Multiple prediction through inversion: Theoretical advancements and real data application

Yanghua Wang¹

ABSTRACT

Wave-equation-based multiple attenuation seismic methods may be divided into the two distinct phases of multiple modeling and multiple subtraction. These two are interrelated and must be optimized in order to produce an optimal final result. The multiple prediction through inversion (MPI) scheme updates the multiple model iteratively, as we usually do in a linearized inverse problem. The scheme models the multiple wavefield without an explicit knowledge of surface and subsurface structures or of the source signature; both are generally unknown in seismic surveys. However, compared to a conventional surface-related multiple attenuation method, the accuracy of the multiple model is improved both kinematically and dynamically. It is because the MPI scheme implicitly takes account of the spatial variation of the surface reflectivity, the source signature, the detector patterns and receiver ghosts, and other effects included in the so-called surface operator. When the MPI scheme is used in the first phase it also significantly reduces the nonlinearity of the problem in the second phase that involves attenuating multiples without removing or altering primaries. The effectiveness of the MPI scheme is demonstrated by examples involving real marine seismic data.

INTRODUCTION

The attenuation of multiple reflections in surface seismic data has been a high priority in research and development in both operating and service companies for many years. Many partial solutions exist that are able to ameliorate the problem in specific circumstances. This is particularly the case for marine seismic data. There is, however, no method or group of methods that works in all cases; in many cases, multiple contamination is still the controlling limit on noise

and resolution. This is true for surface-related multiples on land seismic data and for interbed multiples on all data.

Among various multiple attenuation techniques, a suite of free-surface multiple attenuation (SMA) methods has been developed that seeks to predict the multiple wavefield directly from the seismic field data. These methods use minimal assumptions about the structure of the subsurface and then seek to subtract this (imperfect) prediction from the field data in some optimum manner (Verschuur et al., 1992; Verschuur and Berkhout, 1997; Weglein et al., 1997; Dragoset and Jerićević, 1998; Luo et al., 2003; Guitton, 2005). In many circumstances, particularly those involving moderately two-dimensional structures on marine data without strong interbed multiples (as conventional SMA methods treat the interbeds as primaries), these methods work well (Hadidi et al., 2002). On many other data sets, especially those acquired on land (Kelamis and Verschuur, 2000), those that are monotonously one-dimensional with strong interbed multiples, and those that are highly three-dimensional, existing multiple-suppression methods are often inadequate (Baumstein et al., 2005). This is particularly important when the utility of the interpretation requires maximum spatial resolution of weak primaries or depends upon the interpretation of subtle characteristics of the data extracted prestack.

For SMA methods, the multiple attenuation procedure may be divided into the two distinct phases of multiple prediction and multiple subtraction. These two parts are interrelated, and both must be optimized in order to produce an optimal final result. Wang (2003a, 2004) has proposed two complementary, novel, and promising concepts for multiple attenuation: (1) multiple prediction through inversion for multiple modeling and (2) expanded multichannel matching for multiple subtraction. These yield both improved prediction and improved subtraction. Together they appear to overcome some of the limitations of the previously mentioned existing methods.

The core of this series of multiple attenuation techniques is the multiple prediction through inversion (MPI) concept for modeling the multiple wavefield. The MPI scheme refines the multiple model through iteration, just as we usually do in an iterative inverse problem. Further, I offer two theoretical justifications of the MPI con-

Manuscript received by the Editor February 12, 2006; revised manuscript received August 29, 2006; published online January 11, 2007.
¹Imperial College London, Department of Earth Science and Engineering, Centre for Reservoir Geophysics, London, United Kingdom. E-mail: yanghua.wang@imperial.ac.uk.
© 2007 Society of Exploration Geophysicists. All rights reserved.

cept. First, consider the multiple attenuation as an optimization problem. For each iteration, the cost function is optimized from one level to the next by updating the current model (wavefield). A desired updating direction should be normal to the next level (contour) of the cost function. In the MPI concept, searching for the desired updating direction can be implemented in two steps: (1) setting a preliminary updating direction that emits normally from the current level (contour) of the cost function and (2) then adjusting the direction by a proper adaptive subtraction, so that the final updating direction has a normal incident angle impinging at the next level (contour) of the cost function.

The second theoretical justification is that the MPI scheme takes into account the surface operator implicit in multiple modeling. The surface operator consists of effective source characteristics, reflectivity of the multiple-generating interface, and many other effects (Berkhout, 1982, 1984). Almost all methods now in use require explicit knowledge of these effects, which are problematic and highly variable spatially. Because the MPI scheme implicitly takes account of these effects, it generates a multiple model with both kinematical and dynamically improved accuracy. It also significantly reduces the nonlinearity of the problem in the subsequent adaptive subtraction phase.

MULTIPLE PREDICTION THROUGH INVERSION

The MPI concept

This section provides an alternative but concise derivation to the MPI formula presented in Wang (2004) for multiple modeling.

Consider the multiple attenuation problem as an optimization problem and denote $\mathbf{P}$ to represent the data set consisting of primaries and multiples and $\mathbf{P}_0$ to represent the desired data set without any multiples. The optimization is performed iteratively, and $\mathbf{P}_0^{(n)}$ is the multiple attenuation result after the n th iteration.

Suppose that after the $(n-1)$ th iteration the multiple attenuation result $\mathbf{P}_0^{(n-1)}$ is obtained from the previous iteration result $\mathbf{P}_0^{(n-2)}$; a

Figure 1. In the MPI scheme, the modeling phase is represented by vector $\mathbf{P}_0^{(n-1)} \rightarrow \tilde{\mathbf{P}}_0^{(n)}$, which has a normal emitting angle emerging from the contour $C^{(n-1)}$, and the adaptive subtraction phase is represented by vector $\tilde{\mathbf{P}}_0^{(n)} \rightarrow \mathbf{P}_0^{(n)}$, which finally makes vector $\mathbf{P}_0^{(n-1)} \rightarrow \mathbf{P}_0^{(n)}$ normal to the contour $C^{(n)}$. The $\mathbf{T}^{(n-1)}$ operator represents the previous iteration $\mathbf{P}_0^{(n-2)} \rightarrow \mathbf{P}_0^{(n-1)}$, which has a normal incident angle impinging at the contour $C^{(n-1)}$.

convolution operator $\mathbf{T}^{(n-1)}$ may be introduced to link $\mathbf{P}_0^{(n-1)}$ and $\mathbf{P}_0^{(n-2)}$:

$$\mathbf{P}_0^{(n-1)} = \mathbf{T}^{(n-1)} \mathbf{P}_0^{(n-2)}. \quad (1)$$

The updating operator $\mathbf{T}^{(n-1)}$ can be estimated as

$$\mathbf{T}^{(n-1)} = \mathbf{P}_0^{(n-1)} (\mathbf{P}_0^{(n-2)})^H [\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H]^{-1}. \quad (2)$$

This $\mathbf{T}^{(n-1)}$ operator is borrowed for the n th iteration to predict $\mathbf{P}_0^{(n)}$,

$$\tilde{\mathbf{P}}_0^{(n)} = \mathbf{T}^{(n-1)} \mathbf{P}_0^{(n-1)}, \quad (3)$$

where $\tilde{\mathbf{P}}_0^{(n)}$ represents an approximation because $\mathbf{T}^{(n-1)}$ is estimated from the previous iteration result and is not the accurate $\mathbf{T}$ operator for the current iteration ($\mathbf{T}^{(n)}$).

Equation 3 for primary prediction can easily be transferred to an equation for multiple prediction. Following Berkhout and Verschuur (1997), the free-surface multiple wavefield $\mathbf{M}$ may be defined as

$$\mathbf{M} = \mathbf{P}_0 \mathbf{A} \mathbf{P}, \quad (4)$$

where $\mathbf{A}$ is a so-called surface operator, physically a convolution of the inverse source signature, the free-surface reflectivity, effects of detector patterns and receiver ghosts, and many other effects (Berkhout, 1982, 1984). Equation 4 may be used to rewrite equation 3 as

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}^{(n-1)} \mathbf{M}^{(n-1)}, \quad (5)$$

where $\tilde{\mathbf{M}}^{(n)} = \tilde{\mathbf{P}}_0^{(n)} \mathbf{A} \mathbf{P}$ and $\mathbf{M}^{(n-1)} = \mathbf{P}_0^{(n-1)} \mathbf{A} \mathbf{P}$. If equation 2 is substituted into equation 5, the MPI formula is obtained as

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} (\mathbf{P}_0^{(n-2)})^H [\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}), \quad (6)$$

for $n \geq 2$.

Once the multiple model $\tilde{\mathbf{M}}^{(n)}$ is generated, an adaptive subtraction scheme can be used to obtain the multiple attenuation result as

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \mathbf{A} \tilde{\mathbf{M}}^{(n)}, \quad (7)$$

where $\mathbf{A}$ is the adaptive filter presented in the frequency domain, but commonly is estimated in the time domain as a multichannel shaping filter (Wang, 2003a). In an iterative multiple attenuation scheme, equations 6 and 7 are the two phases of modeling and subtraction that form one iteration.

The justification for the use of the $\mathbf{T}^{(n-1)}$ operator is illustrated in Figure 1. The previous iteration has made the best effort to obtain a result ($\mathbf{P}_0^{(n-1)}$). The best effort means that vector $\mathbf{P}_0^{(n-2)} \rightarrow \mathbf{P}_0^{(n-1)}$ is a normal incident vector impinging on $C^{(n-1)}$, the contour of the cost function that is optimized locally. Vector $\mathbf{P}_0^{(n-2)} \rightarrow \mathbf{P}_0^{(n-1)}$ is the $\mathbf{T}^{(n-1)}$ operator to which I refer. In the current iteration, the multiple modeling phase $\tilde{\mathbf{P}}_0^{(n)} = \mathbf{T}^{(n-1)} \mathbf{P}_0^{(n-1)}$ means that vector $\mathbf{P}_0^{(n-1)} \rightarrow \tilde{\mathbf{P}}_0^{(n)}$ has a normal emitting angle emerging from the contour $C^{(n-1)}$, and the adaptive subtraction phase means wavefield moderation from $\tilde{\mathbf{P}}_0^{(n)}$ to $\mathbf{P}_0^{(n)}$. The latter determines the step length and also tunes the update direction, so that a new vector $\mathbf{P}_0^{(n-1)} \rightarrow \mathbf{P}_0^{(n)}$ (i.e., the $\mathbf{T}^{(n)}$ operator) finally impinges on contour $C^{(n)}$ with a normal incident angle.

Improvements to conventional SMA

A conventional SMA assumes that the surface operator $\mathbf{A}$ in equation 4 is a scalar and predicts multiples by using the following spatial convolution (Verschuur and Berkhout, 1997):

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{P}. \quad (8)$$

Then an adaptive subtraction phase partially recovers the effect of the surface operator $\mathbf{A}$ omitted in equation 8. The next iteration throws away this effort and again starts multiple modeling with the same assumption. By contrast, the effort made in the previous iteration for recovering the spatial variation in the surface operator $\mathbf{A}$ has been kept in the MPI scheme.

The MPI scheme modifies the multiple wavefield obtained in the previous iteration to build a multiple model for a new iteration, as shown in equation 5. The MPI scheme can be understood as

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P}, \quad (9)$$

and

$$\mathbf{A}^{(n-1)} \approx [\mathbf{P}_0^{(n-2)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}) \mathbf{P}^{-1} \approx [\mathbf{P}_0^{(n-1)}]^{-1} - \mathbf{P}^{-1}. \quad (10)$$

Detailed derivation can be found in Wang (2004). Even though the $\mathbf{A}^{(n-1)}$ operator reconstructed from the previous iteration is not sufficiently accurate, including it in multiple modeling will improve the accuracy of the modeled multiple wavefield $\tilde{\mathbf{M}}^{(n)}$ kinematically and dynamically, if compared with the conventional SMA in equation 8. Meanwhile, it significantly reduces the nonlinearity of the problem for the adaptive subtraction. In MPI, the surface operator is not explicitly presented in multiple modeling, but is included implicitly in equations 5 and 6.

MPI starts from $n = 2$, for which $\mathbf{P}_0^{(0)}$ and $\mathbf{P}_0^{(1)}$ are needed for estimating the first $\mathbf{T}$ operator $\mathbf{T}^{(n-1)} = \mathbf{T}^{(1)}$. In practice, $\mathbf{P}_0^{(0)} = \mathbf{P}$, the original data matrix, is often assumed. For $\mathbf{P}_0^{(1)}$, either a conventional SMA or moveout equation-based method (Wang, 2003a) can be used to model the multiple, followed by an adaptive subtraction. Other methods such as Radon-domain multiple attenuation (Wang, 2003b) can also be used to obtain $\mathbf{P}_0^{(1)}$ directly. Although a conventional SMA method could be used to initiate the iteration, the central idea of the MPI scheme is that at each iteration the accuracy of multiple modeling is maximized, so that the subsequent subtraction can be optimized or linearized.

In MPI, an operator $\mathbf{T}$ is used to modify the multiple wavefield obtained in the previous iteration. Within the $\mathbf{T}$ operator (equation 2), the calculation involves auto- and crosscorrelations of common-receiver gathers, rather than the spatial convolution between shot gathers (equation 8) used in a conventional SMA method. Because the MPI scheme is not formulated explicitly in terms of a spatial convolution, the scheme manages to minimize the edge effects that appear in a conventional SMA multiple prediction (Wang, 2004). This leads to greater stability and accuracy and is of particular relevance to land acquisition, where the shooting geometry is often controlled by the terrain rather than by the requirements of a particular processing algorithm. Because of this property, the MPI scheme also eliminates the need to synthesize near-offset traces, required by a conventional SMA scheme, and so it can work for a seismic data set with missing or noisy near-offset traces (Wang, 2004).

Figure 2a displays a data matrix $\mathbf{P}$ in the frequency domain. Each column represents a shot record, and each row a common-receiver

gather (Berkhout, 1982). Figure 2a shows the amplitude values of all traces for a single-frequency component. Note that the dashed line indicates the zero-offset position; in this example, five near-offset traces were missing. A conventional SMA method would require filling out the near offsets (i.e., the data around the main diagonal) and even some of the offsets at the other side of the diagonal. In the MPI method, the $\mathbf{T}$ operator is nearly a diagonal-band matrix, as shown in Figure 2b-d, the $\mathbf{T}^{(n-1)}$ operators for $n = 2, 3$, and 4, respectively. Equation 5 indicates that such a narrow-band diagonal matrix acts as complex-valued weighting on the multiple-wavefield matrix; therefore, there is no edge effect, and missing near-offset data does not affect the $\mathbf{T}$ operator.

The $\mathbf{T}$ operator, calculated by using equation 2, is nearly a narrow-band matrix. Wang (2004) suggests approximating the $\mathbf{T}$ operator as a tridiagonal matrix and then speeding up the computation significantly. In addition, the $\mathbf{T}$ operator converges to a diagonal matrix. Such convergence from $n = 3$ to $n = 4$ is marginal (see Figure 2c and d); thus three iterations are usually sufficient for real data processing, as indicated by Wang (2004).

Figure 3a is a sample shot record, a column vector of data matrix $\mathbf{P}$ in the frequency domain. Figure 3b-d displays the predicted multiple model of the first, second, and third iterations, a column vector of $\tilde{\mathbf{M}}^{(1)}$, $\tilde{\mathbf{M}}^{(2)}$, and $\tilde{\mathbf{M}}^{(3)}$, respectively. Figure 3e-g shows the multiple attenuated gather after the first, second, and third iterations, a column vector of $\mathbf{P}_0^{(1)}$, $\mathbf{P}_0^{(2)}$, and $\mathbf{P}_0^{(3)}$, respectively. Note that the residuals at $t = 1500$ and 4000 ms at near offsets are suppressed through iterations. Figure 3h is the difference between Figure 3a and 3g (i.e., the multiple energy attenuated after the third iteration, a column vector of $\mathbf{M}^{(3)}$ in the frequency domain). This shot-gather example is arbitrarily selected from a synthetic data set, Pluto 1.5. Stacked sections before and after MPI multiple attenuation were shown in Wang (2004).

Figure 2. (a) A data matrix in which each column represents a shot record in the frequency domain and each row is a common receiver gather. In this example, five near-offset traces are missing, and a diagonal line indicates the zero-offset positions. (b-d) The $\mathbf{T}^{(n-1)}$ operators for $n = 2, 3$, and 4, respectively.

REAL DATA EXAMPLES

This section demonstrates the effectiveness of the MPI scheme by using two real data examples from marine seismic surveys. In the first real data set, the original shot interval is 50 m, four times the receiver interval, which is 12.5 m. In practice, an SMA technique requires a shot at every receiver station. Shot interpolation is performed by using the f - x - y domain trace-interpolation method (Wang, 2002), where f is the frequency and x and y are the two spatial axes representing the shot location and the source-receiver offset, respectively. The f - x - y domain trace-interpolation method can relax the requirement that events must be linear in an f - x domain method. Events that are nonlinear in one direction but linear in another may be interpolated exactly with an f - x - y interpolation filter.

Figure 4a displays a sample shot gather, consisting of 14 missing near-offset traces (the offset between 0 and 162.5 m) and 48 live traces (the offset between 175 and 762.5 m). Figure 4b and c shows a multiple attenuation gather and the attenuated multiple energy. Gather in the zoomed-in version in Figure 4 show clearly that overlapping primaries and multiples can be well separated in this processing.

It is worthwhile to point out that although MPI provides an accurate multiple prediction, the effectiveness of multiple attenuation depends upon the adaptive subtraction scheme adopted. Here, we use the expanded multichannel matching method (Wang, 2003a). For a

normal multichannel matching, where an original seismic trace is matched by a group of multiple-prediction traces, the lateral coherence of adjacent traces is typically exploited to discriminate between overlapping multiple and primary reflections. In the expanded multichannel matching, a seismic trace is matched not only by a group of predicted multiple traces, but also by a number of mathematically related traces. If the convolution coefficients associated with the normal multichannel filter are presented as a 2D operator in the time-space domain, this 2D operator is now expanded with an additional spatial dimension to improve the robustness of the adaptive subtraction.

In the expanded multichannel matching, the mathematically related traces include the first derivatives, the Hilbert transforms, and the first derivatives of their Hilbert transforms. Differentiation with respect to time is equivalent to multiplication by frequency in the frequency domain. Because multiplication by frequency enhances higher relative to lower frequencies, it can sometimes change the appearance of a seismic trace dramatically. Therefore, use of the expanded traces will improve the matching resolution in time.

Figure 5a is the stack section of the first real data set, which shows a complicated structure under the water bottom. The water depth is about 525–675 m (700–900 ms in two-way traveltime). Figure 5b presents the stack section after multiple attenuation, and Figure 5c displays the attenuated multiple energy. On close examination of the multiple attenuation result between 1500 and 2100 ms, it can be seen

that the spatial structure of weak reflections with lateral continuation has been revealed. Free-surface multiples have been cleanly attenuated, without removing the primary. The three sections in Figure 5 are plotted by using the same amplitude scale.

Figure 6 shows another real data example. Each shot gather consists of 100 channels with 12.5-m intervals; the shot spacing is 25 m. The f - x - y trace interpolation is used again to generate a new shot gather between a pair of shots. This makes the final shot interval 12.5 m, the same as the receiver interval. Figure 6a shows the brute stack before multiple attenuation, Figure 6b displays the multiple attenuation result using the MPI scheme presented in this paper, and Figure 6c illustrates the energy removed by MPI [i.e., the difference between the original stack section (Figure 6a) and the multiple attenuation result (Figure 6b)].

DISCUSSION

Examples have demonstrated that the MPI scheme can work for seismic data with missing near-offset traces. This is one of the advantages compared to conventional SMA methods. For marine seismic reflection data, if the source signature and the near-surface reflectivity can be assumed to be approximately constant, another difference between MPI and conventional SMA methods is that MPI effectively includes deghosting, whereas SMA does not. A conventional SMA needs a proper deghosting prior to multiple modeling (Verschuur et al., 1992). For a deghost-

Figure 3. (a) A sample shot record with five missing near-offset traces. (b, c, d) Predicted multiple models of the first, second, and third iteration. (e, f, g) Multiple-attenuated gathers after the first, second, and third iteration. (h) The multiple energy attenuated after the third iteration, i.e., the difference between (a) and (g).

Figure 4. (a) An original shot gather. (b) The shot gather after multiple attenuation. (c) The attenuated multiple wavefield, i.e., the difference between (a) and (b).

ing example, readers may refer to Kragh et al. (2004). For the source signature, Ziolkowski et al. (1999) suggest measuring the array-source wavefield during marine seismic data acquisition to remove sea-surface multiples.

In the derivation of the MPI formula, the prerequisite surface operator is represented in terms of the original data set, which in theory contains all the information constituting the surface operator. Thus the MPI scheme takes account of the unknown and potentially complicated spatial variation of those effects composed in the surface operator — both the source characteristics and the multiple-generating interface. The potential of the MPI concept may be exploited in the following two areas: land seismic data and internal multiple attenuation.

Figure 5. (a) Brute stack of a real seismic data set, before multiple attenuation. (b) Stack section after MPI multiple attenuation. (c) The section of attenuated multiples, i.e., the difference between (a) and (b).

Land seismic data, in most cases, do not have available information about the source signature, directivity, and polarization, which are needed in the surface operator and are highly variable from shot to shot. The other component required for the surface operator is surface reflectivity; in land surveys, the surface operator must properly include elastic effects, which vary with angle, azimuth, polarization, and from shot-to-shot. All of these components cannot easily be measured in the field, nor easily removed through preprocessing. Therefore, the MPI concept, which eliminates the need for an explicit surface operator, provides a most attractive formulation for attack-

ing surface-related multiples in data acquired on land.

The extension of the MPI concept from surface-related multiple attenuation to internal multiple attenuation is much less problematic than the similar extension of conventional SMA approaches (Weiglein et al., 1997; Jakubowicz, 1998; Berkhout and Verschuur, 2005). This is because the MPI approach does not need an explicit surface operator; thus, when applied to internal multiple attenuation, it does not need an explicit internal operator. In practice, schemes that do have such a requirement struggle to determine the operator, and pass most of their uncertainty forward to be dealt with by improved adaptive subtraction. In monotonously layered data, where interbed multiples are most often problematic, an adaptive subtraction cannot reliably discriminate between multiples and primaries, and the results are poor. In the MPI scheme, no such limitation arises. Although the MPI scheme does not require an explicit source, the spatial variation of such an internal operator is still taken into account in the multiple prediction.

CONCLUSIONS

The MPI scheme considers multiple attenuation as an iterative optimization problem. Within each iteration an update vector, the so-called **T** operator, is searched to model the multiple wavefield approximately, and then the multiple model is refined by a proper adaptive subtraction.

It can be understood that the MPI multiple modeling effectively includes the surface operator recovered from the previous iteration. Compared with a conventional SMA method, which assumes the surface operator to be a diagonal matrix, the accuracy of the MPI multiple modeling has been improved kinematically and dynamically.

The MPI multiple modeling significantly reduces the nonlinearity of the problem in the subsequent phase and facilitates robust application of the expanded multichannel matching for adaptive subtraction. The adaptive filter will be more diagonal, so that work done by the filter will be easier.

Real data examples have demonstrated that the MPI method is able to separate multiples from primaries, even for shot gathers with missing near-offset traces. This effectiveness results from the combination of accurate multiple modeling and robust adaptive subtraction.

ACKNOWLEDGMENTS

I thank associate editor Eric Verschuur, Rodney Johnston, and two anonymous reviewers for constructive reviewing and Xuxuan Li and Xu Chang for stimulating discussions. I also acknowledge the sponsors of the Centre for Reservoir Geophysics, Imperial College London, for supporting this research.

REFERENCES

- Baumstein, A., M. T. Hadidi, D. L. Hinkley, and W. S. Ross, 2005, A practical procedure for application of 3D SRME to conventional marine data: *The Leading Edge*, **24**, 254–258.
- Berkhout, A. J., 1982, *Seismic migration — Imaging of acoustic energy by wave field extrapolation*: Elsevier Science Publishing Co., Inc.
- , 1984, *Seismic migration — Imaging of acoustic energy by wave field extrapolation, Part B, Practical aspects*: Elsevier Science Publishing Co., Inc.
- Berkhout, A. J., and D. J. Verschuur, 1997, Estimation of multiple scattering by iterative inversion, Part I, Theoretical considerations: *Geophysics*, **62**,

Figure 6. Another real data example of MPI multiple attenuation. (a) Brute stack of the real data set before multiple attenuation. (b) Stack section after MPI multiple attenuation on shot gathers. (c) The attenuated multiple energy, i.e., the difference between (a) and (b).

- 1586–1595.
- , 2005, Removal of internal multiples with the common-focus-point (CFP) approach, Part 1, Explanation of the theory: *Geophysics*, **70**, no. 3, V45–V60.
- Dragoset, W. H., and Z. Jerićević, 1998, Some remarks on multiple attenuation: *Geophysics*, **63**, 772–789.
- Guittou, A., 2005, Multiple attenuation in complex geology with a pattern-based approach: *Geophysics*, **70**, no. 4, V97–V107.
- Hadidi, M. T., A. Baumstein, and Y. C. Kim, 2002, Surface-related multiple elimination on wide-tow marine data: *The Leading Edge*, **21**, 787–790.
- Jakubowicz, H., 1998, Wave equation prediction and removal of interbed multiples: 68th Annual International Meeting, SEG, Expanded Abstracts, 1527–1530.
- Kelamis, P. G., and D. J. Verschuur, 2000, Surface-related multiple elimination on land seismic data — Strategies via case studies: *Geophysics*, **65**, 719–734.
- Kragh, E., J. Robertsson, R. Laws, L. Amundsen, T. Røsten, T. Davies, K. Zerouk, and A. Strudley, 2004, Rough-sea deghosting using wave heights derived from low-frequency pressure recordings — A case study: 74th Annual International Meeting, SEG, Expanded Abstracts, 1309–1312.
- Luo, Y., P. G. Kelamis, and Y. Wang, 2003, Simultaneous inversion of multiples and primaries — Inversion versus subtraction: *The Leading Edge*, **20**, 814–819.
- Verschuur, D. J., and A. J. Berkhout, 1997, Estimation of multiple scattering by iterative inversion, Part II: Practical aspects and examples: *Geophysics*, **62**, 1596–1611.
- Verschuur, D. J., A. J. Berkhout, and C. P. A. Wapenaar, 1992, Adaptive surface-related multiple elimination: *Geophysics*, **57**, 1166–1177.
- Wang, Y., 2002, Seismic trace interpolation in the f - x - y domain: *Geophysics*, **67**, 1232–1239.
- , 2003a, Multiple subtraction using an expanded multichannel matching filter: *Geophysics*, **68**, 346–354.
- , 2003b, Multiple attenuation: Coping with the spatial truncation effect in the Radon transform domain: *Geophysical Prospecting*, **51**, 75–87.
- , 2004, Multiple prediction through inversion: A fully data-driven concept for surface-related multiple attenuation: *Geophysics*, **69**, 547–553.
- Weglein, A. B., F. A. Gasparotto, P. M. Carvalho, and R. H. Stolt, 1997, An inverse scattering series method for attenuating multiples in seismic reflection data: *Geophysics*, **62**, 1975–1989.
- Ziolkowski, A. M., D. B. Taylor, and R. G. Johnston, 1999, Marine seismic wavefield measurement to remove sea-surface multiples: *Geophysical Prospecting*, **47**, 841–871.