Vibrational pair cat states

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We propose a scheme for generating mesoscopic superpositions of two distinguishable pair coherent states of motion in a two-dimensional ion trap. In our scheme, the trapped ion is excited bichromatically by five laser beams along different directions in the X - Y plane of the ion trap. Four of these have the same frequency and can be derived from the same source, reducing the demands on the experimentalist. We show that if the initial vibrational state is given by a two-mode Fock state, as demonstrated in recent experiments, these highly correlated two-mode ‘‘Schrödinger cat’’ states are realized when the system reaches its steady state, which is indicated by the extinction of the fluorescence emitted by the ion. [S1050-2947(96)02611-X]

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Ever since Schrödinger suggested his famous cat experiment in 1935 [1], superpositions of macroscopically distinguishable quantum states (which are also known as Schrödinger cat states) have become a longstanding exemplar of the peculiarities which occur in the interpretation of quantum reality. To explore this subtlety and to gain insight into the fundamental issues of quantum theory, a number of schemes have been proposed for the realization of such states. In quantum optics, the Schrödinger cat states are usually described as superpositions of different coherent states [2], as coherent states are the closest quantum states to a classical description of the field of definite complex amplitude. Specifically, the archetype of a Schrödinger cat state is given by the superposition $|\Psi\rangle_{\text{cat}} = \mathcal{N}[\alpha + \exp(i\phi)|-\alpha\rangle]$, where $|\alpha\rangle$ is a coherent state of the single-mode quantized field and $\mathcal{N}$ is a normalization coefficient. In particular, these states are referred to the even, odd, and Yurke-Stoler [3] coherent states when $\phi=0, \pi$, and $\pi/2$, respectively. They have been extensively studied and shown to exhibit nonclassical properties such as squeezing and sub-Poissonian statistics [2].

In a recent paper Gerry and Grobe [4] have proposed a two-mode generalization of Schrödinger cat states which are defined as superpositions of two different pair coherent states (PCS). For two independent boson annihilation operators $\hat{a}$ and $\hat{b}$, a pair coherent state $|\xi, q\rangle$ is defined as an eigenstate of both the pair annihilation operator $\hat{a}\hat{b}$ and the number difference operator $\hat{Q} = \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}$ [5], i.e.,

$$\hat{a}\hat{b}|\xi, q\rangle = \xi|\xi, q\rangle, \quad \hat{Q}|\xi, q\rangle = q|\xi, q\rangle, \quad (1)$$

where ξ is a complex number and q is the ‘‘charge’’ which is a fixed integer. Without loss of generality, we may set $q \geq 0$ and the PCS can be explicitly expanded as a superposition of the two-mode Fock states, i.e.,

$$|\xi, q\rangle = \mathcal{N}_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n+q, n\rangle_F, \quad (2)$$

where $\mathcal{N}_q = [|\xi|^{-q} I_q(2|\xi|)]^{-1/2}$ is the normalization constant (I_q is the modified Bessel function of the first kind of order q). Pair coherent states are regarded as an important type of correlated two-mode state with prominent nonclassical prop-

erties such as sub-Poissonian statistics, strong intermode correlation in the number fluctuations, squeezing of quadrature variances, and violations of Cauchy-Schwarz inequalities [5].

The correlated two-mode Schrödinger cat states $|\xi, q, \phi\rangle$ are defined as superpositions of two PCS separated in phase by π [4], i.e.,

$$|\xi, q, \phi\rangle = \mathcal{N}_\phi [|\xi, q\rangle + e^{i\phi} |-\xi, q\rangle], \quad (3)$$

where the normalization constant $\mathcal{N}_\phi$ is given by

$$\mathcal{N}_\phi = \frac{1}{\sqrt{2}} \left[1 + \mathcal{N}_q^2 \cos \phi \sum_{n=0}^{\infty} \frac{(-1)^n |\xi|^{2n}}{n!(n+q)!} \right]^{-1/2}. \quad (4)$$

Gerry and Grobe have shown that these superposed states are characterized by additional nonclassical features beyond those of the PCS. It is easy to verify that the states $|\xi, q, \phi\rangle$ are eigenstates of the operator $(\hat{a}\hat{b})^2$ with eigenvalue ξ^2 . The problem remains of how such states may be generated. In this paper we address this issue.

Recently, due to the remarkable advances in laser cooling of a single trapped ion [6], it has become possible to realize nonclassical states of the center-of-mass (c.m.) motion of a single trapped ion. When an ion is trapped in a harmonic potential and driven by external laser beams, its internal and external degree of freedom are coupled via the momenta exchange between the field and the ion. Thus one can manipulate the external motion of the ion by controlling the configurations of the driving lasers. In particular, if the vibrational amplitude of the ion is much smaller than the laser wavelength (Lamb-Dicke limit), and at the same time the driving field is resonant with one of the vibrational sidebands of the atomic transition, then this model can be simplified to a form similar to the Jaynes-Cummings model (JCM) in which the quantized radiation field is replaced by the quantized c.m. motion of the ion [7]. Dissipative effects which are inevitable from cavity damping in the optical or microwave regime can be significantly suppressed for the ion motion by virtue of the extremely weak coupling between the vibrational modes and the external environment. This unique feature thus makes it possible to realize cavity QED experiments without using an optical cavity. Following this

FIG. 1. Configuration to generate the superpositions of vibrational PCS. The carrier field LC excites the ion on resonance along the negative X axis while the other four lasers are all resonant with the fourth lower vibrational sideband. The laser pairs (LX, LY) and (LX', LY') whose amplitudes and phases are given by $(E_1, 0)$ and $(2E_1, \pi)$ drive the ion along the X - Y and X' - Y' directions, respectively.

approach, nonclassical vibrational states of the trapped ions such as Fock [8], squeezed [9], and single-mode Schrödinger cat states [10,11] have been proposed and observed [12].

We have previously proposed a scheme [13] to generate vibrational PCS of a trapped ion. In this paper we extend this to propose a scheme to generate the correlated two-mode Schrödinger cat states of motion described by Eq. (3) for $|\psi\rangle_e = |\xi, q, \phi = 0\rangle$ and $|\psi\rangle_o = |\xi, q, \phi = \pi\rangle$, i.e., the even and odd PCS, respectively.

Consider the quantized motion of a two-level ion of mass M which is trapped in a two-dimensional (2D) isotropic harmonic potential characterized by the trap frequency ν . The annihilation (creation) of vibrational quanta in the X and Y directions is described by the operators $\hat{a}(\hat{a}^\dagger)$ and $\hat{b}(\hat{b}^\dagger)$, respectively. Accordingly, the position operators are given by $\hat{x} = \eta(\hat{a} + \hat{a}^\dagger)$ and $\hat{y} = \eta(\hat{b} + \hat{b}^\dagger)$, where $\eta = \sqrt{\hbar/2\nu M}$. In our scheme, the ion is bichromatically driven by five laser beams in the X - Y plane as shown in Fig. 1. Four of these have the same frequency and can be derived from a common source. The frequency of the carrier beam LC is resonant to the atomic transition while the other two pairs of laser beams are all tuned to the fourth lower vibrational sideband. The laser pairs (LX, LY) and (LX', LY') whose amplitudes and phases are given by $(E_1, 0)$ and $(2E_1, \pi)$ excite the ion along the X - Y and X' - Y' directions, respectively. The amplitude and phase of LC are chosen as (E_0, φ_0) , and the direction of LC can be arbitrary in general. However, for simplicity we assume that LC drives the ion along the negative X axis. Thus, in the rotating-wave approximation, the total Hamiltonian describing the coherent system evolution is given by

$$\hat{H} = \hbar\nu(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}) + \frac{\hbar\omega_0}{2}\hat{\sigma}_z - [\mathcal{D}\mathcal{E}^{(-)}(\hat{x}, \hat{y}, t)\hat{\sigma}_- + \text{H.c.}], \quad (5)$$

where the first two terms describe the free evolution of the external and internal degrees of freedom of the ion and the last indicates the atom-field interaction. The atomic operators are defined as $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, and

$\hat{\sigma}_- = |g\rangle\langle e|$, where $|e\rangle$ and $|g\rangle$ are the atomic excited and ground states, respectively. The transition in the two-level ion is characterized by the dipole matrix element $\mathcal{D}$ and the transition frequency ω_0 . The negative frequency part of the classical electric driving field is given by

$$\mathcal{E}^{(-)}(\hat{x}, \hat{y}, t) = E_1[e^{i(\omega_4 t - k_4 \hat{x})} - 2e^{i(\omega_4 t - k_4 \hat{x}')} + e^{i(\omega_4 t - k_4 \hat{y})} - 2e^{i(\omega_4 t - k_4 \hat{y}')}] + E_0 e^{i(\omega_0 t + k_0 \hat{x} + \varphi_0)}, \quad (6)$$

where $\omega_4 = \omega_0 - 4\nu$ and k_0, k_4 are the wave vectors of the driving lasers. Here we have introduced the position operators $\hat{x}'$ and $\hat{y}'$ which are related to $\hat{x}$ and $\hat{y}$ by a $\pi/4$ rotation in the X - Y plane as shown in Fig. 1, so that for annihilation (creation) operators $\hat{A}$ ($\hat{A}^\dagger$) and $\hat{B}$ ($\hat{B}^\dagger$), defined in the X' and Y' directions, respectively, we obtain the transformation

$$\begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}. \quad (7)$$

Vogel and de Matos Filho [10,14] have pointed out that in the well resolved sideband limit where the trapping frequency ν is much larger than the other characteristic frequencies, the ion-laser interaction can be described as a nonlinear Jaynes-Cummings model provided that the driving laser is resonant with one of the vibrational sidebands. Following the approach of Refs. [10,14], the Hamiltonian of Eq. (5) can be written in the interaction picture as

$$\begin{aligned} \hat{H}_I = & -\hbar e^{-\varepsilon^2/2} \left[\sum_{j=0}^{\infty} \frac{\Omega_j (i\varepsilon)^{2j+4}}{j!(j+4)!} [\hat{a}^j (\hat{a}^\dagger)^{j+4} - 2\hat{A}^j (\hat{A}^\dagger)^{j+4} \right. \\ & \left. + \hat{b}^j (\hat{b}^\dagger)^{j+4} - 2\hat{B}^j (\hat{B}^\dagger)^{j+4}] \right] \\ & + \Omega_0 e^{-i\varphi_0} \sum_{j=0}^{\infty} \frac{(i\varepsilon)^{2j}}{j!j!} \hat{a}^j (\hat{a}^\dagger)^j \hat{\sigma}_- + \text{H.c.}, \end{aligned} \quad (8)$$

where $\Omega_j = -DE_j/\hbar$ are the Rabi frequencies, and the Lamb-Dicke parameter $\varepsilon = k\eta$ has been defined assuming $k_4 \approx k_0 = k$.

As the damping of vibrational quanta can be significantly suppressed in an ion trap, the dominant decay process is the spontaneous emission from the two-level ion, and the time evolution of the system in the interaction picture can be described by a density operator $\hat{\rho}$ obeying the master equation [8–10,13]

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_I, \hat{\rho}] + \frac{\Gamma}{2}(2\hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} - \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_-) \quad (9)$$

where Γ is the spontaneous decay rate of the excited state of the ion, and the modified density operator

$$\hat{\rho} = \frac{1}{4} \int_{-1}^1 \int_{-1}^1 W(u, v) \hat{D}^\dagger \hat{\rho} \hat{D} du dv \quad (10)$$

accounts for the momentum transfer in the X - Y plane due to spontaneous emission, where $\hat{D} = \exp[-ik(u\hat{x} + v\hat{y})]$ is the displacement operator and $W(u, v)$ is the angular distribution of the spontaneous emission.

FIG. 2. The excitation number distribution $P(n, m)$ of the two vibrational modes at different times. The system evolves from (a), the initial state $|\Psi\rangle_{\text{ini}} = |e\rangle|1, 0\rangle_F$ ($\Gamma t = 0$), through (b) and (c), two intermediate states ($\Gamma t = 125/4$) and ($\Gamma t = 125$), into (d), its steady state which is the pure product of the internal ground state and even PCS: $|\Psi\rangle_s = |g\rangle|\psi\rangle_e$. The data shown are obtained from a Monte Carlo simulation which includes 1000 trajectories and was performed in a truncated Fock state basis ($|0, 0\rangle_F \dots |20, 20\rangle_F$). Parameters: $\lambda = 0.2, \xi = 2, \Gamma = 10$.

In the Lamb-Dicke regime, $\varepsilon \ll 1$, the master equation, Eq. (9), can be well approximated by expanding Eq. (10) up to first order in ε and Eq. (8) up to fourth order in ε . In this case, only the leading terms, i.e., $j=0$ in $\hat{H}_I$, are retained, and the modified density operator $\hat{\rho}$ is replaced by $\hat{\rho}$. Consequently, the master equation is simplified to

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}'_I, \hat{\rho}] + \frac{\Gamma}{2}(2\hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} - \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_-), \quad (11)$$

with the effective Hamiltonian $\hat{H}'_I$ given by

$$\hat{H}'_I = \lambda[(\hat{a}\hat{b})^2 - \xi^2]\hat{\sigma}_+ + \text{H.c.}, \quad (12)$$

where $\lambda = \hbar\Omega_1\varepsilon^4\exp(-\varepsilon^2/2)/4$, and $\xi = 2(\Omega_0/\Omega_1)^{-1/2}\varepsilon^{-2}\exp(-i\varphi_0/2)$.

Essentially, the correlated two-mode states of motion are generated via the process involving pair annihilation or creation in $\hat{H}'_I$. This implies that the number difference operator $\hat{Q}$ is a constant of motion as $[\hat{H}'_I, \hat{Q}] = 0$. However, the constancy of $\hat{Q}$ can be disturbed both by coherent and incoherent processes. The coherent disturbance results from the incorporation of higher-order terms into the effective Hamiltonian Eq. (12): that leads to the nonvanishing commutator $[\hat{H}'_I, \hat{Q}] \neq 0$, which indicates that $\hat{Q}$ is no longer conserved. On the other hand, the incoherent effects of spontaneous emission described in Eq. (10) destroy the correlations of the two modes as they change the motional states in different directions independently. In the master equation, Eq. (9), however, if only the lowest terms in $\hat{H}_I$ are considered while the density operator is fully described by Eq. (10), the time dependence of the mean value of $\hat{Q}$ is determined by

FIG. 3. Same as Fig. 2 except $|\Psi\rangle_{\text{ini}} = |e\rangle|2, 1\rangle_F$ and $|\Psi\rangle_s = |g\rangle|\psi\rangle_o$.

$$\frac{d}{dt}\langle\hat{Q}\rangle = \Gamma\left\langle\hat{\sigma}_+\hat{\sigma}_-\left[\frac{1}{4}\int_{-1}^1\int_{-1}^1 dudv W(u, v)(\hat{D}\hat{Q}\hat{D}^\dagger - \hat{Q})\right]\right\rangle. \quad (13)$$

In this case, it is interesting to point out that if the spontaneous emission is symmetric in the X - Y plane then $d\langle\hat{Q}\rangle/dt = 0$, indicating that $\langle\hat{Q}\rangle$ remains unchanged although the fluctuations in the number difference in the two modes do not vanish in the system evolution. All these analyses above therefore suggest that the assumption of a small Lamb-Dicke parameter which leads us from Eq. (9) to Eq. (11) is important. Nevertheless, we note that ε should not be too small in real experiments, for the finite size of ε is necessary to allow the fourth sideband excitations.

In the long time limit, the steady-state solution $\hat{\rho}_s$ for the master equation, Eq. (11), is given by $\hat{\rho}_s = |g\rangle\hat{\rho}_v\langle g|$ as a consequence of the atomic spontaneous decay, where $\hat{\rho}_v$ is the density operator describing the vibronic steady state of the ion. If we substitute the steady state $\hat{\rho}_s$ into the equation $d\hat{\rho}_s/dt = 0$, we find the condition $[\hat{H}'_I, \hat{\rho}_s] = 0$, or equivalently,

$$(\hat{a}\hat{b})^2\hat{\rho}_v = \xi^2\hat{\rho}_v. \quad (14)$$

The steady-state solution $\hat{\rho}_v$ is not uniquely determined by Eq. (14). In order to have the steady-state solution as superpositions of even and odd PCS, additional restrictions on the initial states are required. Specifically, the initial state of motion is required to have the form

$$|\chi\rangle_{\text{ini}} = \sum_{m=0}^{\infty} c_m |m+q, m\rangle_F \quad (15)$$

to satisfy the conserved “charge” condition Eq. (1). This means q must be retained as a fixed integer. Furthermore, as the time evolution does not mix even and odd two-mode states, the state $|\chi\rangle_{\text{ini}}$ can be separated into states having different parities, i.e., with

$$|\chi\rangle_e = \sum_{m=0}^{\infty} c_{2m} |2m+q, 2m\rangle_F \quad (16)$$

as the state of even parity and

FIG. 4. Same as Fig. 2. except $|\Psi\rangle_{\text{ini}} = 2^{-1/2} |e\rangle[|1,0\rangle_F + |2,1\rangle_F]$ and the steady state is described by a statistical mixture $\hat{\rho}_s = 2^{-1} |g\rangle[\langle\psi|_{ee}\langle\psi| + |\psi\rangle_{oo}\langle\psi|]\langle g|$.

$$|\chi\rangle_o = \sum_{m=0}^{\infty} c_{2m+1} |2m+1+q, 2m+1\rangle_F \quad (17)$$

as the state of odd parity. For any initial state described by Eq. (15) whose parity is mixed, the state solution of Eq. (14) can be described as a statistical mixture of even and odd PCS [10]

$$\hat{\rho}_v = P_e |\psi\rangle_{ee}\langle\psi| + P_o |\psi\rangle_{oo}\langle\psi|, \quad (18)$$

where P_e and P_o are the weights of the even and odd parities in the initial states. However, if the initial state is given in either even or odd parity, then the steady state can be a pure state. It is straightforward to verify that if the initial state is given as $|\chi\rangle_e$, the steady state is a superposed state with $\phi=0$, i.e., an even PCS. Similarly, for the initial state $|\chi\rangle_o$, the system evolves to the odd PCS with $\phi=\pi$. For

FIG. 5. The scaled 2D spatial distribution functions of the pair coherent state $|\xi, q\rangle$ with (a) $\xi=2, q=1$; (b) $\xi=-2, q=1$.

FIG. 6. The scaled 2D spatial distribution function of the ion, $P(\tilde{x}, \tilde{y})$, given by the steady-state solution shown in Fig. 2(d). The dimensionless variables $\tilde{x}$ and $\tilde{y}$ are given by $x = \sqrt{2} \eta \tilde{x}$ and $y = \sqrt{2} \eta \tilde{y}$.

simplicity, $|\chi\rangle_e$ and $|\chi\rangle_o$ can be prepared in the Fock state $|q, 0\rangle_F$ and $|q+1, 1\rangle_F$ ($q>0$) which, according to recent work, can be prepared with very high efficiency [12].

In the following, we investigate the time evolution of the vibrational states by solving the master equation, Eq. (11), numerically using a Monte Carlo state-vector technique [15]. Our numerical analysis was performed using a high-order unravelling technique [16] in a finite (truncated) Fock state basis with a cutoff chosen such that an increase of this cutoff does not alter the result of our integration. In Figs. 2–4, the excitation number distributions $P(n, m)$ in the two vibrational modes at different times are depicted with initial vibrational states prepared in $|1, 0\rangle_F$, $|2, 1\rangle_F$, and $(|1, 0\rangle_F + |2, 1\rangle_F)/\sqrt{2}$, respectively. It is found that in Fig. 2 and Fig. 3 the number distribution $P(n, m)$ starts from the given initial Fock state and subsequently evolves to the steady states with only the components $P(n+1, n)$ (n is even for Fig. 2 and odd for Fig. 3) developing in the time evolution. The oscillation in number distribution does not occur in Fig. 4, as the initial state of motion is a superposition of Fock states with even and odd parity and thus leads to an incoherent superposition of even and odd components of $P(n+1, n)$ in the time evolution. The purity of the steady state is indicated by calculating the trace of the square of the density operator $\text{Tr}(\hat{\rho}_s^2)$ in our numerical integration. We find that $\text{Tr}(\hat{\rho}_s^2) = 1, 1$, and 0.79 for the steady state $\hat{\rho}_s$ in Figs.

FIG. 7. The scaled 2D spatial distribution function of the ion, $P(\tilde{x}, \tilde{y})$, given by the steady-state solution shown in Fig. 3(d). The dimensionless variables $\tilde{x}$ and $\tilde{y}$ are given by $x = \sqrt{2} \eta \tilde{x}$ and $y = \sqrt{2} \eta \tilde{y}$.

FIG. 8. The scaled 2D spatial distribution function of the ion, $P(\tilde{x}, \tilde{y})$, given by the steady-state solution shown in Fig. 4(d). The dimensionless variables $\tilde{x}$ and $\tilde{y}$ are given by $x = \sqrt{2} \eta \tilde{x}$ and $y = \sqrt{2} \eta \tilde{y}$.

2–4, respectively, demonstrating the evolution to pure or mixed states as expected from our steady-state analysis. The scaled 2D spatial probability distributions corresponding to the steady states given in Figs. 2–4 are shown in Figs. 6–8, respectively. We find that, in comparison with the spatial distributions of the pure PCS described by $\xi=2, q=1$ and $\xi=-2, q=1$ in Fig. 5, the quantum interferences of the coherent superpositions are indicated by the fringe structures in Figs. 6–8.

In conclusion, we have proposed a scheme for the realiza-

tion of even and odd pair coherent states of the c.m. motion of a trapped ion. In the present scheme, five laser beams, one of which is tuned to the carrier frequency and the other four to the fourth lower vibrational sideband, are used to drive the ion trapped in a 2D isotropic harmonic potential well. In appropriate limits, the system will relax to a steady state due to the spontaneous emission from the ion. The steady state of the system is indicated by the extinction of the fluorescence emitted by the ion, and can be described as a product of the atomic ground state with a quantum superposition of two pair coherent states of motion. For general initial states of motion, the vibrational steady state of the ion is a statistical mixture. However, if the initial state of motion of the ion is prepared in the Fock state $|q, 0\rangle_F (|q+1, 1\rangle_F)$, then the vibrational steady state of the ion is described by an even (odd) PCS. In this case, the three parameters ξ , q , and ϕ that characterize the superposed PCS are determined by the intensity and phases of the driving lasers and by the “charge” and “parity” of the initial motional state, respectively.

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