 2008; Liew and Gardner, in press) and its application to aluminum alloy structures is explained. The CSM for indeterminate structures (Gardner et al., 2011) combines the merits of traditional plastic analysis (i.e. considering a plastic collapse mechanism) and an accurate assessment of cross-section moment capacity allowing for strain hardening. The cross-section moment capacity M_{csm} is determined on a deformation basis, with the limiting deformation ε_{csm} defined as a continuous function of cross-section plate slenderness $\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}}$, where σ_{cr} is the elastic buckling stress of the cross-section. The relationship between cross-section deformation capacity and cross-section slenderness is referred to as the ‘base curve’. The ultimate collapse load of the indeterminate system can be derived from the principle of virtual work by equating the external work done by the applied loads to the internal work resulting from the hinge rotations. The CSM utilizes a bi-linear strain hardening material model with a strain hardening slope E_{sh} , defined, for aluminum alloys, by Equation (1), where ε_y is the yield strain equal to f_y/E and ε_u is the strain at the ultimate tensile stress, predicted by means of Equation (2).

$$E_{sh} = \frac{f_u - f_y}{0.5\varepsilon_u - \varepsilon_y} \quad (1)$$

$$\varepsilon_u = 0.13(1 - f_y / f_u) + 0.059 \quad (2)$$

The resulting CSM design procedure for determining the design strengths of indeterminate aluminum alloy structures is summarized in the following seven steps:

- (1) Identify the location of the plastic hinges, of number i , in a manner similar to traditional plastic design and determine the respective hinge rotations θ_i . In the case of five-point bending, $\theta_1 = 2\delta/L_2$ and $\theta_2 = \delta/L_1 + \delta/L_2$ are derived, as illustrated in Fig. 1.

- (2) Calculate the cross-section slenderness $\bar{\lambda}_p$ at each hinge position:

$$(\bar{\lambda}_p)_i = \left(\sqrt{\frac{f_y}{\sigma_{cr}}} \right)_i \quad (3)$$

where σ_{cr} may be determined numerically (e.g. by means of CUFSM (Li and Schafer, 2010)) or using simplified analytical expressions (Seif and Schafer, 2010). Alternatively, $\bar{\lambda}_p$ may be conservatively determined on an element by element basis, and taking the cross-section slenderness as that of its most slender constituent plate.

- (3) Determine the level of strain that a cross-section can endure (ε_{csm}) from the base curve at each hinge:

$$\left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_i = \frac{0.25}{(\bar{\lambda}_p)_i^{3.6}} \quad \text{but} \quad \left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_i \leq \text{lesser} \left(15, \frac{0.5\varepsilon_u}{\varepsilon_y} \right) \quad \text{for } (\bar{\lambda}_p)_i \leq 0.68 \quad (4)$$

- (4) For a given hinge rotation θ_i , section height h_i and strain ratio $(\varepsilon_{csm}/\varepsilon_y)_i$, calculate the corresponding hinge demands α_i :

$$\alpha_i = \frac{\theta_i h_i}{(\varepsilon_{csm} / \varepsilon_y)_i} \quad (5)$$

The critical hinge is identified as the one with the highest hinge demand $\alpha_{max} = \max\{\alpha_i\}$, with the strain ratio at the critical hinge now labeled $(\varepsilon_{csm}/\varepsilon_y)_{max}$.

- (5) The final strain ratios at each hinge location $(\varepsilon_{csm}/\varepsilon_y)_{hinge,i}$ are then assigned in proportion to the hinge rotation ratios:

$$\left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_{hinge,i} = \frac{\alpha_i}{\alpha_{max}} \left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_{max} \quad \text{but} \quad \left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_{hinge,i} \leq \left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)_i \quad (6)$$

- (6) Calculate the cross-section bending moment capacity M_i at each plastic hinge based on the corresponding strain ratio $(\varepsilon_{csm}/\varepsilon_y)_{hinge,i}$:

$$\left(\frac{M_{csm}}{M_{pl}} \right)_i = \left(1 + \frac{E_{sh}}{E} \frac{W_{el}}{W_{pl}} \left(\frac{\varepsilon_{csm}}{\varepsilon_y} - 1 \right) - \left(1 - \frac{W_{el}}{W_{pl}} \right) \left(\frac{\varepsilon_{csm}}{\varepsilon_y} \right)^{-2} \right)_i \quad (7)$$

- (7) Determine the collapse load of the system by means of virtual work, whereby the external work done by the applied loads F_j acting through virtual displacements δ_j and equated to that of the internal work resulting from the hinge rotations θ_i .

$$\sum_j F_j \delta_j = \sum_i M_i \theta_i \quad (8)$$

Note that if $(\varepsilon_{csm}/\varepsilon_y)_{max}$ derived from step (4) is less than 3.6 for SHS/RHS, global plastic analysis is not recommended (Gardner et al., 2011) and global elastic analysis should be used; $\varepsilon_{csm}/\varepsilon_y = 3.6$ corresponds to the Class 2 limit according to the classification system of EC9 (2007). This results in a step in resistance at the boundary between Class 2 and 3 cross-sections, as shown in Fig. 2.

The results derived from the above described approach are compared to the experimental and numerical collapse loads in the following section.

COMPARISON OF TEST AND NUMERICAL STRENGTHS WITH DESIGN STRENGTHS

In this section, the experimental and numerical ultimate loads F_u obtained from this study are compared with the design strengths predicted by the American (F_{AA}) (AA, 2010), Australian/New Zealand ($F_{AS/NZS}$) (AS, 1997) and European (F_{EC9}) (EC9, 2007) specifications for aluminum alloy structures, as well as the capacities calculated based on the traditional plastic design method (F_{pl}), the plastic hinge method (F_{EC9-H}) and the CSM for indeterminate structures (F_{csm}). The curves showing the design capacities of the traditional plastic design method, the plastic hinge method and the CSM, normalized by the theoretical plastic load F_{coll} , are plotted in Fig. 2 as a function of cross-section slenderness $\bar{\lambda}_p$. These illustrative curves are derived based on loading configuration I and the average measured material properties and cross-sectional dimensions. Calculation concepts and design treatments of the aforementioned methods are presented in Table 2, where the following symbols are used: c_{cf} is the distance from the centerline of the compression flange to the neutral axis of the cross-section, c_{cw} is the distance from the centerline of the web group's extreme compressive fiber to the neutral axis of the cross-section, c_{tf} is the distance from the extreme tensile fiber to the neutral axis of the cross section, c_{tw} is the distance from the web group's extreme tensile fiber to the neutral axis of the cross-section, f_c is the local buckling stress of the flat elements in uniform compression, f_t is the yield stress in tension, f_b is the stress corresponding to the strength of flat elements in flexure, f_{bc} and f_{bt} are the weighted average compressive/tensile stress in accordance with the ratio of the area of each elements, I_f and I_w are the moments of inertia of the flange/web group about the neutral axis of the cross section, M_{nc} and M_{nt} are the nominal weighted average moment for compression/tension, W_{pl} , W_{el} and W_{eff} are the

plastic section modulus of the gross section, elastic section modulus of the gross section and elastic section modulus of the effective section, α is the shape factor, η is the correction factor and ε_{csm} is the CSM limiting strain. The measured material properties and cross-sectional dimensions were used in all the design calculations.

Comparisons between the calculated design values and experimental ultimate loads obtained from the continuous beam tests are shown in Table 3, where the conservatism in the current international design specifications may be seen for each individual specimen. Comparisons covering both experimental and numerical results are summarized in Table 4 and plotted in Fig. 3. Only specimens within the CSM applicability limits (i.e. $\bar{\lambda}_p \leq 0.68$) are discussed in this section.

International design specifications

The AA (2010), AS/NZS (1997) and EC9 (2007) specifications have similar approaches for indeterminate structures, i.e. the global elastic design. The design collapse load is determined when the first hinge forms. However, these three specifications have different approaches and parameters to calculate the cross-sectional flexural resistance, finally leading to different design capacities for continuous beams. The mean values of the experimental-to-predicted ultimate load ratios F_u/F_{AA} , $F_u/F_{AS/NZS}$ and F_u/F_{EC9} are 1.81, 2.00 and 1.70, with the corresponding coefficients of variation (COV) of 0.230, 0.236 and 0.200, respectively, where F_{AA} , $F_{AS/NZS}$ and F_{EC9} are the design capacities determined according to the AA (2010), the AS/NZS (1997) and EC9 (2007) provisions. Among the considered specifications, the AS/NZS (1997) specification provides the most conservative predictions. The predictions of all three specifications may all be seen to be rather conservative, particularly for the more stocky sections, as indicated in Fig. 3.

Plastic hinge method (EC9, 2007)

The plastic hinge method is included in the Annex H of EC9 (2007) as an alternative plastic design method for indeterminate structures; the method applies principally to plastic (Class 1) sections but can also be used for compact (Class 2) and semi-compact (Class 3) sections provided specific account is taken of local buckling. The plastic hinge method (F_{EC9-H}) is applied to Class 1 sections herein; that is to say, predictions of F_{EC9} and F_{EC9-H} only differ for Class 1 sections and are the same for Classes 2, 3 and 4 sections. The cross-sectional ultimate moment capacity M_u for the plastic hinge method is defined by Eq. (9), where η is a correction factor to the conventional yield stress to take into consideration the available hardening behavior of the material and α_ξ is the shape factor depending on the alloy ductility features as required in Annex G (EC9, 2007). The cross-sectional ultimate bending moment is calculated as a fully plastic moment with allowance for strain hardening. The plastic hinge method also takes benefits from global plastic analysis at the system level.

$$M_u = \eta \alpha_\xi f_y W_{el} \quad (9)$$

The predictions from the plastic hinge method are much more accurate compared to the existing international specifications. The ratio of experimental-to-predicted ultimate loads F_u/F_{EC9-H} is 1.43, with a coefficient of variation (COV) of 0.213, as shown in Table 4.

Traditional plastic design method

Traditional plastic design is conventionally applied to continuous beams with Class 1 sections. The collapse load F_{pl} is the theoretical load causing a collapse mechanism based on the formation of plastic hinges at their full plastic moment capacities. It is determined by means of a global plastic design with the plastic moment capacity $W_{pl}f_y$ at each hinge, and therefore takes

consideration of moment redistribution for continuous beams with Class 1 sections. Continuous beams with Classes 2, 3 and 4 sections are designed excluding redistribution, and the capacity is determined when the capacity of the most heavily loaded cross-section is reached, i.e. using elastic global analysis with cross-section capacities $W_{pl}f_y$, $W_{eff}f_y$ and $W_{eff}f_y$ for Classes 2, 3 and 4 sections, respectively, all calculated in accordance with EC9 (2007).

The mean ratio of experimental and numerical results to predicted values F_u/F_{pl} is, on average, 1.53 with a COV of 0.170, using the traditional plastic design method, as shown in Table 4. This indicates that the capacity of non-slender sections can still continue to rise significantly after the plastic hinge attains the prescribed moment capacity. The key diversion between the traditional plastic design method and the plastic hinge method in EC9-Annex H is the calculation of the capacity of Class 1 sections, with the latter allowing for strain hardening and hence achieving more accurate predictions.

Continuous strength method (CSM) for indeterminate aluminum alloy structures

The comparison of the CSM predicted capacities with the test and numerical results gives a mean value of 1.31 with a corresponding COV of 0.155, as shown in Table 4. The CSM for indeterminate structures provides the most precise predictions of the test and numerical results, with the mean strength prediction ratios being closest to unity and the COV being the lowest. The improved predictions are related to the deformation based design approach, the allowance for moment redistribution and consideration of strain hardening. Moreover, since the cross-section deformation capacity can be explicitly determined by the CSM according to the continuous relationship with the cross-section slenderness $\bar{\lambda}_p$, it is not necessary to classify the cross-sections into the discrete classes, and thus the CSM avoids the limitations of the conventional classification systems.

RELIABILITY ANALYSIS

The reliability of the AA, AS/NZS and EC9 specifications, the traditional plastic design analysis method, the plastic hinge method in Annex H (EC9, 2007) and the CSM for aluminum alloy continuous beams are evaluated by means of statistical analysis in this section. The reliability analysis followed the AISC approach (2005), with the parameters for aluminum alloy beams taken from Clause 1.3.2 of Appendix 1, Part I of the Aluminum Design Manual (AA, 2010). The reliability index β is used as an indicator of the design safety level. Key parameters adopted herein include: the mean values and COV for material properties $M_m = 1.10$ (for behavior governed by the yield stress) and 1.00 (for behavior governed by the ultimate tensile stress), $V_M = 0.06$ as well as the mean values and COV for fabrication variables $F_m = 1.00$ and $V_F = 0.05$. The statistical parameters P_m and V_p are the mean value and COV of the ratio of the experimental and numerical to predicted ultimate load, and are given in Table 4.

The considered design specifications (i.e. the AA, the AS/NZS and EC9) use different load combinations and the resistance factors in the determination of reliability index β . For aluminum alloy beams, the AA (2010) specification uses a resistance factor ϕ of 0.90 and a load combination of 1.2DL + 1.6LL (where DL and LL signify Dead loads and Live loads, respectively), the AS/NZS (1997) specification uses a resistance factor ϕ of 0.85 and a load combination of 1.25DL + 1.50LL, while EC9 employs a resistance factor ϕ of 0.91 and a load combination of 1.35DL + 1.50LL (Rogers and Hancock, 1996). The target reliability index for aluminum alloy beams is 2.50, as required in the AA (2010). In this study, a value of 2.50 is adopted as the target value to determine whether a design method can be deemed to be reliable or not. In addition, in order to have a direct comparison among different design approaches, all parameters were set to those given in the AA (2010) specification to calculate the reliability index. For this direct comparison, the calculated reliability index is denoted as β_2 .

The reliability indices β determined on the basis of the experimental and numerical data for the AA (2010) specification, the AS/NZS (1997) specification, EC9 (2007) and Annex H of EC9 (2007) for specimens with $\bar{\lambda}_p \leq 0.68$ are 3.71, 4.05, 3.65 and 2.85, respectively, while the reliability indices become 3.34, 3.53, 3.53 and 2.92 for the aforementioned specifications when considering the full set of specimens. Clearly, all exceed the target reliability of 2.50. In the direct comparison with $\phi = 0.90$ and a load combination of 1.2DL + 1.6LL, the six design approaches discussed herein all displayed a calculated reliability index β_2 greater than 2.50, and even 3.00. A reliability index of 3.06 was obtained for the CSM. Therefore, it may be concluded that the CSM can provide reliable design resistance predictions for aluminum alloy continuous beams, using a resistance factor $\phi = 0.9$.

CONCLUSIONS

Following validation of the finite element models in the companion paper, a parametric study was performed herein, generating a total of 120 numerical results on aluminum alloy continuous beams. The combined experimental and numerical data set included a wide range of cross-sectional slenderness, aspect ratios and moment gradients on SHS and RHS members. These data were then used to investigate the design efficiency of the American (2010), Australian/New Zealand (1997) and European (2007) provisions, as well as the traditional plastic design method, the plastic hinge method given in Annex H of EC9 (2007) and the continuous strength method for indeterminate aluminum alloy structures. The three aforementioned specifications were found to be overly conservative in predicting the capacity of aluminum alloy continuous beams, especially for stocky (plastic and compact) sections. The other three design methods - the traditional plastic design, the Annex H plastic hinge method and the continuous strength method - were found to estimate the test and FE ultimate loads more accurately, due partly to their adoption of global plastic analysis for stocky cross-sections. Overall, for the

comparisons made herein, the continuous strength method was shown to provide the most accurate and consistent predictions. Besides the employment of global plastic design, the explanation for the good predictions relate to the systematic exploitation of strain hardening at the cross-sectional level and the deformation-based design approach. Reliability analyses were also conducted and all methods considered, including the continuous strength method was found to be safely applicable to the design of aluminum alloy continuous beams. Further investigations are however needed into other indeterminate configurations before reaching more general conclusions in the future.

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NOTATION

b	= Flat width of flange
c_{cf}	= Distances from the centerline of the compression flange to the neutral axis of the cross section
c_{cw}	= Distances from the centerline of the compression web group's extreme compression fiber to the neutral axis of the cross section
c_{tf}	= Distances from the extreme tension fiber group's extreme tension fiber to the neutral axis of the cross section
c_{tw}	= Distances from the extreme tension web group's extreme tension fiber to the neutral axis of the cross section

COV	= Coefficient of variation
E	= Young's modulus
E_{sh}	= Strain hardening modulus
f_b	= Local buckling/tensile stress of the flat elements in flexure
f_{bc}	= Weighted average factored limit state compressive stress in accordance with the ratio of the area of each element
f_{bt}	= Weighted average factored limit state tensile stress in accordance with the ratio of the area of each element
f_c	= Local buckling stress of the flat elements in uniform compression
f_t	= Yield stress in tension
f_y	= Yield strength, taken as the 0.2% proof strength
f_u	= Ultimate tensile strength
F	= Applied load
F_{AA}	= Ultimate load predicted by the AA (2010)
$F_{AS/NZS}$	= Ultimate load predicted by the AS/NZS (1997)
F_{coll}	= Ultimate load level at which the plastic collapse mechanism forms (with cross-sectional capacity at the hinge equal to $W_{pl}f_y$)
F_{csm}	= Ultimate load predicted by the CSM
F_{design}	= Predicted ultimate load

F_{EC9}	= Ultimate load predicted by EC9 (2007)
F_{EC9-H}	= Ultimate load predicted by the plastic hinge method in Annex H of EC9 (2007)
F_{exp}	= Experimental total ultimate load
F_{FE}	= Ultimate load for simulated models
F_{hl}	= Ultimate load level at which the first hinge forms (with cross-sectional capacity at the hinge equal to $W_{pl}f_y$)
F_m	= Mean value of fabrication factor
F_{pl}	= Ultimate load predicted by the traditional plastic design method
F_u	= Experimental and numerical ultimate loads
I_f	= Moment of inertia of the flange group about the neutral axis of the cross sections
I_w	= Moment of inertia of the web group about the neutral axis of the cross section
L	= Member length
M	= Moment
M_{csm}	= Ultimate moment capacity predicted by the CSM
M_m	= Mean value of material factor
M_{nc}	= Nominal strength moment for compression
M_{nt}	= Nominal strength moment for tension

M_{pl}	= $W_{pl}f_y$ is the plastic moment capacity
P_m	= Mean value of test (or numerical)-to-predicted load ratios
t	= Wall thickness
V_F	= Coefficient of variation of fabrication factor
V_M	= Coefficient of variation of material factor
V_P	= Coefficient of variation of test (or numerical)-to-predicted load ratios;
W_{eff}	= Elastic modulus of effective section
W_{el}	= Elastic section modulus
W_{pl}	= Plastic section modulus
β	= Reliability index
ε_{csm}	= CSM limiting strain
ε_f	= Material fracture strain
ε_u	= Strain at ultimate tensile stress
ε_y	= f_y/E is the yield strain
ϕ	= Resistance factor
$\bar{\lambda}_p$	= Cross-section/plate slenderness
θ	= Rotation at hinge point from tests
α	= Hinge rotation demand

α_{ξ}	= Shape factor
δ	= Displacement at hinge point
η	= Correction factor of the conventional yield stress

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Table 1. Ultimate loads from FE parametric study on aluminum alloy continuous beams

Specimen	F_{FE} (kN)	Specimen	F_{FE} (kN)	Specimen	F_{FE} (kN)	Class (EC9)
H180×180×12.0 B5I	946	H180×180×12.0 B5II	1070	H180×180×12.0 B5III	940	2
H180×180×9.0 B5I	690	H180×180×9.0 B5II	722	H180×180×9.0 B5III	678	3
H180×180×6.0 B5I	428	H180×180×6.0 B5II	375	H180×180×6.0 B5III	420	4
H180×180×3.5 B5I	182	H180×180×3.5 B5II	149	H180×180×3.5 B5III	203	4
H160×200×12.0 B5I	1006	H160×200×12.0 B5II	1084	H160×200×12.0 B5III	988	2
H160×200×9.0 B5I	718	H160×200×9.0 B5II	728	H160×200×9.0 B5III	700	2
H160×200×6.0 B5I	432	H160×200×6.0 B5II	408	H160×200×6.0 B5III	390	4
H160×200×3.5 B5I	183	H160×200×3.5 B5II	177	H160×200×3.5 B5III	193	4
H140×100×8.0 B5I	378	H140×100×8.0 B5II	454	H140×100×8.0 B5III	373	2
H140×100×6.5 B5I	306	H140×100×6.5 B5II	363	H140×100×6.5 B5III	302	3
H140×100×5.0 B5I	232	H140×100×5.0 B5II	266	H140×100×5.0 B5III	224	4
H140×100×3.5 B5I	151	H140×100×3.5 B5II	167	H140×100×3.5 B5III	142	4
H130×50×8.0 B5I	221	H130×50×8.0 B5II	258	H130×50×8.0 B5III	206	2
H130×50×6.5 B5I	183	H130×50×6.5 B5II	212	H130×50×6.5 B5III	168	3
H130×50×5.0 B5I	143	H130×50×5.0 B5II	164	H130×50×5.0 B5III	142	4
H130×50×3.5 B5I	99	H130×50×3.5 B5II	114	H130×50×3.5 B5III	97	4
H50×130×8.0 B5I	521	H50×130×8.0 B5II	591	H50×130×8.0 B5III	448	1
H50×130×6.5 B5I	420	H50×130×6.5 B5II	458	H50×130×6.5 B5III	345	1
H50×130×5.0 B5I	317	H50×130×5.0 B5II	328	H50×130×5.0 B5III	285	1
H50×130×3.5 B5I	208	H50×130×3.5 B5II	217	H50×130×3.5 B5III	180	2
N180×180×12.0 B5I	561	N180×180×12.0 B5II	671	N180×180×12.0 B5III	639	1
N180×180×9.0 B5I	381	N180×180×9.0 B5II	451	N180×180×9.0 B5III	438	1
N180×180×6.0 B5I	222	N180×180×6.0 B5II	256	N180×180×6.0 B5III	256	4
N180×180×3.5 B5I	105	N180×180×3.5 B5II	102	N180×180×3.5 B5III	122	4
N160×200×12.0 B5I	596	N160×200×12.0 B5II	697	N160×200×12.0 B5III	670	1
N160×200×9.0 B5I	392	N160×200×9.0 B5II	474	N160×200×9.0 B5III	447	1
N160×200×6.0 B5I	221	N160×200×6.0 B5II	257	N160×200×6.0 B5III	262	2
N160×200×3.5 B5I	99	N160×200×3.5 B5II	112	N160×200×3.5 B5III	123	4
N140×100×8.0 B5I	252	N140×100×8.0 B5II	301	N140×100×8.0 B5III	248	1
N140×100×6.5 B5I	199	N140×100×6.5 B5II	235	N140×100×6.5 B5III	194	1
N140×100×5.0 B5I	142	N140×100×5.0 B5II	168	N140×100×5.0 B5III	140	3
N140×100×3.5 B5I	86	N140×100×3.5 B5II	101	N140×100×3.5 B5III	86	4
N130×50×8.0 B5I	138	N130×50×8.0 B5II	167	N130×50×8.0 B5III	134	1
N130×50×6.5 B5I	111	N130×50×6.5 B5II	134	N130×50×6.5 B5III	105	1
N130×50×5.0 B5I	83	N130×50×5.0 B5II	100	N130×50×5.0 B5III	81	2
N130×50×3.5 B5I	55	N130×50×3.5 B5II	66	N130×50×3.5 B5III	55	4
N50×130×8.0 B5I	327	N50×130×8.0 B5II	352	N50×130×8.0 B5III	311	1
N50×130×6.5 B5I	244	N50×130×6.5 B5II	258	N50×130×6.5 B5III	231	1
N50×130×5.0 B5I	166	N50×130×5.0 B5II	174	N50×130×5.0 B5III	160	1
N50×130×3.5 B5I	97	N50×130×3.5 B5II	117	N50×130×3.5 B5III	111	1

Table 2. Design Concepts for different design approaches

Design approach	Section capacity and analysis type	Class 1	Class 2	Class 3	Class 4
AA	Cross-section capacity	Lesser of $(M_{nc}=f_c I_f/c_{cf}+f_b I_w/c_{cw}$, $M_{nt}=f_t I_f/c_{tf}+f_b I_w/c_{tw})$, where f_c, f_b and f_t are slenderness dependent.			
	Global analysis	Elastic			
AS/NZS	Cross-section capacity	Lesser of (W_{elfbc}, W_{elfbt}) , where f_{bc} and f_{bt} are slenderness dependent			
	Global analysis	Elastic			
EC9	Cross-section capacity	$W_{pl}f_y$	$W_{pl}f_y$	$W_{elf}f_y$	$W_{eff}f_y$
	Global analysis	Elastic			
EC9-Annex H	Cross-section capacity	$\alpha\eta W_{elf}f_y$	$W_{pl}f_y$	$W_{elf}f_y$	$W_{eff}f_y$
	Global analysis	Plastic	Elastic		
Traditional plastic design	Cross-section capacity	$W_{pl}f_y$	$W_{pl}f_y$	$W_{elf}f_y$	$W_{eff}f_y$
	Global analysis	Plastic	Elastic		
CSM	Cross-section capacity	M_{csm} for cross-section			
	Global analysis	Plastic, but with allowance for ratio of hinge rotations. (for $\varepsilon_{csm}/\varepsilon_y < 3.6$: elastic)			

Table 3. Comparison between five-point bending test results and design capacities

Specimen	b/t	F_{exp} (kN)	$\frac{F_{exp}}{F_{AA}}$	$\frac{F_{exp}}{F_{AS}}$	$\frac{F_{exp}}{F_{EC9}}$	$\frac{F_{exp}}{F_{EC9-H}}$	$\frac{F_{exp}}{F_{pl}}$	$\frac{F_{exp}}{F_{csm}}$
H55×70×4.2B5I	11.4	114.1	1.58	1.77	1.45	1.46	1.46	1.20
H55×70×4.2B5I-R	11.5	112.3	1.57	1.75	1.40	1.40	1.40	1.15
H70×55×4.2B5I	15.1	84.9	1.41	1.52	1.29	1.29	1.29	1.12
H50×95×10.5B5I	2.9	329.9	2.18	2.58	1.87	1.58	1.67	1.44
H95×50×10.5B5I	7.1	188.2	2.24	2.38	1.83	1.49	1.63	1.40
H64×64×3.0B5I	20.4	65.3	1.41	1.54	1.55	1.55	1.55	1.37
N50×95×10.5B5I	2.9	306.7	2.21	2.61	1.96	1.73	1.74	1.49
N70×120×10.5B5I	4.7	532.9	2.45	2.84	2.19	1.75	1.95	1.65
N120×70×10.5B5I	9.7	362.0	2.64	2.82	2.22	1.80	1.97	1.66
N120×120×9.0B5I	11.5	655.2	1.79	1.98	1.64	1.27	1.46	1.24
H55×70×4.2B5II	11.4	141.5	1.90	2.12	1.74	1.74	1.74	1.18
H55×70×4.2B5II-R	11.5	130.6	1.81	2.02	1.66	1.66	1.66	1.13
H70×55×4.2B5II	15.1	120.2	1.97	2.14	1.80	1.80	1.80	1.28
H50×95×10.5B5II	2.9	436.0	2.61	3.11	2.31	1.70	1.66	1.47
H95×50×10.5B5II	7.2	222.1	2.64	2.80	2.15	1.42	1.55	1.37
H64×64×3.0B5II	20.4	80.8	1.75	1.91	1.91	1.91	1.91	1.69
N70×120×10.5B5II	4.8	693.9	3.20	3.71	2.87	1.86	2.07	1.80
N120×70×10.5B5II	9.7	450.8	3.16	3.39	2.73	1.75	1.96	1.69
N120×120×9.0B5II	11.5	657.8	1.74	1.92	1.58	1.00	1.14	0.99
H55×70×4.2B5III	11.4	91.6	1.48	1.66	1.36	1.36	1.31	1.23
H55×70×4.2B5III-R	11.4	109.6	1.41	1.57	1.29	1.29	1.29	1.16
H70×55×4.2B5III	15.0	72.1	1.49	1.62	1.36	1.36	1.36	1.27
H50×95×10.5B5III	2.9	346.2	1.90	2.27	1.68	1.66	1.62	1.42
H95×50×10.5B5III	7.2	191.4	1.92	2.05	1.57	1.54	1.51	1.32
H64×64×3.0B5III	20.4	64.3	1.29	1.41	1.41	1.41	1.41	1.25
N70×120×10.5B5III	4.8	589.7	2.47	2.86	2.22	1.92	2.14	1.84
N120×70×10.5B5III	9.6	377.7	2.45	2.63	2.11	1.81	2.04	1.74
Mean			2.02	2.26	1.82	1.57	1.64	1.39
COV			0.263	0.231	0.233	0.148	0.165	0.164

Table 4. Summary of comparisons between experimental and numerical results and design strengths

Parameters	$\frac{F_u}{F_{AA}}$	$\frac{F_u}{F_{AS/NZS}}$	$\frac{F_u}{F_{EC9}}$	$\frac{F_u}{F_{EC9-H}}$	$\frac{F_u}{F_{pl}}$	$\frac{F_u}{F_{csm}}$
Number of specimens	117(147)	117(147)	117(147)	117(147)	117(147)	117
Mean, P_m	1.81(1.71)	2.00(1.88)	1.70(1.67)	1.43(1.45)	1.53(1.47)	1.31
COV, V_p	0.230(0.256)	0.236(0.275)	0.200(0.207)	0.213(0.213)	0.170(0.196)	0.155
ϕ	0.90	0.85	0.91	0.91	---	---
β	3.71(3.34)	4.05(3.53)	3.65(3.53)	2.85(2.92)	---	---
ϕ_2 (direct comparison)	0.90	0.90	0.90	0.90	0.90	0.90
β_2 (direct comparison)	3.71(3.34)	3.98(3.47)	3.76(3.64)	2.96(3.03)	3.66(3.31)	3.06

Note that only 117 specimens are within the limits of applicability of the CSM, while a total of 147 test and numerical results are covered by the other design methods. Values in brackets refer to the full database of 147 data.

Fig. 1: Plastic collapse mechanism for two-span continuous beam (Gardner et al., 2011)

(a)

(b)

Fig. 2: Curves indicating design capacities of different design approaches for continuous beams of (a) normal-strength and (b) high-strength aluminum alloys

(a)

(b)

(c)

Fig. 3: Comparison between test and numerical results of configuration (a) I, (b) II and (c) III with design strengths