The effects of ion temperature on the energy spectra of $T + T \rightarrow 2n + \alpha$ reaction products

B. Appelbe and J. Chittenden

*Centre for Inertial Fusion Studies, The Blackett Laboratory, Imperial College, London,
SW7 2AZ, United Kingdom*

Abstract

The effects of ion temperature on the energy spectra of products of the $T + T \rightarrow 2n + \alpha$ reaction are modeled and analysed. A model is derived by assuming that the spectra in the centre of mass (CM) frame for a given reaction energy are known. The model is then applied to two different sets of data for the energy spectra in the CM frame. In both cases, it is shown that varying the ion temperature causes significant changes in the shapes of the n and α spectra. For the n spectrum, the apparent intensity of sequential decay through the ground state of ${}^5\text{He}$ decreases with increasing temperature. For the α spectrum, the sharp edge in the CM frame spectrum near 3.75 MeV caused by the dineutron reaction channel results in a thermally broadened spectrum with a high-energy tail at energies $> 4\text{ MeV}$. Knowledge of such features may help to interpret data from experiments designed to investigate the $T + T$ reaction at low reaction energies.

1. Introduction

The $T + T \rightarrow 2n + \alpha$ (or TT) reaction is of interest for a number of reasons. It is one of the principal sources of neutrons in inertial confinement fusion (ICF) experiments, together with the $D + T \rightarrow n + \alpha$ (DT) and $D + D \rightarrow n + {}^3\text{He}$ (DD) reactions. The TT reaction is also related to the ${}^3\text{He} + {}^3\text{He} \rightarrow 2p + \alpha$ via isospin symmetry. This reaction is the dominant energy-producing step in the solar proton-proton chain,[1] and so studying the TT reaction can improve our knowledge of stellar nucleosynthesis.

Email address: `b.appelbe07@imperial.ac.uk` ()

The TT reaction produces three particles in the final state. This results in product energy spectra that are broad with a number of features that depend on the intermediate states through which the reaction proceeds. This is in contrast to the DT and DD reactions which are two-body reactions and so neutrons are produced in the CM frame with a specific energy. The only source of spectral broadening for these reactions is due to motion of the reactants. The shape of the TT spectra depends on both the motion of reactant particles and the intermediate states of the reaction. The reaction channels of interest in the TT reaction are as follows

where (GS) denotes the ground state of a ${}^5\text{He}$ nucleus and (ES) denotes the first excited state. The Q value for the reaction is 11.33 MeV . The neutrons produced from each channel have different spectral features. The $n + n + \alpha$ channel (referred to as the dineutron channel) results in neutrons distributed broadly between $0 - 9.4 \text{ MeV}$ with a peak just below 4 MeV . The ${}^5\text{He} (GS)$ channel produces a spectrum peak at 8.7 MeV while the ${}^5\text{He} (ES)$ channel produces a broad, near-elliptical neutron spectrum. Further details of the different spectral features, for both the n and α particles, may be found in the recent work of Brune et al.[2]

A major question regarding the TT reaction is the relative strengths of the reaction channels and how the relative strength varies with reaction energy. This has been the focus of much research. Experiments using an accelerator suggested that at a reaction energy of 250 keV the relative strengths of the different channels were 0.7, 0.2 and 0.1 for the dineutron, ${}^5\text{He} (GS)$ and ${}^5\text{He} (ES)$, respectively,[3] while experiments at a reaction energy of 110 keV suggested that the relative strengths to two significant figures were 0.95, 0.05 and 0.00.[4] More recently, the TT reaction has been investigated using laser-driven spherical capsule implosion experiments. In one set of such experiments it was concluded that there was a negligible ${}^5\text{He} (GS)$ channel at a mean reaction energy of 23 keV since no peak in the neutron spectrum near 8.7 MeV was detected.[5] However, another set of experiments at a mean reaction energy of 16 keV did observe such a peak[6] and this result was supported by theoretical calculations.[2] The lack of clear trends in existing experimental data highlights the need for further investigation of the TT

reaction.

The use of inertial confinement implosions for the investigation of nuclear reactions is a relatively new technique that offers a number of advantages over conventional accelerator based approaches.[7] The reactions take place in a thermonuclear plasma rather than the beam-target system of an accelerator. This means that particle fluxes are significantly higher and it is easier to access lower reaction energies. Furthermore, free electron screening in the thermonuclear plasma means that inertial confinement implosions are a much better model of the stellar environment than beam-target systems. However, the thermonuclear plasma environment also has a number of complications that are not present in accelerator experiments. The thermonuclear plasma contains a Maxwellian distribution of the reactant particle energies (characterised by the ion temperature) and, therefore, nuclear reactions cannot be investigated at a specific reaction energy but, instead, at a distribution of reaction energies. This also means that the energy spectra of product particles are affected by thermal broadening which must be accounted for when interpreting experimental data.

In addition, some of the key diagnostics in ICF experiments involve measurements of neutrons in the $3 - 10 \text{ MeV}$ range. For example, images of scattered DT neutrons with energies between $6 - 12 \text{ MeV}$ are used to study the shape and structure of cold fuel at stagnation,[8] while the fuel areal density (ρR) can be inferred from DT neutrons that are scattered to energies below 6 MeV . [9] Accurate interpretation of the data from these diagnostics requires a precise knowledge of the TT spectrum shape and intensity so that the contribution of TT neutrons to measured signals can be distinguished from scattered DT neutrons.

In this work we study the effect of thermal broadening on the TT product particle energy spectra. It is shown that thermal broadening can have a number of effects on both the spectra of neutron and α particles emitted by the reaction. Most notably, it is shown that thermal broadening can affect the apparent intensity of different features of the spectra even if the temperature does not change the relative strengths of the reaction channels. These effects may complicate the accurate identification of the relative strengths.

The contents of the paper are as follows. In section 2 a semi-analytic expression for thermal broadening of the TT product particle spectrum is derived. In sections 3 and 4 the major effects of thermal broadening on the neutron and α spectrum, respectively, are identified and quantified. Section 5 studies spectra produced by a multi-temperature plasma. Finally, some

conclusions are discussed in section 6.

2. Calculating thermal broadening of the TT product particle spectra

Thermal broadening in a Maxwellian plasma is due to two separate effects. First, the Maxwellian distribution of reactant velocities gives rise to a distribution of centre of mass (CM) frame velocities. This means that a given spectrum in the CM frame will have to be transformed into the lab frame according to this distribution of CM frame velocities in order to obtain the thermally broadened spectrum.

Secondly, the Maxwellian distribution of reactants gives rise to a distribution of reaction energies in the CM frame. The reaction energy, E_r , is defined by

$$E_r = \frac{1}{2}\mu v_r^2, \quad (1)$$

where μ is the reduced mass of the two reactant species and v_r is the relative velocity between a given pair of reactants. As mentioned in section 1, the shape of the product particle spectra in the CM frame is a function of the reaction energy. Therefore, the spectrum in the CM frame is a weighted mean of the spectra corresponding to each reaction energy in the distribution. We note that in thermonuclear plasmas, a mean value for the reaction energy is often used when calculating reaction rates and product spectra. This mean reaction energy is referred to as the Gamow energy and is obtained by weighting the Maxwellian distribution with the barrier penetration factor.[1] In this work we use the distribution of reaction energies rather than the Gamow energy in order to improve accuracy of the calculations.

We quantify these two effects in the calculation that follows. The calculation is of a similar nature to those previously carried out for the neutron spectra from DT and DD reactions.[10] However, due to the presence of three particles in the final state, the calculation for spectra from TT reactions differs in a key manner. For reactions with two particles in the final state then, for a given pair of reactants, the product particles have a unique value of energy in the CM frame that is given by conservation of mass-energy. For reactions with three particles in the final state, the product particles have a distribution of energies in the CM frame for a given pair of reactants. We introduce a term $F_{E_r}(E'_p, \theta_s)$ to account for this fact. This term represents the energy spectrum of product particles in the CM frame as a function of

reaction energy and direction of emission. It can be obtained either from theoretical calculations or from existing experimental data. It is normalized such that

$$\int F_{E_r} (E'_p, \theta_s) dE'_p d\Omega_s = 1. \quad (2)$$

Here, E'_p represents product particle energy in the CM frame, θ_s represents the scattering angle of a product particle in the CM frame and $d\Omega_s$ is the unit of solid angle for this vector.

We start with the general expression

$$\frac{dR}{dE'_p d\Omega'_p} = \frac{n_1 n_2}{1 + \delta_{12}} v_r \sigma(E_r) F_{E_r} (E'_p, \theta'_p) f_1(\mathbf{v}_1) f_2(\mathbf{v}_2) d^3\mathbf{v}_1 d^3\mathbf{v}_2. \quad (3)$$

This expression represents the number of reactions occurring between reactants with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ per unit volume per unit time resulting in a product particle with an energy in the CM frame of E'_p and direction Ω' . Here, $\sigma(E_r)$ represents the cross-section for the reaction. The variables n_1 and n_2 are the reactant species number densities, $f_1(\mathbf{v}_1) d^3\mathbf{v}_1$ and $f_2(\mathbf{v}_2) d^3\mathbf{v}_2$ are the reactant distribution functions and $\delta_{12} = 1$ for intra-species reactions, 0 otherwise. We use non-relativistic kinematics throughout. Although, it has been shown that a relativistic treatment introduces non-negligible corrections,[11, 12] these corrections are significantly smaller than the effects of thermal broadening that we identify here.

We choose both reactant distribution functions to be Maxwellian with the form

$$f(\mathbf{v}) = \left(\frac{m_t}{2\pi T_i} \right)^{\frac{3}{2}} \exp \left(-\frac{m_t v^2}{2T_i} \right), \quad (4)$$

where m_t is the tritium particle mass and T_i is the ion temperature. We therefore have

$$\begin{aligned} \frac{dR}{dE'_p d\Omega'_p} = & \frac{n_1 n_2}{2} \left(\frac{m_t}{2\pi T_i} \right)^3 v_r \sigma F_{E_r} \times \\ & \exp \left(-\frac{m_t}{2T_i} (v_1^2 + v_2^2) \right) d^3\mathbf{v}_1 d^3\mathbf{v}_2. \end{aligned} \quad (5)$$

The velocities of the reactants are related to the relative velocity $\mathbf{v}_r$ and

velocity of the CM frame $\mathbf{v}_{\text{cm}}$ by

$$\mathbf{v}_1 = \mathbf{v}_{\text{cm}} + \frac{1}{2}\mathbf{v}_{\text{r}}, \quad (6)$$

$$\mathbf{v}_2 = \mathbf{v}_{\text{cm}} - \frac{1}{2}\mathbf{v}_{\text{r}}. \quad (7)$$

These relations, together with (1), allow us to carry out a change of variables of (5) from $(\mathbf{v}_1, \mathbf{v}_2)$ to $(\mathbf{v}_{\text{cm}}, dE_r, d\theta_r, d\phi_r)$. This transformation has a Jacobian determinant of

$$d^3\mathbf{v}_1 d^3\mathbf{v}_2 = \sqrt{\frac{2E_r}{\mu^3}} d^3\mathbf{v}_{\text{cm}} dE_r d\Omega_r \quad (8)$$

and results in

$$\begin{aligned} \frac{dR}{dE'_p d\Omega'_p} = & \frac{n_1 n_2}{2} \frac{m_t}{(\pi T_i)^3} E_r \sigma F_{E_r} \times \\ & \exp\left(-\frac{m_t}{T_i} v_{cm}^2\right) \exp\left(-\frac{E_r}{T_i}\right) d^3\mathbf{v}_{\text{cm}} dE_r d\Omega_r. \end{aligned} \quad (9)$$

Now, a given product particle will have velocity $\mathbf{v}_{\text{p}}$ in the lab frame and $\mathbf{v}'_{\text{p}}$ in the CM frame which are related by

$$\mathbf{v}_{\text{cm}} = \mathbf{v}_{\text{p}} - \mathbf{v}'_{\text{p}}. \quad (10)$$

We can use these relations to transform from variables $\mathbf{v}_{\text{cm}}$ to $\mathbf{v}_{\text{p}}$ (which has a Jacobian determinant of 1). We can then convert from the product particle velocity co-ordinates $(\mathbf{v}_{\text{p}})$ to energy co-ordinates (E_p, Ω_p) which has the following Jacobian determinant

$$d^3\mathbf{v}_{\text{p}} = \sqrt{\frac{2E_p}{m_p^3}} dE_p d\Omega_p, \quad (11)$$

where m_p is the mass of the product particle (a neutron or α). We note that (10) also gives

$$v_{cm}^2 = \frac{2}{m_p} (E_p + E'_p) - \frac{4\sqrt{E_p E'_p}}{m_p} \cos \theta_{pp'}, \quad (12)$$

where $\theta_{pp'}$ is the angle between $\mathbf{v}_p$ and $\mathbf{v}_{p'}$. After these operations (9) becomes

$$\begin{aligned} \frac{dR}{dE_p d\Omega} &= \frac{n_1 n_2 m_t}{2\pi^3 T_i^3} \sqrt{\frac{2E_p}{m_p^3}} E_r \sigma_{F_{E_r}} \exp\left(-\frac{E_r}{T_i} - \frac{2m_t}{m_p T_i} (E_p + E_p')\right) \times \\ &\quad \exp\left(\frac{4m_t \sqrt{E_p E_p'}}{m_p T_i} \cos \theta_{pp'}\right) dE_p' d\Omega_p' dE_r d\Omega_r, \end{aligned} \quad (13)$$

where we have also exchanged $dE_p d\Omega$ and $dE_p' d\Omega_p'$.

We now integrate over the angular variables. We first note that the CM frame scattering angle can be defined by $\cos \theta_s = \hat{\mathbf{v}}_r \cdot \hat{\mathbf{v}}_p'$. Here, $\hat{\mathbf{v}}_r$ denotes the direction of the first reactant in the CM frame. With this definition, integration over $d\Omega_s$ is equivalent to integration over $d\Omega_r$. The only term in (13) that is a function of Ω_r is $F_{E_r}(E_p', \theta_r)$. We denote the result of this integral by

$$\int F_{E_r}(E_p', \theta_r) d\Omega_r = \bar{F}_{E_r}(E_p'), \quad (14)$$

where $\bar{F}_{E_r}$ represents the angular-averaged distribution of particle energies in the CM frame. Only limited experimental data on the isotropy of emission of particles in the CM frame exists.[3] However, the significance of (14) is that, even if the CM frame spectrum contains anisotropic features, the thermal broadening does not depend on angular variations in the CM frame spectrum but only on the angular-averaged spectrum. Therefore, anisotropic emission in the CM frame or effects such as the angular correlation of primary and secondary neutrons emitted by the ${}^5\text{He}(GS)$ [2] cannot be observed by varying the temperature of a reacting plasma. This may not be the case for reactant distributions other than Maxwellian, even if the distributions are isotropic.

Next, we integrate over $d\Omega_p'$. The only term that depends on this variable is $\cos \theta_{pp'}$. For a given direction of $\mathbf{v}_p$, the integral over the solid angle $d\Omega_p'$ ($= \sin \theta_{pp'} d\theta_{pp'} d\phi_{pp'}$) will be

$$\int_0^{2\pi} \int_0^\pi \sin \theta_{pp'} \exp(\alpha \cos \theta_{pp'}) d\theta_{pp'} d\phi_{pp'} = \frac{2\pi}{\alpha} [\exp(\alpha) - \exp(-\alpha)], \quad (15)$$

where

$$\alpha = \frac{4m_t \sqrt{E_p E_p'}}{m_p T_i}. \quad (16)$$

Substituting the results of the angular variable integration into (13) gives

$$\begin{aligned} \frac{dR}{dE_p d\Omega} = & \frac{\Lambda E_r}{\sqrt{E'_p}} \sigma \bar{F}_{E_r} \exp\left(-\frac{E_r}{T_i}\right) \left[\exp\left(-\frac{2m_t}{T_i m_p} \left(\sqrt{E_p} - \sqrt{E'_p}\right)^2\right) \right. \\ & \left. - \exp\left(-\frac{2m_t}{T_i m_p} \left(\sqrt{E_p} + \sqrt{E'_p}\right)^2\right) \right] dE'_p dE_r, \end{aligned} \quad (17)$$

with

$$\Lambda = \frac{n_1 n_2}{2\sqrt{2}\pi^2 T_i^2 \sqrt{m_p}}. \quad (18)$$

For $E_p, E'_p \gg T_i$, the last exponential term in (17) is negligible and can be ignored. The limits of integration for dE'_p and dE_r are $[0, \infty)$. These integrals cannot be solved without specifying the form of σ (which is a function of E_r) and $\bar{F}_{E_r}$ (which is a function of E_r and E'_p). However, by taking each integral in turn we can see the physical significance of each. First, integration over E_r gives

$$G(E'_p) = \int \sigma \bar{F}_{E_r} E_r \exp\left(-\frac{E_r}{T_i}\right) dE_r, \quad (19)$$

where $G(E'_p)$ represents an aggregate energy spectrum in the CM frame that accounts for the distribution of reaction energies arising from the Maxwellian plasma. The second integral then accounts for the transformation of spectra from CM frame to the lab frame according to the distribution of CM frame velocities

$$\frac{dR}{dE_p d\Omega_p} = \Lambda \int \frac{G(E'_p)}{\sqrt{E'_p}} \exp\left(-\frac{2m_t}{T_i m_p} \left(\sqrt{E_p} - \sqrt{E'_p}\right)^2\right) dE'_p. \quad (20)$$

This equation represents a semi-analytic expression for the energy spectrum of product particles per unit solid angle produced by tt reactions in a Maxwellian plasma. The relationship between this result and simpler approximations is discussed in Appendix A. In the next two sections we study the behaviour of (19) and (20) with changing ion temperature for both the neutron and α particles produced from the TT reaction.

3. The neutron spectrum

In order to apply (19) and (20) to the neutron spectrum it is necessary to know the shape of the neutron spectrum in the CM frame as a function

of reaction energy. A number of sources of such data exist and these are not always in agreement due to the limited experimental data and uncertainties regarding the strengths of different channels. Therefore, we use two separate sets of data for the CM frame spectra that are available in the literature.

The first set of data for the CM frame neutron spectra that we use is that which results from a recent R-matrix calculation.[2] This data includes contributions from the ${}^5\text{He}$ (GS), ${}^5\text{He}$ (ES) and dineutron channels as well as interference between the different reaction channels. The R-matrix calculation was carried out for a reaction energy of 16 keV only and parameter values were chosen so that the spectrum agreed with experimental data.[6] Therefore, in order to apply (19) to the R-matrix data it is assumed that $\bar{F}_{E_r}$ is independent of E_r . This approximation means that the integral (19) affects only the yield of TT neutrons, not the spectrum shape. Therefore, thermal broadening is due only to the transformation of spectra from the CM frame to the lab frame according to (20).

Thermal broadening using the R-matrix data is shown in fig. 1. The most notable feature is the decrease in the intensity of the ${}^5\text{He}$ (GS) peak relative to the intensity of the peak near 4.0 MeV as the temperature increases (i.e. a decrease in apparent intensity of the ${}^5\text{He}$ (GS) peak). There are two reasons why this occurs. First, when a spectrum in the CM frame is transformed into the lab frame for a given $\mathbf{v}_{\text{cm}}$ then the change in energy is larger for particles with a larger energy in the CM frame. For a Maxwellian plasma we have a Maxwellian distribution of $\mathbf{v}_{\text{cm}}$ and corresponding frame transformations. Secondly, the peak near 4.0 MeV is much broader than the peak near 8.7 MeV . The combination of these features means that when transforming a spectrum from the CM frame to the lab frame, the magnitude of the spectrum shift relative to the peak width is far higher for the ground state peak than for the dineutron peak. Therefore, the thermal broadening effect is larger for peaks that occur at a higher energy in the CM frame and increases with temperature since the Maxwellian distribution of $\mathbf{v}_{\text{cm}}$ is getting larger with temperature.

Another notable feature of the spectrum in fig. 1 is that the apparent energy at which the ${}^5\text{He}$ (GS) peak occurs moves to a lower energy with increasing temperature. This occurs due to the asymmetry of the spectrum near the peak, where the high-energy side of the peak drops off much more sharply than the lower-energy side. Both of these features are quantified in the lower diagram of fig. 1.

The second set of data is that contained in the ENDF database.[13]

This data was calculated using a three-body resonance model, including the $n + {}^5\text{He}$ (GS) and $n + n + \alpha$ resonances, taking into account exchange contributions that arise from symmetrizing in the identical neutrons. Experimental measurements were used to determine the relative amplitudes of the resonant contributions.[3] Unlike the R-matrix data, the data in the ENDF database gives values for $\bar{F}_{E_r}$ that vary with E_r . Therefore, both (19) and (20) affect the shape of the spectrum.

The results of applying these integrals to the ENDF data for a number of different ion temperatures are shown in fig. 2. The most notable trend is again that, as temperature increases, the intensity of the peak corresponding to the ${}^5\text{He}$ (GS) decreases relative to the intensity of the dineutron peak. The ${}^5\text{He}$ (GS) peak also displays significant broadening as temperature increases. These two features have been quantified in the lower diagram of fig. 2 and it is clear that both of these features change significantly over the temperature range considered here.

Finally, in order to provide a more direct comparison between the R-matrix and ENDF data a calculation was carried out in which it was assumed $\bar{F}_{E_r}$ was independent of E_r for the ENDF data (by selecting $\bar{F}_{16\text{ keV}}$ from the ENDF data) and so spectra shapes depended only on (20). However, it was found that for all values of ion temperature the resulting spectra were very similar in shape to those shown in fig. 2, suggesting that the thermal broadening effect is due mainly to the transformation from CM frames to the lab frame and not the distribution of reaction energies. This may not be the case if $\bar{F}_{E_r}$ varies strongly with temperature.

4. The α spectrum

As in the case of the neutron spectrum, the three channels of the TT reaction each result in a distinctive shape of the CM spectrum for the α product. The energy range of the α spectrum is approximately $0 - 4\text{ MeV}$. The dineutron channel contributes a sharp edge in the spectrum near 3.75 MeV . The spectrum is also highly asymmetric with a relatively flat shape between $1 - 2\text{ MeV}$.

In order to study the thermal broadening of the α spectrum we again use data for the CM spectrum from two sources and use (19) and (20) to calculate broadening. Thermal broadening of the α spectrum using data from the R-matrix calculation is shown in fig. 3. The CM frame α spectrum contains

Figure 1: Thermally broadened neutron spectra calculated using R-matrix data. Top: The energy spectrum of neutrons emitted by the TT reaction for a number of different ion temperatures. Each spectrum is independently normalized. Bottom: The left axis shows the variation with ion temperature of the ratio of the neutron spectrum intensity at the ${}^5\text{He}$ (GS) peak to the intensity of the spectrum at the 4.0 MeV peak. The right axis shows the energy at which the ${}^5\text{He}$ (GS) peak occurs.

Figure 2: Thermally broadened neutron spectra calculated using ENDF data. Top: The energy spectrum of neutrons emitted by the TT reaction for a number of different ion temperatures. Each spectrum is independently normalized. Bottom: The left axis shows the variation with ion temperature of the ratio of the neutron spectrum intensity at the ^5He (GS) peak to the intensity of the spectrum at the peak near 4.0 MeV. The right axis shows the full width at 3/4 of maximum intensity of the ^5He (GS) peak as a function of ion temperature.

Figure 3: Thermally broadened α spectra calculated using R-matrix data for a number of different ion temperatures. Each spectrum is independently normalized. The most notable feature here is the increase in the high-energy tail of the spectrum as the temperature increases.

a very sharp feature at approximately 3.75 MeV due to the dineutron contribution. Above this energy the CM frame spectrum drops rapidly to zero. However, thermal broadening causes a smoothing of this feature. Therefore, as ion temperature increases the α spectrum displays a growing high-energy tail extending above 4 MeV .

The results for thermal broadening using the ENDF evaluation are illustrated in fig. 4. The shapes of the spectra here are significantly different to those in fig. 3. This is mainly due to an increase in the weighting of the dineutron channel in the ENDF data compared to the R-matrix calculation. Results for the ENDF data show that the peak of the spectrum becomes broader and the energy at which the peak occurs decreases with increasing temperature. These two features are quantified in the lower diagram of fig. 4 in which it is shown that the energy at which the peak occurs decreases by almost 250 keV as ion temperature is increased from 1 keV to 40 keV . The width of the peak at $3/4$ maximum intensity is also shown here as a measure of broadening. This width increases by more than 700 keV over the same temperature range. Finally, fig. 4 shows that, as with the R-matrix results, increasing temperature causes the high-energy tail of the α spectrum to become less steep and extend to higher energies.

Figure 4: Thermally broadened α spectra calculated using ENDF data. Top: The energy spectrum of α emitted by the TT reaction for a number of different ion temperatures. Each spectrum is independently normalized. Bottom: The energy at which the peak of the α spectrum is located (left axis) and the full width of the peak at 3/4 of the maximum intensity are shown as a function of ion temperature.

5. The effect of inhomogeneous temperature

The plasmas that are produced in ICF experiments can be spatially and temporally inhomogeneous, with a range of temperatures and densities making contributions to the total TT reaction yield. In this section we investigate the effects on the TT spectrum when it is produced by multiple different ion temperatures. To do this, we use a spherically-symmetric hotspot plasma with a radial temperature profile given by

$$T_i(r) = T_{peak} - (T_{peak} - T_{low}) r^x, \quad (21)$$

where r is the normalized radius, $T_{low} = 1 \text{ keV}$ is ion temperature at hotspot edge, T_{peak} is ion temperature at hotspot centre and x is a free parameter. We choose values of $x = 1.5$ and $x = 4$ here. We assume that the hotspot is isobaric. This model results in a significant yield of TT neutrons from a broad range of ion temperatures since the high temperature, high reactivity regions near the hotspot centre have a relatively small volume and low density compared to the lower temperature, lower reactivity regions near the hotspot edge. This is illustrated in fig. 5 with $T_{peak} = 20 \text{ keV}$ in which neutron yield as a function of ion temperature is shown. The corresponding TT neutron spectra will be a composite of the spectra for $T_i = 1 - 20 \text{ keV}$, weighted according to the distribution shown in fig. 5.

The composite spectra produced from these hotspot profiles will be similar to the spectra produced by some measure of the average ion temperature. However, it is not obvious how this average ion temperature should be calculated. Three different estimates for the average ion temperature are shown in fig. 6. The first is the burn-averaged TT temperature, defined as

$$\langle T_i \rangle_{TT} = \frac{\int T_i Y_{TT}(T_i) dT_i}{\int Y_{TT}(T_i) dT_i}, \quad (22)$$

where $Y_{TT}(T_i)$ is the yield of TT neutrons as a function of ion temperature. The burn-averaged TT temperature is labelled $TT_{ave.}$ in fig. 6. The second temperature measure is the ion temperature inferred from the FWHM of a DT neutron spectrum produced by the same hotspot profile (labelled DT_{FWHM}). This temperature measure has previously been used to measure mean ion temperature in experiments investigating the TT spectrum.[5, 6] The third temperature measure shown in fig. 6 is obtained by taking the ratio of the intensity of the peaks of the composite neutron spectrum and

using fig. 2 to estimate the average ion temperature (labelled TT_{irat}). We have used ENDF data for the TT neutron spectrum in the CM frame in this calculation.

As can be seen in fig. 6, these three different measures can give noticeably different estimates for the ion temperature, particularly for higher values of T_{peak} . The largest discrepancy is between the $TT_{ave.}$ and DT_{FWHM} . There are two reasons for this. First, differences in the variation of DT and TT reactivities with temperature. Although the reactivity for both reactions increases rapidly with temperature, the rate of increase peaks at approximately $T_i = 14\text{ keV}$ for DT reactions, as illustrated in fig. 7. This means that TT reactions are more strongly weighted by higher temperature components of the hotspot profile than the DT reactions, particularly when the peak ion temperature exceeds 14 keV . Secondly, the rate of increase of the FWHM of the DT neutron spectrum decreases with increasing temperature.[11] This causes the DT reactions from higher temperature components to contribute proportionally less to the broadening of the DT spectrum than the lower temperature components.

Figure 6 also shows a discrepancy between the burn-averaged TT temperature and the temperature inferred from the relative intensity of peaks of the TT neutron spectrum. This discrepancy arises because, as can be seen in fig. 2, the relative intensity of the peaks does not change linearly with ion temperature. At higher temperatures the relative intensity is less sensitive to changes in ion temperature. Therefore, higher temperature components contribute proportionally less to the decrease in the relative intensity measure than the lower temperature components.

Effects such as time evolution of the hotspot and bulk fluid motion (discussed in Appendix B) mean that (21) represents a very simplified model of the hotspot. However, it is useful here for illustrating that caution needs to be taken when fitting a TT spectrum from an ICF experiment using just a single temperature value.

6. Conclusions

In this work we have studied the effects of thermal broadening on the energy spectra of product particles from TT reactions. We have derived a model for thermal broadening that is accurate for all points of the spectra. This model was used to show that thermal broadening can have significant

Figure 5: The yield of TT neutrons as a function of ion temperature for two isobaric hotspot profiles with $T_{peak} = 20 \text{ keV}$ and radial variation given by (21).

effects on the spectral features of both the neutron and α spectra. The most notable effects identified are

1. For the neutron spectrum, the decrease in the apparent intensity of the ${}^5\text{He}$ (GS) peak near 8.7 MeV as the ion temperature increases. This is due to a spreading out of the ${}^5\text{He}$ (GS) peak and not changes in the neutron yield.
2. For the α spectrum, the presence of a high-energy tail in the spectrum at energies $> 4 \text{ MeV}$.

These features were present in both data sets considered for the CM frame spectra. Such features could have important implications for interpreting experimental data of the TT product spectra emitted by a plasma or for identifying the relative strengths of each reaction channel for different reaction energies. However, current uncertainties in the CM frame spectra data (particularly at lower reaction energies) limit the predictive capability for thermal broadening of TT spectra produced by plasmas.

For both data sets considered, the shape of the CM frame spectra showed little variation with changing reaction energy for the temperature range considered. This suggests that if variation with temperature of the spectral features highlighted here are not observed in experiments, this may be due to variations of the CM frame spectra with reaction energy that are not contained in the data sets.

Figure 6: Variation of different measures of ion temperature for isobaric hotspot profiles with $x = 1.5$ (top) and 4.0 (bottom) as a function of the ion temperature at the centre of the hotspot (T_{peak}).

Figure 7: Reactivities for the TT and DT reactions as a function of ion temperature. The reactivities are normalized to the value at $T_i = 40 \text{ keV}$ ($3.04 \times 10^{-24} \text{ m}^3 \text{ s}^{-1}$ for TT and $8.0 \times 10^{-22} \text{ m}^3 \text{ s}^{-1}$ for DT). The normalized rate of change of reactivities with respect to ion temperature are also shown. This quantity has a peak near 14 keV for DT but increases monotonically for TT .

We finally note that the model for thermal broadening derived in section 2 is also suitable for other reactions that produce three particles in the final state, such as ${}^3\text{He} + {}^3\text{He} \rightarrow 2p + \alpha$ and $T + {}^3\text{He} \rightarrow n + p + \alpha$.

Appendix A. Comparison with other models for thermal broadening

Existing models of thermal broadening for reaction product spectra in plasmas have been derived for two-body reactions (such as DT and DD). [14, 15, 12, 11] In the two-body case the range of energies of product particles in the CM frame is sufficiently small that $G(E'_p)$ in (19) is well approximated by a delta function. The delta function is located at

$$E'_p \equiv E_Q = E_G + \frac{m_4}{m_3 + m_4} Q, \quad (\text{A.1})$$

where E_G is the Gamow energy, Q is kinetic energy liberated in the reaction, m_3 is the mass of the product particle whose spectrum we wish to calculate and m_4 is the mass of the second product particle. In this case, integration over dE'_p in (20) results in

$$\frac{dR}{dE_p d\Omega_p} = \frac{\Lambda}{\sqrt{E_Q}} \exp \left(-\frac{2m_t}{T_i m_p} \left(\sqrt{E_p} - \sqrt{E_Q} \right)^2 \right). \quad (\text{A.2})$$

Expansion of the exponential function about $E_p = E_Q$ then gives

$$\frac{dR}{dE_p d\Omega_p} \approx \frac{\Lambda}{\sqrt{E_Q}} \exp \left(-\frac{m_t}{2m_p T_i E_Q} (E_p - E_Q)^2 \right), \quad (\text{A.3})$$

This equation represents the familiar Gaussian approximation that is used to model neutron spectra from DD and DT reactions.[14]

An approximate model for thermal broadening of product spectra from three-body reactions can be obtained by convolving a spectrum in the CM frame with the Gaussian function in (A.3). In the case of the neutron spectrum from TT reactions, this approximate model agrees well with the model derived in section 2. In the temperature range that we have considered here ($T_i = 1 - 40 \text{ keV}$) the approximate model has an error of less than 3% with the exact model. However, the approximate model is not suitable for application to the α spectrum. Figure A.8 illustrates the discrepancy between the model for thermal broadening derived in section 2 and the Gaussian approximation. For a plasma ion temperature of 10 keV , we have obtained the CM frame spectrum, $G(E'_p)$, using (19) and the ENDF data. We have then used (20) to produce an exact model for thermal broadening and also obtained a Gaussian approximation for broadening by convolving $G(E'_p)$ with (A.3). Clearly, there are significant differences between the two results.

Appendix B. The effect of fluid motion

It has recently been shown that bulk motion of a thermal plasma can affect the shape of the dd and dt neutron spectra.[10, 16] For the tt spectrum, this effect can be modeled by using a shifted Maxwellian distribution for the distribution of reactants

$$f(\mathbf{v}) = \left(\frac{m_t}{2\pi T_i} \right)^{\frac{3}{2}} \exp \left(-\frac{m_t (\mathbf{v} + \mathbf{v}_f)^2}{2T_i} \right), \quad (\text{B.1})$$

in which $\mathbf{v}_f$ represents the bulk fluid velocity. A similar derivation to that described in section 2 then results in the following expression for the lab frame energy spectrum of product particles

$$\frac{dR}{dE_p d\Omega} = \Lambda \sqrt{\frac{E_p}{E_p^f}} \int \frac{G(E'_p)}{\sqrt{E'_p}} \exp \left(-\frac{2m_t}{T_i m_p} \left(\sqrt{E_p^f} - \sqrt{E'_p} \right)^2 \right) dE'_p, \quad (\text{B.2})$$

Figure A.8: A comparison of thermal broadening models on the α spectrum for a plasma with $T_i = 10 \text{ keV}$ and using ENDF data for the CM frame spectrum. The exact broadening curve is obtained with the model derived in section 2 while the Gaussian broadening curve results from convolving the CM frame spectrum with the Gaussian function given by (B.3).

where

$$E_p^f = E_p + \frac{1}{2}m_p v_f^2 + \sqrt{2m_p E_p} v_f \cos \theta_f, \quad (\text{B.3})$$

and $G(E_p')$ is given by (19). Here, E_p^f represents the energy of the product particle in the rest frame of the fluid and θ_f is the angle between the direction of fluid flow and the direction of emission of the product particle.

Acknowledgements

The authors would like to thank Maria Gatu-Johnson, Daniel Sayre and Dan Casey for informative discussions and Carl Brune for providing the R-matrix data.

- [1] D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis*. University of Chicago Press (1983).
- [2] C. R. Brune, et al., *Phys. Rev. C* **92**, 014003 (2015).
- [3] C. Wong, J. Anderson, J. McClure, *Nuclear Physics* **71**(1), 106 (1965).
- [4] K. W. Allen, E. Almqvist, J. T. Dewan, T. P. Pepper, J. H. Sanders, *Phys. Rev.* **82**, 262 (1951).
- [5] D. T. Casey, et al., *Phys. Rev. Lett.* **109**, 025003 (2012).

- [6] D. B. Sayre, et al., *Phys. Rev. Lett.* **111**, 052501 (2013).
- [7] A. Bacher, et al., *Few-Body Systems* **54**(7-10), 1599 (2013).
- [8] G. P. Grim, et al., *Physics of Plasmas* **20**(5) (2013).
- [9] C. J. Forrest, et al., *Review of Scientific Instruments* **83**(10) (2012).
- [10] B. Appelbe, J. Chittenden, *Plasma Physics and Controlled Fusion* **53**(4), 045002 (2011).
- [11] B. Appelbe, J. Chittenden, *High Energy Density Physics* **11**(0), 30 (2014).
- [12] L. Ballabio, J. Kallne, G. Gorini, *Nuclear Fusion* **38**(11), 1723 (1998).
- [13] M. B. Chadwick, et al., *Nuclear Data Sheets* **107**, 2931 (2006).
- [14] H. Brysk, *Plasma Physics* **15**(7), 611 (1973).
- [15] G. Lehner, *Zeitschrift fr Physik* **232**(2), 174 (1970).
- [16] B. K. Spears, et al., *Physics of Plasmas* **22**(5), (2015).