Multiple prediction through inversion: A fully data-driven concept for surface-related multiple attenuation

Yanghua Wang*

ABSTRACT

This paper introduces a fully data-driven concept, multiple prediction through inversion (MPI), for surface-related multiple attenuation (SMA). It builds the multiple model not by spatial convolution, as in a conventional SMA, but by updating the attenuated multiple wavefield in the previous iteration to generate a multiple prediction for the new iteration, as is usually the case in an iterative inverse problem. Because MPI does not use spatial convolution, it is able to minimize the edge effect that appears in conventional SMA multiple prediction and to eliminate the need to synthesize near-offset traces, required by a conventional scheme, so that it can deal with a seismic data set with missing near-offset traces. The MPI concept also eliminates the need for an explicit surface operator, which is required by conventional SMA and is comprised of the inverse source signature and other effects. This method accounts implicitly for the spatial variation of the surface operator in multiple-model building and attempts to predict multiples which are not only accurate kinematically but are also accurate in phase and amplitude.

INTRODUCTION

This paper proposes a fully data-driven concept of multiple-model building for surface-related multiple attenuation (SMA). SMA is an effective demultiple technique used in seismic data processing. The earliest application of SMA, based on the autoconvolution of a single seismic trace, is found in Anstey and Newman (1966). Kennett (1979) proposed a frequency-wavenumber domain procedure for multioffset data in horizontally layered elastic media. Berkhout (1982) redefined the problem for laterally varying acoustic media. Wapenaar et al. (1990), Verschuur et al. (1992), and Verschuur and Berkhout (1997) developed an adaptive version, which they demonstrated with marine seismic examples. Kelamis and Verschuur (2000) attempted to apply SMA to land seismic data.

The concept introduced in this paper is multiple prediction through inversion (MPI). In a conventional SMA scheme, a multiple-model trace is generated by spatial convolution between the multiple removed traces in a common receiver gather and the original data traces in a common shot gather. In contrast, the MPI concept for multiple-model building is to update the multiple wavefield that was initially attenuated from the original data set to generate the multiple prediction for a new iteration, as is usually the case in an iterative inverse problem. Using this concept for multiple prediction may minimize the edge effect in conventional SMA multiple modeling and eliminate the need to synthesize near-offset traces, required by a conventional SMA scheme, so that it can handle a seismic data set with missing near-offset traces. These two weaknesses in conventional SMA are caused directly by spatial convolution in the multiple prediction.

As we know, multiple prediction in SMA is wave theory based and does not require any information about the subsurface structure below the receivers. However, conventional SMA requires a so-called surface operator to be included in multiple-model building. The surface operator is composed of the inverse source signature and other effects and is not usually available in practice. The MPI concept eliminates the need for an explicit surface operator in the multiple prediction and attempts to predict multiples accurately not only in time (kinematically) but also in phase and in amplitude.

In this paper I first introduce the MPI concept and summarize its differences from a conventional SMA scheme. I then show that the MPI formula can also be derived from the same equation used in conventional SMA. Subsequently, I compare it with conventional SMA and analyze the characteristics of the MPI concept. Finally, I demonstrate the effectiveness of the new concept using synthetic data example Pluto 1.5.

THE MPI CONCEPT

Multiple prediction through inversion

SMA is implemented iteratively and consists of a multiple-model building step and an adaptive subtraction step within each iteration. In the MPI concept, the multiple model for the

Manuscript received by the Editor January 31, 2003; revised manuscript received October 18, 2003.

*Imperial College London, Centre for Reservoir Geophysics, Department of Earth Science and Engineering, South Kensington, London SW7 2AZ, United Kingdom. E-mail: yanghua.wang@imperial.ac.uk.

© 2004 Society of Exploration Geophysicists. All rights reserved.

current iteration is built by updating the multiple wavefield attenuated in the previous iteration:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}\mathbf{M}^{(n-1)}, \quad (1)$$

where $\mathbf{M}^{(n-1)}$ is the multiple wavefield attenuated in the $(n-1)$ th iteration, $\tilde{\mathbf{M}}^{(n)}$ is the multiple model for the n th iteration, and $\mathbf{T}$ is an updating operator, as used in an iterative, linearized inverse problem.

Suppose the multiple wavefield $\mathbf{M}^{(n-1)}$ is not the final multiple and needs to be updated further. Equivalently, the demultiple result, $\mathbf{P}_0^{(n-1)}$, needs to be updated since

$$\mathbf{M}^{(n-1)} = \mathbf{P} - \mathbf{P}_0^{(n-1)}, \quad (2)$$

where $\mathbf{P}$ is the original data set. The updating steplength and direction from $\mathbf{P}_0^{(n-1)}$ to $\mathbf{P}_0^{(n)}$ can be approximated using those from $\mathbf{P}_0^{(n-2)}$ to $\mathbf{P}_0^{(n-1)}$. Therefore, the updating operator $\mathbf{T}$ is expressed as

$$\mathbf{T} = \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1}. \quad (3)$$

Combining equations (1), (2), and (3), we obtain the following MPI formula:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}), \quad (4)$$

for $n \geq 2$. Note that $\tilde{\mathbf{M}}^{(n)}$ with “ $\sim$ ” indicates the multiple model $\tilde{\mathbf{M}}$ is an approximation, differentiating from $\mathbf{M}$ without “ $\sim$ ”, the attenuated multiple wavefield. Such an approximation is the result of the approximation involved in the updating operator $\mathbf{T}$ in equation (3).

The multiple model is then subtracted from the original data matrix:

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \tilde{\mathbf{M}}^{(n)}, \quad (5)$$

where Λ is a shaping operator and $\mathbf{M}^{(n)} = \Lambda \tilde{\mathbf{M}}^{(n)}$ is the attenuated multiple wavefield after the n th iteration. Multiple subtraction (5) may be performed adaptively in the time-space domain, in which the spatial-variant operator Λ is implemented by localized adaptation (trace by trace, for example) in the subtraction phase. Because Λ is a frequency-dependent matrix, it is also time variant in the time domain.

Comparison with the conventional method

In conventional SMA, a multiple model is generated using

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A} \mathbf{P} \quad (\text{conventional SMA}), \quad (6)$$

where $\mathbf{A}$ is the surface operator. It is a convolution model, that is, a multiple-model trace is generated by the spatial convolution of $\mathbf{P}_0^{(n-1)}$ and $\mathbf{P}$. Comparing the MPI method for multiple prediction with the conventional SMA method, we observe two differences. First, the multiple model is not predicted using spatial convolution but is built by updating an actually attenuated multiple wavefield. Second, multiple prediction eliminates the need for an explicit surface operator $\mathbf{A}$.

The first difference is certainly an advantage over conventional SMA, specifically in the case of attenuating free-surface multiples in marine streamer seismic data. If a data set is recorded with a shot-leading geometry (even if a shot record

has live traces starting from zero offset), conventional SMA cannot properly predict the multiples in near-offset traces (Figure 1). An ideal seismic data set for the application of a conventional SMA scheme is that recorded with a split-spread acquisition geometry. The MPI scheme does not show such edge effects on near-offset multiple traces (Figure 1). In addition, a conventional SMA scheme is not applicable to seismic shot records with missing near-offset traces. If the shot record is obtained with a shot-leading acquisition geometry, it is necessary to synthesize near-offset traces prior to SMA processing. The MPI scheme, however, does not require that near-offset traces be synthesized (Figure 2).

The second difference is multiple prediction with or without an explicit surface operator. The surface operator $\mathbf{A}$ is physically a convolution of the inverse source signature, the free-surface reflectivity, and the effects of detector patterns and receiver ghosts (Berkhout and Verschuur, 1997). Among these items the most necessary component is the inverse source signature, since the free-surface reflectivity can be well approximated and the receiver ghost can be easily removed. A common practice in conventional SMA is to assume that the prerequisite surface operator is a scalar and to ignore its spatial variation and nonstationarity (Verschuur et al., 1992; Verschuur and Berkhout, 1997). I envision a fully data-driven SMA scheme that relaxes such a scalar assumption and estimates the surface operator implicitly in multiple prediction.

The MPI formula [equation (4)] can also be derived from the same equation used in a conventional SMA method. In the derivation shown in the next section, the surface operator $\mathbf{A}$ is estimated approximately and implicitly using the original data set $\mathbf{P}$ and the demultiple results of two iterations. Its dependency on $\mathbf{P}$ can be understood physically in that the original data set $\mathbf{P}$ contains not only information about the subsurface structure but also all of the information that constitutes the surface operator. Such a relationship between $\mathbf{A}$ and $\mathbf{P}$ may well justify our attempt to take account of the nonstationarity of the surface operator in the new SMA scheme.

DERIVATION OF THE MPI FORMULA

The MPI formula [equation (4)] may also be derived from the equation used in conventional SMA (Berkhout, 1982; Berkhout and Verschuur, 1997). This equation is expressed as

$$\mathbf{P} = [\mathbf{I} - \mathbf{P}_0 \mathbf{A}]^{-1} \mathbf{P}_0, \quad (7)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{P}_0$ is the primary response. If both $\mathbf{P}_0$ and the surface operator $\mathbf{A}$ are known, equation (7) can be used to model the seismic record $\mathbf{P}$ that consists of primary and multiple reflections recorded at the acquisition surface. In this forward modeling procedure, the primary response $\mathbf{P}_0$ acts as a subsurface model whose components are medium structure, reflectivity, raypath, etc., and $[\mathbf{I} - \mathbf{P}_0 \mathbf{A}]^{-1}$ is the operator to predict the seismic response, including all surface-related multiples.

Multiple attenuation means retrieving the primary response $\mathbf{P}_0$ from equation (7) through inversion. However, because we do not have any knowledge about the surface operator $\mathbf{A}$, the fundamental problem faced in conventional SMA is that two unknown variables, $\mathbf{A}$ and $\mathbf{P}_0$, are within one basic equation. This problem is the motivation for the derivation of the new MPI scheme.

Equation (7) can be rewritten as

$$\mathbf{P}_0 = \mathbf{P} - \mathbf{P}_0 \mathbf{A} \mathbf{P}, \quad (8)$$

which indicates the exact multiple wavefield is given by

$$\mathbf{M} \equiv \mathbf{P}_0 \mathbf{A} \mathbf{P}. \quad (9)$$

An iterative procedure is adopted by rewriting equation (8) as

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P}, \quad (10)$$

where

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P} \quad (11)$$

is the multiple-model prediction in the n th iteration and Λ is the undetermined shaping operator given in equation (5).

The surface operator $\mathbf{A}^{(n-1)}$ in equation (10) is derived as follows. First, following Berkhout and Verschuur (1997), equation

(8) is rewritten as

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n)} \mathbf{P} \quad (12)$$

to improve the accuracy of $\mathbf{P}_0$ through iteration, if the surface operator $\mathbf{A}$ is supplied. Then $\mathbf{A}^{(n)}$ is derived explicitly as

$$\mathbf{A}^{(n)} = [\mathbf{P}_0^{(n-1)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n)}) \mathbf{P}^{-1}. \quad (13)$$

Substituting $\mathbf{A}^{(n-1)}$ for the $(n-1)$ th iteration in equation (10), we finally obtain the following expression:

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}). \quad (14)$$

This is the key equation in the MPI scheme; it clearly indicates that, in the two-step SMA procedure, the multiple model $\tilde{\mathbf{M}}^{(n)}$ [equation (4)] is predicted by the multiple wavefield after the previous iteration $\mathbf{M}^{(n-1)} \equiv \mathbf{P} - \mathbf{P}_0^{(n-1)}$ and is adjusted by the operator $\mathbf{T} \equiv \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1}$.

Figure 1. The MPI method minimizes the edge effect that appears in conventional SMA. (a) A sample shot record with shot-leading geometry. (b) The gather of multiples attenuated by SMA. (c) The multiple model predicted by conventional SMA, which cannot properly predict multiples in the near-offset traces. (d) The multiple model predicted by the MPI method does not have such an edge effect.

CHARACTERISTICS

This section compares three characteristics of the MPI scheme with a conventional SMA scheme.

Accounting for spatial variations in the surface operator

The derivation of the MPI scheme starts with equation (10). In a conventional SMA scheme, which assumes the surface operator is a scalar, $\mathbf{A} = \lambda \mathbf{I}$, the corresponding equation is approximated to (Verschuur and Berkhou, 1997)

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \mathbf{P}_0^{(n-1)} \mathbf{P} \quad (\text{conventional SMA}), \quad (15)$$

where the multiple model is predicted by $\mathbf{P}_0^{(n-1)} \mathbf{P}$ and the scalar λ is included in Λ , the shaping filter for the adaptive subtraction. The MPI method [equation (10)] differs from conventional SMA because of the surface operator $\mathbf{A}^{(n-1)}$, used in building the multiple model. This is the first characteristic of the MPI method.

A conventional SMA scheme considers the surface operator as a scalar and assumes that the properties of sources and receivers do not vary during the seismic survey. When the assumption of stationary source and receiver characteristics is invalid, we must supply a prior surface operator for a good multiple prediction. In the derivation of the MPI scheme, the prerequisite surface operator $\mathbf{A}^{(n-1)}$ is replaced with the original data set $\mathbf{P}$ and the demultiple results of two previous iterations, $\mathbf{P}_0^{(n-1)}$ and $\mathbf{P}_0^{(n-2)}$. Thus, the MPI scheme takes into account the spatial variation of the surface operator implicitly in the multiple prediction.

Within each iteration of SMA, the second step is an adaptive subtraction in the time-space domain, which is typically performed within sliding windows moving along the time and space directions. Such a localized adaptation is, to a large extent, aimed at dealing with the spatial and temporal variations in the predicted multiples caused by neglecting the surface operator $\mathbf{A}$ in conventional SMA. In MPI, the iteration procedure takes advantage of the local adaptation of the multiples in the previous SMA iteration, thus building up the knowledge on

surface operator $\mathbf{A}$; conventional SMA starts all over again for each next iteration. By including $\mathbf{A}^{(n-1)}$ in the multiple model $\mathbf{M}^{(n)}$, the most nonlinear component of the problem is considered in the multiple prediction phase, reducing the nonlinearity of the problem in the subsequent multiple-subtraction phase.

Minimizing the edge effect

The second characteristic is demonstrated in Figure 1. The input shot record (Figure 1a) has live traces from zero offset to a maximum offset of about 1350 m. The attenuated multiple wavefield (Figure 1b) is the adaptation result of the MPI model. A significant difference between the two multiple models (Figures 1c and 1d) is that the model generated by a conventional SMA scheme has a strong edge effect at near-offset traces.

The edge effect is the result of the shot-leading acquisition geometry of the data set. Each of the two data matrices $\mathbf{P}$ and $\mathbf{P}_0$ represents all shot records in the frequency domain: each column is a shot record, each row is a common receiver point gather, and the main diagonal is the zero-offset section (Berkhout, 1984). In practice, the defined square data matrix is a diagonal band matrix. With the shot-leading acquisition geometry, only the upper half of the band in the diagonal band matrix is filled with data. In a conventional SMA method, the multiple model given by the matrix product $\mathbf{P}_0 \mathbf{P}$, which represents the time-domain convolution of a common receiver gather and a common shot record, produces the edge effect shown in Figure 1c. The MPI scheme, in which the multiple model is predicted by equation (4), elegantly eliminates the edge effect.

Applicability to shot records with missing near-offset traces

The MPI scheme minimizes the edge effect in the multiple model. This feature leads to the third characteristic: being applicable to shot records with missing near-offset traces.

In a conventional SMA scheme, as shown in equation (15), a multiple-model trace is generated by the convolution of a

Figure 2. The MPI method can handle a seismic data set with missing near-offset traces. (a) A sample shot record with five missing near-offset traces. (b) Predicted multiple model.

common receiver gather (a row of $\mathbf{P}_0$) and a shot gather (a column of matrix $\mathbf{P}$). Thus, one of the basic requirements for conventional SMA application is that we should have live traces at all near-offset locations, including zero offset. Any missing near-offset trace in a shot record must be preinterpolated. If a near-offset trace is missing, the first few live traces will be missed out completely, producing a much more severe edge effect in multiple prediction.

Using the MPI concept, which modifies the attenuated multiple wavefield in order to generate a new multiple model, the updating operator $\mathbf{T}$ in equation (3) is estimated by

$$\mathbf{T} = \mathbf{P}_0^{(n-1)} \mathbf{P}_0^{(n-2)H} \left[\mathbf{P}_0^{(n-2)} \mathbf{P}_0^{(n-2)H} \right]^{-1}, \quad (16)$$

which involves the matrix product $\mathbf{P}_0 \mathbf{P}_0^H$, i.e., the correlation between common receiver gathers. This matrix multiplication produces a band matrix with nonzero elements in the principal diagonal. Two such band matrices are involved in building the operator $\mathbf{T}$. Therefore, the MPI scheme is also applicable to seismic shot records with missing near-offset traces. Figure 2a displays a sample shot record in which five near-offset traces are missing. Figure 2b is the predicted multiple model. It is generated using expression (4) for each frequency component, followed by inverse Fourier transformation, producing the multiple model in the time domain. Even with five missing near-offset traces, the multiple model does not show any edge effect.

The efficiency of multiple modeling in the MPI scheme relies, to a high degree, on the calculation of the matrix $\mathbf{T}$, a modification operator indicating the updating direction of the multiple prediction. In the full solution of $\mathbf{T}$ given by equation (16), the matrix products $\mathbf{P}_0^{(n-1)} \mathbf{P}_0^{(n-2)H}$ and $\mathbf{P}_0^{(n-2)} \mathbf{P}_0^{(n-2)H}$ represent time-domain crosscorrelation and autocorrelation of

common receiver gathers. Thus, to improve the efficiency, we can assume $\mathbf{T}$ to be a diagonal band matrix and regularize it as a narrow band such as a pentadiagonal or a tridiagonal matrix. Such regularization may not only improve the computational efficiency but also stabilize the solution.

APPLICATION EXAMPLES

In this section, I demonstrate the effectiveness of the MPI demultiple technique by applying it to a synthetic data set, Pluto 1.5.

In the MPI scheme [equation (4)] where iteration starts from $n=2$, we need to know at least $\mathbf{P}_0^{(0)}$ and $\mathbf{P}_0^{(1)}$. In practice, we assume $\mathbf{P}_0^{(0)} = \mathbf{P}$. To estimate $\mathbf{P}_0^{(1)}$, we can use a less ambitious demultiple approach such as a simple and efficient moveout equation-based demultiple method (Wang, 2003). Alternatively, we can use equation (15) as in the conventional SMA, in which we need to extrapolate live traces to fill the near-offset gap (Kabir and Verschuur, 1995). The examples shown in this paper use equation (15), in which a simple moveout correction method projects the first live trace to the offset locations within the gap. In this case, because of using the MPI concept, a more accurate extrapolation method is unnecessary, since these extrapolated traces will be discarded after the first iteration.

In SMA, the iteration is performed merely to build a more accurate multiple model. The final demultiple result is the subtraction output in the last iteration. It is not a result in which multiples have been attenuated gradually through iteration. In the third iteration, we already have the estimates $\mathbf{P}_0^{(1)}$ and $\mathbf{P}_0^{(2)}$, adequate for the calculation of operator $\mathbf{T}$. Therefore, for real data processing, three iterations are usually sufficient.

As a demonstration, I apply the MPI demultiple technique to a 2D synthetic data set, Pluto 1.5, designed for benchmarking multiple attenuation algorithms and supplied by the SMAART

Figure 3. Stack section of a synthetic seismic data set in which each shot record consists of 55 live traces with five near-offset traces missing, as shown in Figure 2.

joint venture consortium (Bishop et al., 2001; Stoughton et al., 2001). Figure 3 displays the stack section of the synthetic seismic. As shown in Figure 2, each shot record consists of 55 live traces, with five near-offset traces missing.

Figure 4 shows the multiple attenuation result, using the MPI scheme. Figure 4a is the brute stack after multiple attenuation. Figure 4b shows the energy removed by MPI, i.e., the difference between the original stack section (Figure 3) and the demultiple result (Figure 4a). It shows clearly that the water-bottom multiple and the multiples of the three salt bodies have been attenuated effectively.

Figure 5 shows a real data example in which each shot record consists of 14 missing near-offset traces (the offset between 0 and 162.5 m) and 48 live traces (the offset between 175 and 762.5 m). On close examination of the demultiple result between 1500 and 3100 ms, we can see that free surface multiples are cleanly attenuated, without removing the primary. Although SMA provides an accurate multiple prediction, the effectiveness of multiple attenuation depends, to a high degree, upon the adaptive subtraction scheme adopted. I use an expanded multichannel matching filter, presented in Wang (2003).

Figure 4. (a) Stack section after multiple attenuation. (b) The multiple energy that is attenuated. Free-surface multiples of the water bottom and three salt bodies have been attenuated effectively without removing the primary reflections.

Figure 5. A real data example in which each shot record has 14 missing near-offset traces (the offset between 0 m and 162.5 m) and contains 48 live traces (the offset between 175 and 762.5 m). (a) A shot record before multiple attenuation. (b) The gather of multiples that are attenuated. (c) The shot record after multiple attenuation.

CONCLUSIONS

The concept of multiple prediction through inversion may be presented as the follows:

$$\begin{aligned}\bar{\mathbf{M}}^{(n)} &= \mathbf{T}\mathbf{M}^{(n-1)} \\ &= \mathbf{P}_0^{(n-1)}[\mathbf{P}_0^{(n-2)}]^{-1}(\mathbf{P} - \mathbf{P}_0^{(n-1)}).\end{aligned}$$

It builds a multiple model by updating the multiple wavefield that was removed from the previous demultiple iteration, rather than by the spatial convolution used in conventional SMA. It predicts a multiple model without the surface operator, which is required explicitly in conventional SMA.

In contrast to the conventional SMA method, the proposed MPI demultiple technique has the following three characteristics:

- 1) it accounts for the spatial variation of those effects constituting the surface operator by including the surface operator implicitly in multiple-model building;
- 2) it minimizes the edge effect that appears in conventional SMA multiple prediction; and
- 3) it eliminates the need to synthesize near-offset traces and can handle a data set with missing near-offset traces.

Further investigation is needed to exploit the potential of the first characteristic. Nevertheless, partially decoupling the surface-operator estimation and multiple subtraction in the adaptive subtraction phase may reduce the nonlinearity of the problem.

REFERENCES

- Anstey, N. A., and Newman, P., 1966, Part I—The sectional autocorrelation; Part II—The sectional retro-correlation: *Geophysical Prospecting*, **14**, 389–426.
- Berkhout, A. J., 1982, *Seismic migration—Imaging of acoustic energy by wave field extrapolation*: Elsevier Science Publishing Company, Inc.
- 1984, *Seismic migration—Imaging of acoustic energy by wave field extrapolation (part B, practical aspects)*: Elsevier Science Publishing Company, Inc.
- Berkhout, A. J., and Verschuur, D. J., 1997, Estimation of multiple scattering by iterative inversion, part I—Theoretical considerations: *Geophysics*, **62**, 1586–1595.
- Bishop, K., Keliher, J., Paffenholz, J., Stoughton, D., Michell, S., Ergas, R., and Hadidi, M., 2001, Investigation of vendor demultiple technology for complex subsalt geology: 71st Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 1273–1276.
- Kabir, M. M. N., and Verschuur, D. J., 1995, Restoration of missing offsets by parabolic Radon transform: *Geophysical Prospecting*, **43**, 347–368.
- Kelamis, P. G., and Verschuur, D. J., 2000, Surface-related multiple elimination on land seismic data—Strategies via case studies: *Geophysics*, **65**, 719–734.
- Kennett, B. L. N., 1979, The suppression of surface multiples on seismic records: *Geophysical Prospecting*, **27**, 584–600.
- Stoughton, D., Stefani, J., and Michell, S., 2001, 2D elastic model for wavefield investigations of subsalt objectives, deep water Gulf of Mexico: 71st Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 1269–1272.
- Verschuur, D. J., and Berkhout, A. J., 1997, Estimation of multiple scattering by iterative inversion, part II—Practical aspects and examples: *Geophysics*, **62**, 1596–1611.
- Verschuur, D. J., Berkhout, A. J., and Wapenaar, C. P. A., 1992, Adaptive surface-related multiple elimination: *Geophysics*, **57**, 1166–1177.
- Wang, Y., 2003, Multiple subtraction using an expanded multichannel matching filter: *Geophysics*, **68**, 346–354.
- Wapenaar, C. P. A., Herrmann, P., Verschuur, D. J., and Berkhout, A. J., 1990, Decomposition of multicomponent seismic data into primary *P*- and *S*-wave responses: *Geophysical Prospecting*, **38**, 633–661.