Prediction of rotational galloping of rectangular and semi-circular leading edge sections

I. Robertson, S. Sherwin, P.W. Bearman

Department of Aeronautics, Imperial College, Prince Consort Road, London, SW7 2BY

e-mail: s.sherwin@ic.ac.uk

Abstract. In this paper we investigate the rotational galloping instability due to fluid/structure interaction of two bodies of the same aspect ratio but different leading edges using a previously developed algorithm [1]. This numerical technique utilises a two-dimensional spectral/ hp element method and a frame of reference transformation to ensure efficient computations. Rotational motion of idealised bridge deck sections are simulated at reduced velocities of sufficient magnitude to promote a galloping response. Comparison with quasi-steady theory is found to be favourable. The differences, in terms of the flow field and resulting critical reduced velocity, between sections with rectangular and semi-circular leading edge sections are investigated.

Key words: fluid/structure interaction, instability, galloping, spectral/ hp , CFD

1. Introduction

The possibility of fluid/structure interaction leading to structural failure in many situations is high, particularly when long-span bridges are involved. Historically bridge deck designs have been routinely tested in wind tunnels to evaluate optimum aerodynamic and structural parameters. At the early stages of design this is an expensive process in terms of cost and time. Computational Fluid Dynamics (CFD) offers a potentially efficient method to quickly and cheaply determine the general configuration of a safe but economical bridge deck structure which can then be extensively tested using traditional methods. CFD can also provide insights into fluid/structure interactions that may be very difficult, or even impossible, to obtain from conventional wind tunnel experiments. Thereby providing the design engineer with vital information, enabling an informed decision concerning any modifications necessary.

To this end, a two dimensional hp /Spectral fluid solver, *NekTar*[2], has been modified to incorporate the modeling of a body undergoing translational and rotational motion [1]. A moving frame of reference is attached to the body to utilise the efficiency of a fixed mesh solver.

The ultimate aim of this research is the prediction of the critical wind velocity at which the motion of a bridge deck design, free to rotate and translate, will become unstable. The point of instability is characterised by the commencement of increasing amplitude oscillations. Before this is done it is essential to test the CFD code against other numerical, experimental and analytical results. The motion of rectangular structures free to oscillate rotationally or translationally, according to specified structural parameters, when acted upon by a freestream has been extensively experimentally and analytically studied. These results have been used to validate the

results of the numerical algorithm [3]. In this paper these numerical experiments will be extended to investigate the effect of modifications to the leading edge on the flow field, pressure field and ultimately the potential of the body to undergo rotational galloping.

2. Model Equations

We will consider the non-dimensional form of the Navier-Stokes equations;

$$\nabla^* \cdot \mathbf{v}^* = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}^*}{\partial t^*} + (\mathbf{v}^* \cdot \nabla^*) \mathbf{v}^* = -\nabla^* p^* + Re^{-1} (\nabla^*)^2 \mathbf{v}^*, \quad (2)$$

where an asterisk denotes a non-dimensional variable. The variables in the equations above have been non-dimensionalised as follows,

$$\begin{aligned} t^* &= \frac{tU}{D}, & x^* &= \frac{x}{D}, & y^* &= \frac{y}{D}, \\ u^* &= \frac{u}{U}, & v^* &= \frac{v}{U}, & p^* &= \frac{p}{\rho U^2}, \\ Re &= \frac{\rho U D}{\mu} = \frac{UD}{\nu}, \end{aligned} \quad (3)$$

where D is a characteristic length, and U is the freestream velocity. Correspondingly, the forces and moment are non-dimensionalised by,

$$F_x^* = \frac{F_x}{\rho D U^2}, \quad F_y^* = \frac{F_y}{\rho D U^2}, \quad M_\theta^* = \frac{M_\theta}{\rho D^2 U^2}. \quad (4)$$

We note that the usual lift, drag and pitching moment coefficients are given by,

$$C_x = 2F_x^*, \quad C_y = 2F_y^*, \quad C_M = 2M_\theta^*. \quad (5)$$

The governing structural equation for the one degree of freedom torsional mode is

$$I_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = M_\theta, \quad (6)$$

where I_θ is the mass moment of inertia, c_θ is the torsional damping constant, k_θ is the torsional stiffness constant, and θ denotes the rotational angle of the body around the elastic centre. The right-hand side of equation (6) is the moment about the elastic centre. The non-dimensional form of equation (6) is

$$\ddot{\theta} + 2\xi_\theta \left(\frac{2\pi}{U_\theta} \right) \dot{\theta} + \left(\frac{2\pi}{U_\theta} \right)^2 \theta = \frac{M_\theta^*}{n_\theta} = \frac{C_M}{2n_\theta}, \quad (7)$$

where

$$U_\theta = \frac{U}{f_\theta D} \text{ (reduced velocity)}, \quad n_\theta = \frac{I_\theta}{\rho D^4} \text{ (mass moment of inertia ratio)} \quad (8)$$

and f_θ is the structural natural frequency for rotational motion. For the remainder of this paper, the rotational structural equation will be used in its non-dimensionalised form.

Figure 1: Analytical model used in quasi-steady analysis of rotational galloping

3. Quasi-Steady Theory Prediction of Galloping

The use of quasi-steady theory to predict whether a structure has the potential to gallop has been widely used for many years. It was developed by den Hartog [4] and has generally been implemented to predict whether a body has the potential to gallop and, if this is the case, the critical reduced velocity for the onset of translational galloping can be estimated. The theory predicts that a body with a negative variation of lift coefficient with angle of attack will have the potential to vertically gallop, where galloping is an unstable motion of the body caused by the lift force on the body having a component in phase with the velocity.

The implementation of quasi-steady theory to predict the rotational galloping instability of a body is less widely applied. Though this method has been found to predict inaccurate critical reduced velocities for the onset of rotational galloping [3, 5], the potential of a body to undergo rotational galloping can be reliably ascertained using quasi-steady theory [3].

The net damping on a rotating structure, can be approximately evaluated using quasi-steady theory. It will be shown that for a rotating body the relative angle of attack is proportional to its physical angle of attack, θ , and its rotational velocity, $\dot{\theta}$.

The relative angle of attack, α and relative velocity, U_r , of a rotating body can be evaluated as,

$$\alpha = \theta - \tan^{-1} \left[R \dot{\theta} \sin \gamma / (U - R \dot{\theta} \cos \gamma) \right], \quad (9)$$

$$U_r^2 = (R \dot{\theta} \sin \gamma)^2 + (U - R \dot{\theta} \cos \gamma)^2, \quad (10)$$

where the variables are defined in figure 1 and $\gamma = \frac{\pi}{2} - \theta$. The effective angle of attack, α , is obviously dependent on the physical angle of attack, θ , though there is also a contribution due to the rotational velocity of the body.

The rotational motion of the leading edge of the body results in a decrease in the relative free-stream velocity in the horizontal direction and $(U - R \dot{\theta} \cos \gamma)$ is the velocity of the horizontal free stream when the rotational velocity of the body is considered. The vertical motion of the leading edge can be related to the quasi-steady theory for purely heave motion where a positive velocity is shown to be related to a negative effective angle of attack. The leading edge of the body, for small oscillations about a mean displacement, can in a linear sense be said to be

undergoing purely vertical motion. As with the heave theory this vertical motion can then be related to an effective angle of attack.

R is defined as the characteristic radius at which the relative angle of attack due to the rotational velocity of the body is considered. This technique has been used extensively for many different geometries [5, 6, 7] and in the case of rectangular sections, R is taken to be the half-chord length, $\frac{D}{2}$ [5, 6]. This choice of R results in the instantaneous relative angle of attack, α , being evaluated at the leading edge where the most significant forces are generated for rotational motion. For small effective angles of attack, $\alpha \ll 1$, and actual angles of attack, $\theta \ll 1$, equations (9) and (10) reduce to,

$$\alpha \approx \theta - \frac{R\dot{\theta}}{U} , \quad (11)$$

$$U_r \approx U . \quad (12)$$

Therefore C_M can be expanded about $\alpha = 0$, resulting in the relationship,

$$C_M \cong \left. \frac{\partial C_M}{\partial \alpha} \right|_{\alpha=0} \alpha , \quad (13)$$

$$\cong \left. \frac{\partial C_M}{\partial \alpha} \right|_{\alpha=0} \left(\theta - \frac{R\dot{\theta}}{U} \right) . \quad (14)$$

The same method normally adopted for vertical galloping analysis can now be applied to formulate a condition which will determine the potential of a body to undergo torsional galloping. An equation of motion governing the rotation of the body can be formulated by combining the structural equation (7) and the expansion of C_M in equation (14):

$$\ddot{\theta} + 2\xi_\theta \left(\frac{2\pi}{U_\theta} \right) \dot{\theta} + \left(\frac{2\pi}{U_\theta} \right)^2 \theta = \frac{1}{2n_\theta} \left. \frac{dC_M}{d\alpha} \right|_{\alpha=0} \left(\theta - R^*\dot{\theta} \right) , \quad (15)$$

where $R^* = \frac{R}{D}$. Considering equation (15) the net damping of the system, Ψ_θ , is,

$$\Psi_\theta = 2\xi_y \left(\frac{2\pi}{U_\theta} \right) + \frac{R^*}{2n_\theta} \left. \frac{dC_M}{d\alpha} \right|_{\alpha=0} . \quad (16)$$

The condition $\Psi_\theta < 0$ will result in galloping oscillations. As the structural damping for torsional motion, ξ_θ , is always expected to be positive, a necessary condition for instability is,

$$\left. \frac{\partial C_M}{\partial \alpha} \right|_{\alpha=0} < 0 . \quad (17)$$

The variation of C_m with α was investigated for bodies with different leading and trailing edges. The first is a rectangular body with an aspect ratio, Λ , of 4 and is shown in figure 2(a) . The second is of the same aspect ratio, $\Lambda = 4$, and has a semi-circular leading and trailing edge and is shown in figure 2(b). Both figures also show the near-field configuration of the computational mesh. The variation

Figure 2: Bodies used in the investigation of rotational galloping instability (a) rectangular leading edge (b) semi-circular leading edge

of the moment coefficient with increasing effective angle of attack for both designs is illustrated in figure 3. From this figure and considering the quasi-steady theory described above, the rectangular body has the potential to rotationally gallop whilst the body with the semi-circular leading edge does not.

Figure 3: Variation of C_m with α for the bodies with (a) rectangular leading edge and (b) semi-circular leading edge

These predictions are then tested against the results from the dynamic numerical calculations. These experiments were performed by allowing the section to rotate when acted upon by an oncoming free-stream parallel with the x -axis. The dimensional parameters governing the experiments for both bodies were $Re = 250$, $\Lambda = 4$, $U_\theta = 40$, $n_\theta = 400$ and $\xi_\theta = 0.25$. The time history of the rotational displacement of the bodies can be seen in figure 4. The response of the rectangular body is of a much lower frequency and far higher amplitude than that of the semi-circular leading edge body. The rotational frequency of the rectangular body is equal to $\frac{1}{U_\theta}$ which would be expected from a galloping motion, as would the large amplitude response. The high frequency response of the semi-circular leading edge body is indicative of a vortex induced response where the frequency of the oscillation is in phase with the frequency of the shed vortices.

In figure 5 the time history of the applied moment of the fluid the section is shown. It can be seen from the time history in figure 5(a) that the moment applied to the rectangular body is primarily of a low frequency though superimposed on this is a higher frequency oscillation attributable to vortex shedding. The moment applied to the semi-circular leading edge section, shown in figure 5(b), is dominated by the vortex shedding frequency.

The motion of the rectangular section with can be seen in figure 6, where the flow field is represented as vorticity contours and the pressure distribution is also given in terms of C_p . The large pressure variations at time levels (a) and (c) occur when the rotational displacement is at its greatest and considering the time history of the

Figure 4: Rotational response of (a) rectangular body and (b) semi-circular leading edge body to a horizontal free-stream

Figure 5: Moment applied to a (a) rectangular body and (b) semi-circular leading edge body under the influence of a horizontal free-stream

displacement in figure 6(e) it can be seen that the moment and displacement are largely in phase. However, it is the component of the moment which is in phase with the velocity that causes the instability of the motion. The vorticity fields associated with the component of the moment in the direction of motion can be seen at time levels (b) and (d), where it is apparent that the pressure distribution is such that the moment of the section is in the direction of motion of the body, thus causing unstable oscillations.

The motion of the circular leading edge section is shown in figure 7. As with the rectangular section the flow field is represented in terms of vorticity contours and the pressure distribution in terms of C_p . The governing parameters are the same

Figure 6: Vorticity field, pressure distribution (white indicating positive pressure and black negative) and displacement time-history of galloping rectangular section: $\Lambda = 4$, $Re = 250$, $\xi_\theta = 0.25$, $n_\theta = 400$, $U_\theta = 40$

as above, $Re = 250$, $\xi_\theta = 0.25$, $n_\theta = 400$, $U_\theta = 40$, though the body is given an initial angular velocity to promote galloping. The large amplitude motion is quickly damped and vortex induced vibration established. Galloping does not occur as it is apparent from the pressure distribution that there is no component of the force in phase with the velocity. Indeed in figure 7(b) it can be seen that the pressure distribution would cause a moment in the opposite direction to the motion of the body. Therefore there is a component of the moment which is 180° out of phase with the velocity, which would lead to the damping of the oscillations.

Figure 8 illustrates the time history of the rotational displacement of the semi-circular leading edge sections for the same parameters as above though the reduced velocity has been increased to 60. The rotational response of the body is of a very large amplitude. Indeed, the amplitude is of a large enough magnitude to induce numerical breakdown of the calculations. It is believed by the authors that the large amplitude response is due to a negative net torsional stiffness rather than a negative net damping or galloping. From equation (15) it can be shown that the net torsional stiffness of the system, Ω_θ , that is the combined aerodynamic and structural stiffness, is;

$$\Omega_\theta = \left(\frac{2\pi}{U_\theta} \right)^2 - \frac{1}{2n_\theta} \frac{dC_M}{d\alpha} \bigg|_{\alpha=0}. \quad (18)$$

As $\frac{dC_M}{d\alpha}$ is positive for the semi-circular leading edge section, the configuration has the

Figure 7: Vorticity field, pressure distribution (white indicating positive pressure and black negative) and displacement time-history of galloping semi-circular leading edge section: $\Lambda = 4$, $Re = 250$, $\xi_\theta = 0.25$, $n_\theta = 400$, $U_\theta = 40$

potential to diverge, that is have a negative net torsional stiffness. This divergence is shown to occur in figure 8.

4. Conclusion

It has been shown that the shape of the leading edge is of primary importance to the critical reduced velocity of the onset of rotational galloping. A rectangular section has a negative variation of moment coefficient with increasing angle of attack and therefore, by considering quasi-steady theory, the body will undergo rotational galloping at a sufficiently high reduced velocity. However the section with a semi-circular leading edge has a positive variation and according to quasi-steady theory does not have the potential to gallop. Both of these predictions have been shown to be consistent with dynamic, numerical results.

Figure 8: Rotational response of the semi-circular leading edge body: $\Lambda = 4$, $Re = 250$, $\xi_\theta = 0.25$, $n_\theta = 400$, $U_\theta = 60$

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