Sub-optimal Predictive Control for Satellite De-tumbling

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Rate damping in the initial acquisition phase of a magnetically controlled small satellite is a big challenge for the control system. In this phase the main difficulties are dynamic nonlinearities due to high body rates, time-varying control due to the change in the earth's magnetic field, inherent under-actuation and constraints on available power. The control system is required to minimize the de-tumbling time with minimal use of on-board resources. In comparison to the existing control techniques used in the initial acquisition phase, predictive control can be considered a suitable choice for handling such conflicting objectives in the presence of constraints. In this work performance of two existing nonlinear model predictive control (NMPC) schemes that guarantee closed-loop stability are analyzed. NMPC gives improved performance by reducing the de-tumbling time compared to classical control techniques based on the rate of change of the earth's magnetic field; however, the computational requirements are high. Further, it is demonstrated that when the body rates increase, the computational burden of NMPC to reach an optimal point becomes prohibitively large. For these situations, an algorithm is presented that allows early termination of the optimizer by imposing an additional constraint on the cost reduction. The early termination criteria of the optimizer can be chosen based on the available computational resources. The imposed cost reduction constraint also helps in further reducing the de-tumbling time. Extensive numerical simulations show that the presented algorithm works well in practice for a good range of initial body rates.

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Nomenclature

$\mathbb{R}$	set of real numbers
$\ v\ _2$	2-norm of the vector v : $\ v\ _2 := \sqrt{v^T v}$
$\ v\ _P$	weighted 2-norm of the vector v : $\ v\ _P := \sqrt{v^T P v}$
x	state vector
T	prediction horizon
t	initial time
t_f	final time
δ	sample time
$V(\cdot)$	cost function
$\ell(\cdot)$	stage cost
$F(\cdot)$	terminal cost
P, Q	weighing matrices
$x(\cdot), u(\cdot)$	state and input trajectories
$x_{\text{shifted}}(\cdot), u_{\text{shifted}}(\cdot)$	shifted state and input trajectories for warm start
$u(\cdot; x(t), t)$	input trajectory with initial state $x(t)$ at time t
$x^u(\cdot; x(t), t)$	state trajectory for the control trajectory $u(\cdot)$ with initial state $x(t)$ at time t
$u^*(\cdot)$	optimal or sub-optimal control trajectory
$V^*(x^*(\cdot), t)$	optimal or sub-optimal value function for a known control trajectory $u^*(\cdot)$
$\mathbb{U}$	set of inputs
$\mathbb{X}_f$	terminal state set

I. Introduction

The computational burden to optimally solve a nonlinear model predictive control (NMPC) problem is a major concern for practical applications. In this work, the main focus is to analyze the performance of NMPC for the satellite rate damping problem and to study the possibility of real-time implementation, which should take into account the system's computational limitations. The use of NMPC is motivated to address the conflicting objectives, such as reducing the de-tumbling time using minimal on-board power resources in

the presence of dynamic nonlinearities, under-action of magnetic actuators and time-varying control [1, 2].

A small Low-Earth-Orbit (LEO) satellite with only magnetic actuation is considered. Disturbances during the launch phase of the satellite cause a tumbling motion, which results in high body rates. This initial acquisition phase needs to be short since it may restrict many crucial deployments, such as solar panels and communication antennas. Moreover, the dependence on on-board batteries for actuation also enhance the requirement of short de-tumbling phase.

Although use of the earth's magnetic field for satellite attitude control started in the late 1970s [3], it has gained much interest in the last two decades. Magnetic actuators, known as *magnetorquers*, generate a local electromagnetic field by passing current through coils that interact with the earth's magnetic field to generate a control torque. Magnetic actuation is mainly used in LEO satellites [4, 5], because the increasing distance from earth's center decreases the generated torque for control. Moreover, magnetic actuators are easy to manage on-board the satellite due to no fuel requirements, which additionally results in weight saving and making the satellite operational life independent of fuel. These advantages make this concept very attractive, especially for satellites having strict size, weight and budget limitations, e.g. nano- and pico-satellites. However, these benefits are obtained at the price of a more challenging task to control the satellite.

The most commonly used technique for satellite attitude control using the earth's magnetic field, known as β -dot control [6], uses derivatives of the earth's magnetic field and is based on decreasing the kinetic energy of the satellite. Nonlinear control techniques proposed in [5, 7, 8] use state feedback and mainly concentrate on ensuring stability. In [9] sliding mode control has been used for the initial rate damping problem. These techniques, although popular from a practical point of view due to simple on-board implementation, do not directly address the conflicting objectives in an optimal way, such as recovery from a tumbling motion in a minimum time with minimal use of control. Some linear optimal control and MPC techniques have also been proposed in [1, 10–13] for the normal operational phase of the satellite. However, these schemes are not suitable during the tumbling phase due to high nonlinearities. NMPC can address these issues in an optimal way, although it is computationally demanding. Recently some efforts have been made using NMPC [14] for the de-tumbling phase and showed an improved performance. This work follows similar lines, but the main objective here is to make NMPC applicable for real systems with constraints on computational resources.

The performance of two existing NMPC schemes is analyzed in comparison to the β -dot control. The issues that arise in practice while solving an NMPC problem are highlighted, especially when the initial body rates are large. The main issues faced are infeasibility and the large number of iterations that the optimization solver takes to reach optimality. Limiting the maximum number of iterations may result in degraded performance due to the possibility of the cost increasing at each sample time. To overcome this problem, a new sub-optimal formulation is proposed with an additional constraint on cost reduction. Although the imposed constraint adds slightly to the computational burden of the optimizer, it is demonstrated through extensive numerical simulations that, by limiting the maximum iterations of the optimizer to a small number, the cost at each sample time decreases. The imposed cost reduction also gives significant performance improvement by reducing the de-tumbling time compared to other NMPC formulations.

II. Optimal NMPC Formulations

In the MPC literature, most of the research is dominated by establishing stability guarantees, and as a result conditions have been developed for different formulations of MPC to ensure stability [15]. The classical MPC setup uses a terminal equality constraint to guarantee stability [16]. However, such strict stability constraints add to the computational burden of the optimizer. Sometimes satisfaction of the exact equality constraint cannot be achieved in finite computational time, especially for nonlinear systems [17, 18]. Research has been done in relaxing these constraints. One common approach is to relax the strict terminal equality constraint to an inequality, where the terminal state is required to be in a terminal region. A terminal cost is also added to penalize the states at the end of the prediction horizon [19]. The terminal region needs to be invariant for the nonlinear system controlled by a local state feedback control. A similar scheme, known as dual-mode MPC, is proposed in [20]. The reference [18] discusses MPC schemes with a strict terminal equality constraint and a scheme similar to dual-mode MPC. Furthermore, in [21, 22] it has been shown that careful selection of either a terminal cost or horizon length can ensure stability of MPC. They have proposed selection strategies for both the terminal cost and horizon length in a continuous-time setting. In [23], a stabilizing discrete-time NMPC is discussed without terminal constraints, but used a terminal cost with a suitable selection of weighing matrices. Similar considerations for discrete-time systems have recently been reviewed in [24].

In this paper it is assumed that all states are measured that there is no mismatch between the model and the plant and that the control and prediction horizons are the same. Most of the discussions on the performance of the presented NMPC formulations are based on our numerical experience using a specific optimization solver, known as IPOPT [25], which uses a primal-dual interior-point algorithm to solve the optimization problem.

A. Formulation with terminal constraint

For the continuous-time nonlinear satellite attitude dynamics given in Section IV, following NMPC formulation is chosen, which ensures stability, subject to some standard regularity assumptions on the cost function, dynamics and input constraint set [19]

$$\mathbf{P}_1 \left\{ \begin{array}{l} \min_{u(\cdot)} \quad V(x(\cdot), u(\cdot), t) \\ \text{subject to} \\ \quad \dot{x}(s) = f(x(s), u(s), s), \\ \quad u(s) \in \mathbb{U}, \\ \quad x(t+T) \in \mathbb{X}_f, \end{array} \right\} \quad \forall s \in [t, t+T] \quad (1)$$

where $V(\cdot)$ is the cost function, $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n$ with $f(0, 0, s) = 0 \forall s \in [t, t+T]$, the initial state at time t is $x(t)$, which is assumed to be known, $\mathbb{U}$ is a compact subset of $\mathbb{R}^m$ containing the origin in its interior, $x(t+T) := x''(t+T; x(t), t)$, where $x''(\cdot; x(t), t)$ is the state trajectory with a given control trajectory $u(\cdot)$ and initial state $x(t)$ at time t and $\mathbb{X}_f$ is a terminal constraint set. The NMPC cost function is given by

$$V(x(\cdot), u(\cdot), t) := \int_t^{t+T} \ell(x(s), u(s)) ds + F(x(t+T)), \quad (2)$$

where $\ell(\cdot)$ is the stage cost, T is the prediction horizon and $F(\cdot)$ is a terminal cost. For the stage cost, a common choice is a quadratic cost penalizing states and inputs, given as

$$\ell(x(s), u(s)) := \frac{1}{2} (\|x(s)\|_Q^2 + \|u(s)\|_R^2), \quad \forall s \in [t, t+T], \quad (3)$$

where $Q \succeq 0$ and $R \succ 0$ are symmetric weighting matrices. For the satellite rate damping problem the quadratic stage cost is used with a suitable choice of Q and R . The state vector used in the stage cost comprises only the body rates. While evaluating the MPC performance, the case in which $\mathbb{X}_f = \{0\}$ and

$F(x(t+T)) = 0$ is also considered. On the other hand, if an inequality terminal constraint is considered, the set $\mathbb{X}_f$ needs to be an invariant set. It is also required to find a suitable terminal cost and a stabilizing controller for the terminal region. In this work, since the main interest is in bringing the satellite to rest, a rotational kinetic energy of the satellite can also be used as a terminal cost, similar to the one suggested in [14], i.e. $F(x(t+T)) := \frac{1}{2} \omega(t+T)^T J \omega(t+T)$, where J is the inertia matrix of the satellite and ω represents the satellite body rates.

Warm Starting

To speed-up computations while solving problem $\mathbf{P}_1$, a warm start strategy is used [18], i.e. at time $t + \delta$, where δ is the sample time, the input and state trajectories are initialized with the shifted control and state trajectories obtained at time t . For the last interval $[t+T, t+T+\delta]$, the control trajectory from the interval $[t+T-\delta, t+T]$ is repeated. At time t with horizon length T , let the optimal (or early terminated) control trajectory be represented by $u^*(\cdot) := u^*(\cdot; x(t), t)$ and, using this control trajectory, let the state trajectory be represented by $x^{u^*}(\cdot; x(t), t)$. Using this information, the shifted trajectories at time $t + \delta$ for the time interval $[t + \delta, t + \delta + T]$ are written as

$$u_{\text{shifted}}(s) := \begin{cases} u^*(s; x(t), t) & \forall s \in [t + \delta, t + T] \\ u^*(s - \delta; x(t), t) & \forall s \in [t + T, t + T + \delta] \\ 0 & \forall s \in [t + T + \delta, \infty) \end{cases} \quad (4)$$

$$x_{\text{shifted}}(\cdot) := x^{u_{\text{shifted}}}(\cdot; x(t + \delta), t + \delta).$$

The warm starting technique improves the optimizer's performance by providing a good initial guess, which helps the optimizer in reaching an optimal solution in fewer iterations.

B. Formulation without terminal constraint

The problem $\mathbf{P}_1$ given in (1) performs well with small initial body rates ($\leq \pm 5$ deg/s) as shown in Section V. However, when these rates increase, the number of iterations of the optimizer increases considerably. Infeasibility can also arise in these cases due to terminal equality constraint, which is mainly imposed to ensure stability. Since no other state constraint is considered, the terminal constraint is the only one that depends on the predicted state of the system. Thus, removal of the terminal constraint makes the problem

much easier to solve [23]. To ensure stability, the fact that, for a sufficiently long horizon the MPC scheme is stabilizing even without terminal cost and constraints, is utilized. Following the results presented in [22, Theorem 4], a simplified problem without terminal cost and constraint is defined as

$$\mathbf{P}_2 \left\{ \begin{array}{l} \min_{u(\cdot)} \quad V(x(\cdot), u(\cdot), t) \\ \text{subject to} \\ \dot{x}(s) = f(x(s), u(s), s), \\ u(s) \in \mathbb{U}, \end{array} \right\} \forall s \in [t, t+T] \quad (5)$$

where the state vector at initial time t is $x(t)$. The NMPC cost function is given by

$$V(x(\cdot), u(\cdot), t) := \int_t^{t+T} \ell(x(s), u(s)) ds. \quad (6)$$

Due to the removal of the terminal constraint, the optimizer load is significantly reduced at the start, as compared to the problem $\mathbf{P}_1$ with terminal constraint. However, after first few sample iterations both problems take almost the same number of iterations to solve.

III. Sub-optimal NMPC Formulation

For initial body rates less than ± 5 deg/s, both problems $\mathbf{P}_1$ and $\mathbf{P}_2$ results in similar performance if the horizon is sufficiently long (see Figure 2). However, during the de-tumbling phase of the satellite, the initial body rates can be much higher. When initial rates are larger than ± 5 deg/s, the computational effort to find a solution increases. The main reason with $\mathbf{P}_1$ is infeasibility, i.e. with limited control torque it becomes very difficult for the optimizer to find a control sequence that satisfies all constraints. With $\mathbf{P}_2$ the removal of the terminal constraint avoids infeasibility issues; however, the optimizer still needs a large number of iterations to find a solution. This increased computational load is not acceptable for the computational resources normally available in small satellites. With limited computational resources, one possible approach is to fix the maximum number of iterations of the optimizer. However, by limiting the maximum number of iterations, one can obtain a sub-optimal point that may result in an increase in the cost at each sample instant, thereby loosing stability guarantees. If the early terminated points result in an increase in the cost during the first few sample times, one may also lose the benefit of reduced de-tumbling time. This issue is further discussed in Section V.

Sub-optimality in MPC has been discussed in the literature; see for example [18, 23]. Both the references impose an additional constraint on the monotonic decrease in the cost at each sample instant and concentrate on feasibility, instead of optimality. They propose early termination of the optimization process when a feasible point is obtained. This feasible point, which ensures a monotonic decrease in the cost, also ensures closed-loop stability. The references [26, 27] discuss sub-optimality in terms of early termination of the optimizer based on the available time or maximum number of iterations. Both references treat linear systems and propose using primal barrier interior-point methods to solve the optimization problem. The reference [27] has demonstrated through numerical simulations that sub-optimal results with very small number of iterations (e.g. 5-10) perform surprisingly well. The reference [26], however, imposes an extra constraint on the cost reduction. With the availability of a feasible solution at the start, the imposed constraint ensures stability for any available computational time for linear systems, when no disturbances are considered. In the presence of disturbances, they have proposed a robust real-time technique using tube-based MPC, which ensures input-to-state stability. Most of these methods assume availability of a feasible solution at the start to ensure stability; however, this assumption is not always true in the satellite de-tumbling phase when initial body rates are large. Moreover, the proposals to terminate the optimization based on number of iterations or computational time have been tested with linear systems only.

Following similar ideas, a formulation is derived which gives an acceptable performance with early terminated sub-optimal points for the tumbling satellite. The problem $\mathbf{P}_2$ is modified by adding an additional constraint on cost reduction. To pose the cost constraint, the value function at time $t - \delta$ as $V^*(x(t - \delta), t - \delta)$ is defined, which uses the input sequence $u^*(\cdot, x(t - \delta), t - \delta)$ obtained at last time step. The proposed problem is given as

$$\mathbf{P}_3 \left\{ \begin{array}{l} \min_{u(\cdot)} \quad V(x(\cdot), u(\cdot), t) \\ \text{subject to} \\ \quad \dot{x}(s) = f(x(s), u(s), s), \\ \quad u(s) \in \mathbb{U}, \\ \quad V(x(\cdot), u(\cdot), t) \leq V^*(x(t - \delta), t - \delta) - \gamma \|x(t)\|_2^2, \end{array} \right\} \quad \forall s \in [t, t + T] \quad (7)$$

where $V(x(\cdot), u(\cdot), t)$ is the same as defined for problem $\mathbf{P}_2$ and $\gamma > 0$ is a parameter to choose, which defines

Algorithm 1 A Sub-optimal MPC Algorithm

Input/Parameters

- Dynamic equations $f(x(s), u(s), s) \forall s \in [t, t + T]$ and input constraint set $\mathbb{U}$, both satisfying conditions given in Section II A.
- $Q \succeq 0$ and $R \succ 0$ for the quadratic stage cost (3).
- Initial state and input at time t , i.e., $x(t), u(t)$.
- Tuning parameter $\gamma > 0$, maximum iterations $iter_{max}$ and cost tolerance $\epsilon_{cost} > 0$

Output

- Optimal or early-terminated sub-optimal input trajectory $u^*(\cdot)$.

Algorithm

- 1: At initial time t , solve $\mathbf{P}_2$ for $i \leq iter_{max}$, where i represents the number of iterations of the optimizer, using the given initial state and input and find an optimal or sub-optimal control trajectory $u^*(\cdot)$. Use the first δ seconds of $u^*(\cdot)$, i.e., $u^*(s; x(t), t) \forall s \in [t, t + \delta)$ for control.
 - 2: Increment time by δ . Use the warm start scheme (4) to obtain initial state and input trajectories. Solve $\mathbf{P}_3$ optimally or sub-optimally and find $u^*(\cdot)$. Implement the first δ seconds of the obtained control trajectory.
 - 3: If the value function at the last time step $V^* \geq \epsilon_{cost}$, repeat step 2, otherwise solve $\mathbf{P}_2$ instead of $\mathbf{P}_3$ in step 2.
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the required decrease in cost. The decrease in the cost is kept as a function of states only, which is a milder requirement than the stage cost, as proposed in [18].

Using the warm start scheme (4) and problem $\mathbf{P}_3$, a sub-optimal MPC strategy is given in Algorithm 1. The sub-optimal algorithm with a limit on the maximum number of allowed optimizer iterations has shown improved performance by reducing the de-tumbling time compared to other two NMPC problem, especially when initial rates are larger than ± 5 deg/s. Some numerical simulation results to demonstrate the performance are given in Section V. The improved performance with a limited number of optimizer iterations is mainly due to the imposed cost reduction constraint, which is mostly satisfied by the optimizer despite sub-optimality. The 3rd step in Algorithm 1 also gives computational benefit by removing the cost constraint when $V(x(\cdot), u(\cdot), t) < \epsilon_{cost}$. This transition may give an increase in the cost for a small number of iterations, however, since the value of the cost is already small, it does not affect the performance.

IV. Satellite Dynamics

This section describes the attitude dynamics of the satellite with magnetic actuators. In general, to model magnetic actuation, only translational/orbital dynamics is required to estimate the earth's magnetic field at the location of the satellite. The control techniques, such as predictive control, use future information of the earth's magnetic field to calculate the control action at the current time and require knowledge of the satellite position over the horizon length. In this work, since translational dynamics in the NMPC formulation is not considered, the earth's magnetic field mainly rely on a simplified approximate earth's magnetic field model [1]. To present the attitude dynamics, necessary frames required to develop and solve the differential equations are defined now [28, Page 101], [29, Page 20].

A. Frames and Coordinate Systems

Figure 1 shows a basic geometry of a satellite moving in a circular orbit around the earth's center. Here F_b is the body frame, F_o is the orbit frame and F_i is the inertial frame. Coordinate system for these frames are Cartesian, right handed and orthogonal respectively, and are defined with an origin and three bases vectors. Formal definitions for these frames are given below.

Body Frame (F_b)

The body frame is fixed within the body of the satellite with its origin at the satellite centre of mass. The z_b -axis is aligned with the extended boom. The axes x_b and y_b are fixed with respect to some installed instrument. It is also assumed that the body frame is aligned with the principal axes, i.e. the axes about which the moment of inertia matrix is diagonal.

Orbit Frame (F_o)

The orbit frame has its origin at the satellite center of mass, the z_o axis aligns with the line joining the satellite center with the earth center, the x_o axis is tangent to the orbit plane in the direction of orbital angular velocity vector for a circular orbit, as assumed in this case, and the y_o axis completes the right-handed axes system. This frame rotates at a constant rate ω_o around the earth's center.

Fig. 1 Satellite frames.

Inertial Frame (F_i)

The inertial frame, also known as the earth centered inertial (ECI) frame, has its origin at the earth center of mass. The coordinate system is defined with the x_i axis towards the Vernal Equinox, the z_i axis towards the celestial north pole and the y_i axis completes the right-handed axes system.

In the earth pointing equilibrium state, both the body and the orbit frames are aligned. In the post-launch phase, the task of the control system is to bring the satellite to this equilibrium state in a minimum amount of time. In the normal operation, the control system maintains this equilibrium, along with a desired orientation of the body axes.

Frame Transformation

Satellite equations of motion need transformations between different frames. A transformation from a frame F_j to frame F_k is represented using a transformation matrix $C_{k/j}$. All these transformations are orthogonal, i.e., $C_{j/k}^T C_{j/k} = I$ and $C_{k/j} = C_{j/k}^T$. A quaternion is also used to represent a transformation matrix. A quaternion is defined as $q \in \mathbb{R}^4 := \begin{bmatrix} \mathbf{q}^T & q_4 \end{bmatrix}^T$ such that $q^T q = 1$ is satisfied, where $\mathbf{q} \in \mathbb{R}^3 := \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T$ depends on the Euler axis of rotation, while $q_4 \in \mathbb{R}$ depends on the angle of rotation. Use of a quaternion avoids singularity issues in quaternion based kinematic update equations, which is a major concern in the

kinematic update equations based on the Euler angles [29, eq. 1.3-22a]. A transformation matrix C can be written in terms of the quaternion as [30, eq. 41, 42]

$$C := (q_4^2 - \mathbf{q}^T \mathbf{q})I + 2\mathbf{q}\mathbf{q}^T + 2q_4\mathbf{Q}, \quad (8)$$

where

$$\mathbf{Q} := \begin{bmatrix} 0 & q_3 & -q_2 \\ -q_3 & 0 & q_1 \\ q_2 & -q_1 & 0 \end{bmatrix}. \quad (9)$$

B. The State and Control Vectors

The equation of motion is given as a set of first order ordinary differential equations, given as

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{m}(t), t), \quad (10)$$

where the state vector $\mathbf{x}(t) \in \mathbb{R}^{13}$ and control $\mathbf{m}(t) \in \mathbb{R}^3$ are given as

$$\begin{aligned} \mathbf{x}(t) &:= \begin{bmatrix} (\omega_{b/o}^b)^T & (q_{b/o})^T \end{bmatrix}^T, \\ \mathbf{m}(t) &:= \begin{bmatrix} \mathbf{m}_x & \mathbf{m}_y & \mathbf{m}_z \end{bmatrix}^T. \end{aligned} \quad (11)$$

where $\omega_{b/o}^b \in \mathbb{R}^3$ is the angular velocity of F_b with respect to F_o and $q_{b/o}$ represents the quaternion for transformation from F_o to F_b . In the control vector $m(t)$, each m_x , m_y and m_z represents the dipole moment of the magnetic actuator installed about the corresponding axis of the body frame.

C. Rotational Dynamics

Using Newton's law for the rotational momentum of a satellite, i.e. $\sum \tau = {}^i \dot{\mathbf{h}}$, where $\sum \tau$ represents the total torque and ${}^i \dot{\mathbf{h}}$ is the rate of change of the angular momentum of the satellite with respect to F_i , the equation for the body rotational rates is defined with respect to F_i , expressed in the body frame [28, Page 95], namely

$${}^b \dot{\omega}_{b/i}^b = (J^b)^{-1} \left[\sum \tau^b - \omega_{b/i}^b \times J^b \omega_{b/i}^b \right], \quad (12)$$

where $\sum \tau^b$ is the sum of the gravity gradient, control and disturbance torques. The inertia matrix in the body frame can be written as $J^b = \text{diag}(J_x, J_y, J_z)$.

To write the state equations for the satellite body rates with respect to the orbit frame i.e. $\omega_{b/o} := \begin{bmatrix} P & Q & R \end{bmatrix}^T$, firstly $\omega_{b/o}$ is related to $\omega_{b/i}$, both expressed in F_b , as

$$\omega_{b/o}^b = \omega_{b/i}^b - C_{b/o} \omega_{o/i}^o, \quad (13)$$

where $\omega_{i/o}^o := \begin{bmatrix} 0 & -\omega_o & 0 \end{bmatrix}^T$ is the rate of the orbit frame with respect to F_i , and is constant for circular orbits. By taking time derivative of both sides of (13), the rotational state equations with respect to the orbit frame are written as

$$\begin{aligned} {}^b\dot{\omega}_{b/o}^b &= {}^b\dot{\omega}_{b/i}^b - \left(\frac{d}{dt} C_{b/o} \right) \omega_{o/i}^o - C_{b/o} \left(\frac{d}{dt} \omega_{o/i}^o \right) \\ &= (J^b)^{-1} \left[\sum \tau^b - \omega_{b/i}^b \times J^b \omega_{b/i}^b \right] + \Omega_{b/o}^b C_{b/o} \omega_{o/i}^o. \end{aligned} \quad (14)$$

The term $\frac{d}{dt} \omega_{o/i}^o$ is zero, because $\omega_{i/o}^o$ is a constant, while $\frac{d}{dt} C_{b/o} = -\Omega_{b/o}^b C_{b/o}$, where $\Omega_{b/o}^b$ is given as

$$\Omega_{b/o}^b = \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \quad (15)$$

Gravity Gradient Torque

The gravity gradient torque in F_b is given as [28, Page 109]

$$\tau_{gg}^b = \frac{3GM}{r^5} \{ \mathbf{r} \times J^b \mathbf{r} \}. \quad (16)$$

In this expression, $\mathbf{r}$ is the position vector from center of mass of the satellite to the earth center and r is the magnitude of position vector. Since position is not included in the states, an expression for the gravity gradient torque in terms of elements of the transformation matrix is given as [28, Equation 4.8.8, Page 109]

$$\tau_{gg}^b = \frac{3GM}{r^3} \begin{bmatrix} (J_z - J_y) c_{23} c_{33} \\ (J_x - J_z) c_{13} c_{23} \\ (J_y - J_x) c_{13} c_{23} \end{bmatrix}, \quad (17)$$

where c_{ij} is the element of the transformation matrix C at the i^{th} row and the j^{th} column.

Control Torque

The control torque is dependent on the dipole moment m of the three orthogonal coils installed on the body of the satellite and the earth magnetic field vector $\beta \in \mathbb{R}^3$ measured by a magnetometer, and is given as

$$\begin{aligned}\tau_c^b &= m^b \times \beta^b, \\ &= \mathcal{B}(\beta^b)m^b,\end{aligned}\tag{18}$$

where $\mathcal{B}(\beta^b)$ is a skew symmetric matrix in $\mathbb{R}^{3 \times 3}$ with rank equal to two. Due to this fact, magnetic actuators do not provide full controllability of the system at each time instant [7]. Moreover, the earth's magnetic field keeps on changing with the position of the satellite in the orbit. Both of these factors add to the complexity of the control problem.

D. Kinematics

The kinematic equations are required to describe the rigid body orientation, for which a quaternion is used. The differential equations for the quaternion representing the transformation from the orbit frame to the body frame are given as [28, Equation 4.7-13, Page 104]

$$\dot{q}_{b/o} = \frac{1}{2} \begin{bmatrix} 0 & R & -Q & P \\ -R & 0 & P & Q \\ Q & -P & 0 & R \\ -P & -Q & -R & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ q_4 \end{bmatrix}.\tag{19}$$

E. Earth Magnetic Field Models

To compute the control action in NMPC, information of the earth magnetic field at each point in the orbit is required. Sophisticated earth magnetic field models, such as the International Geomagnetic Reference Field (IGRF)[31], give good estimates of the earth magnetic field, but are computationally complex. The lower order models, such as the first order IGRF, are computationally less expensive; however, they require satellite position information. Since translational dynamics is not considered in this work, an approximate model is used which does not use the position information, but instead gives the earth's magnetic field in the

orbit frame, given as [1]

$$\begin{bmatrix} \beta_x^o \\ \beta_y^o \\ \beta_z^o \end{bmatrix} = \frac{GM}{r^3} \begin{bmatrix} \cos \omega_o t \sin i_m \\ -\cos i_m \\ 2 \sin \omega_o t \sin i_m \end{bmatrix}, \quad (20)$$

where i_m is the inclination of orbit plane with respect to the geomagnetic equator and ω_o is the orbit rate. This type of model is very useful for simplified analysis or MPC algorithms, which need the earth magnetic field information for calculating control laws. However, one should carefully take into account the uncertainties arising due to approximate models.

F. State Estimation and Actuation

No direct measurement of attitude and rates is considered in this work. The state estimation mainly relies on static attitude estimation. It is assumed that only two measurements are available, namely the earth magnetic field and the sun vector. The earth magnetic field is measured with magnetometers installed on the satellite body. The sun vector is sensed by a pair of sun sensors also installed on the satellite body. The attitude is obtained by solving the Wahba problem [32] using a standard algorithm, for example QUEST [30], ESOQ [33]. During the initial acquisition phase of these small satellites, the level of uncertainties is large, possibly giving large errors in estimated attitude. In this situation, the robust static attitude algorithm presented in [34] is useful to reduce the errors to a smaller band. Using this estimated attitude in some nonlinear filtering technique, such as a Kalman filter, rates can also be estimated. With only two sensors, attitude estimation is not reliable during an eclipse due to the unavailability of a sun sensor measurement. In such a situation, a modified Kalman filter, as proposed in [35, 36], may be used.

For control, three magnetic actuators, called *magnetorquers*, installed about each body axis, are used. Currents passing through these magnetorquers generate magnetic field, which interact with the earth's magnetic field to generate control torque.

G. Extensions for Elliptic Orbits

The attitude dynamics in Section IV C and the earth magnetic field given in Section IV E assume a circular orbit. The reason for choosing a circular orbit is that orbits of many small earth pointing satellites in LEO are either circular or near-circular. Moreover, the circular orbit assumption makes the NMPC based

attitude control problem formulation simple by considering only attitude dynamics. However, there are some missions in which elliptic orbits with moderate eccentricities are desirable. In elliptic orbits the angular velocity is time-varying, which results in a time-varying pitch torque, given as $\tau_d = J_y \dot{\omega}_o$ [37]. This disturbance torque is proportional to the eccentricity of the orbit. Moreover, the earth magnetic field model will need position information, which requires solution of translational equations in the NMPC formulation, making it computationally more complex. To consider both of these factors, the NMPC formulation needs the following modifications:

- Attitude dynamic equations need to include the pitching torque, which is proportional to the rate of change of the angular velocity of the orbit.
- Orbital dynamic equations need to be solved along with attitude dynamics.
- A position-based earth magnetic field model will be used, while the position information is obtained by solving the orbital dynamic equations. In this regard, a simplified dipole earth magnetic field model in the orbit frame can be used, as in [37, eqn 5]

However, significant simplifications are obtained if an average orbit rotation rate $\bar{\omega}_o$ for an elliptic orbit is assumed. With constant orbit rate there will be no pitch disturbance torque. Moreover, a slightly modified earth magnetic field model can be used, given as

$$\beta^o = \frac{GM}{a^3} \begin{bmatrix} \cos \bar{\omega}_o t \sin i_m \\ -\cos i_m \\ 2 \sin \bar{\omega}_o t \sin i_m \end{bmatrix}, \quad (21)$$

where a is now the semi-major axis of the elliptic orbit.

V. SIMULATION RESULTS

A low cost CubeSat [38] is considered, which is a pico-satellite moving in a circular orbit at an average altitude of 650 km above the earth surface. The nonlinear simulations are based on the data given in Table 1, which is taken from [39]. To solve the NMPC problem, a software package is used for large scale nonlinear optimization problems, known as IPOPT (Interior Point OPTimizer) [25], along with a MATLAB based toolbox ICLOCS (Imperial College London Optimal Control Software) [40]. IPOPT implements a primal-dual interior point method for nonlinear optimization. ICLOCS is used to transcribe the infinite-dimensional

Table 1 Data used in the nonlinear simulation.

Parameter	Value	Unit
Orbit Parameters		
Radius	650	km
Inclination	72	deg
Mass Properties		
Dimensions ($x \times y \times z$)	$10 \times 10 \times 30$	cm^3
Mass	3.0	kg
Moment of inertia about x-axis	0.023001	kgm^2
Moment of inertia about y-axis	0.023565	kgm^2
Moment of inertia about z-axis	0.004197	kgm^2
Control Parameters		
Maximum dipole moment	0.1	Am^2

optimal control problem into a finite-dimensional approximation. It offers three transcription methods, i.e. discrete, multiple-shooting and direct collocation. A direct collocation method is used to discretize the continuous-time system dynamics using one of the three available integration methods in ICLOCS, namely Euler, trapezoidal and Hermite. The results presented in this section are based on simulations that use the trapezoidal method.

In these simulations the control command m at each time step is the first element of the optimal or sub-optimal control trajectory $u^*(\cdot)$, computed by solving an NMPC problem.

A. Performance Comparison Between Solving Problems $\mathbf{P}_1$ and $\mathbf{P}_2$

A performance comparison between solving problems $\mathbf{P}_1$ and $\mathbf{P}_2$ for small initial body rates (less than ± 5 deg/s) is given in Figure 2. A horizon length of 480 seconds has been chosen with a sample period of 3 seconds. The results indicate that the chosen horizon length is sufficiently long and, as a result, solving problem $\mathbf{P}_2$ gives almost similar performance as solving $\mathbf{P}_1$. It can be observed from the second subplot, where the number of iterations is plotted against time, that solving problem $\mathbf{P}_1$ requires around 250 iterations for the first sample time, while $\mathbf{P}_2$ requires around 80. However, once a feasible point is obtained to warm

Fig. 2 Performance comparison between solving P_1 and P_2 .

start the state and control trajectories at the next sample time, the number of iterations drops significantly and is almost the same for both formulations.

B. Comparison with β -dot Control

One of the most commonly used control schemes for satellite rate damping is known as β -dot control [6], which uses the rate of change of the earth's magnetic field and is based on the principle of reducing the rotational kinetic energy of the tumbling satellite. The main reason for the popularity of this scheme is simple measurement requirements, i.e., the rate of change of the earth's magnetic field with respect to the body frame. The earth's magnetic field in the body frame is measured by magnetometers and its derivative is calculated numerically. A comparison between β -dot control and solving NMPC problem P_1 is shown in Figure 3. For β -dot control, a well-tuned controller gain is selected. The initial rates and gain used for β -dot controller and some important parameters for the NMPC are given in Table 2. It is observed that the β -dot control damps the body rates to zero in about half of the orbit time (≈ 3000 seconds), while the NMPC scheme generates an optimal control command, keeping in view the future response of the satellite over the

Fig. 3 Performance comparison between solving P_1 and using β -dot control.

Table 2 Parameters for P_1 and the β -dot control.

Parameter Type	Value
Initial body rates	$P = 0.04 \text{ rad/s}$, $Q = 0.02 \text{ rad/s}$, $R = -0.03 \text{ rad/s}$
β -dot gains	$K_x = 5 \times 10^5$, $K_y = 5 \times 10^5$, $K_z = 5 \times 10^5$
MPC parameters	$T = 480$, $\delta = 3$, $Q_c = I$, $R_c = 1 \times 10^{-6} I$

chosen horizon. It can be observed from Figure 3 that the reduction in the rotational kinetic energy of the satellite is much faster with NMPC as compared to β -dot control. For control energy, it can be observed from Figure 3 that the NMPC controller use more control at start; however, it quickly settles to small values, while the β -dot controller uses more control energy due to a longer de-tumbling time.

C. Suboptimal MPC Performance

With large initial body rates problem P_1 faces infeasibility at the start. To demonstrate this issue two tests are performed with initial rates $P = 6.9 \text{ deg/s}$, $Q = -9.7 \text{ deg/s}$ and $R = 6.3 \text{ deg/s}$. In the first test the

Fig. 4 Performance of the IPOPT solver while solving NMPC problem P_1 and P_2 with initial rates $P = 6.9$ deg/s, $Q = -9.7$ deg/s and $R = 6.3$ deg/s. The comparison is given for two cases, i.e., the maximum number of iterations set to 300 and 50.

maximum number of IPOPT iterations has been fixed to 300, while in the second test to 50. The comparison is given in Figure 4. It can be observed that with 300 iterations, problem P_1 remains infeasible for the first 150 seconds. Problem P_2 without terminal constraint also faces infeasibility for the first few iterations, mainly because of numerical reasons; however, the solution becomes feasible early on. However, during the time when the solution is sub-optimal due to early termination, the cost value increases to almost double the initial value. Afterwards, the decrease in the cost is slow, which results in a degraded rate damping response. A similar response is observed when the maximum number of iterations is limited to 50. However, by imposing the additional cost reduction constraint, a significant improvement in performance is observed. A comparison of the performance given by solving problem P_3 via Algorithm 1 is shown in Figure 5. Due to the additional constraint in Algorithm 1, the cost decreases even when the optimizer terminates early, i.e., the iterate is sub-optimal.

Two more tests have been performed to further analyze the performance. Firstly, the effect of reducing

Fig. 5 Comparison between solving problem P_2 and P_3 with the maximum number of iterations set to 50.

the number of maximum iterations was studied by limiting the maximum number of iterations to 50, 20 and 10. Secondly, the effect of γ in (7) is studied. The results are shown in Figure 6. It can be observed that reducing the maximum number of iterations to a very small number may degrade performance. With decreasing γ , the rate of cost decrease is not as expected, which shows that for large γ the cost reduction constraint is not being satisfied.

Lastly, to show the effect of the cost reduction constraint, a comparison of decrease in the rotational energy of the satellite using β -dot control, solving problem P_2 and problem P_3 via Algorithm 1 is presented in Figure 7. For this test, initial body rates are kept less than ± 5 deg/s. With these initial rates problem P_2 gives an optimal point for all time steps, while in problem P_3 via Algorithm 1 the maximum number of iterations is fixed to 20. It can be observed from the plot that due to the cost reduction constraint, Algorithm 1 results in a faster reduction in the rotational kinetic energy compared to solving problem P_1 and β -dot control, although this is obtained at the expense of more control energy. However, since the states and control go to zero much faster, the overall power consumption is less.

Fig. 6 Performance of sub-optimal MPC with different maximum iterations (left) and γ (right).

Fig. 7 Rotational energy reduction of a well-tuned β -dot controller compared to solving problem P_2 and problem P_3 via Algorithm 1.

VI. Conclusions

Nonlinear Model Predictive Control (NMPC) has been used to damp out the body rates of a tumbling small satellite using magnetic actuators. The NMPC formulations, which guarantee stability via strict terminal equality constraint or sufficiently long horizon, give similar performance for small initial body rates (less than ± 5 deg/s). However, with larger initial rates, which are possible in real satellites, these formulations either face infeasibility or the optimizer needs a large number of iterations, which is not acceptable from a practical point of view due to limited computational resources. To avoid this problem, a sub-optimal formulation with an additional constraint on cost reduction is proposed. Due to the cost reduction constraint, the proposed formulation results in faster rate damping in practice with a fixed limit on the number of iterations. However, due to early termination, no theoretical guarantees can be given on the performance or cost decrease at each sample time. Hence, a further study to establish these guarantees is a possible future research direction. Moreover, extensions of the presented formulations for elliptic orbits is also a possible future work.

This study may also be useful for the designers working with classical control techniques, which are preferred by industry due to simple implementation. The classical algorithms may be redesigned or re-tuned by looking at the control inputs generated by solving the NMPC problems.

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