Inverse- Q filtered migration

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ABSTRACT

An inverse- Q filtered migration algorithm performs seismic migration and inverse- Q filtering simultaneously, in which the latter compensates for the amplitudes and corrects the phase distortions resulting from the earth attenuation effect. However, the amplitudes of high-frequency components grow rapidly in the extrapolation procedure, so numerical instability is a concern when including the inverse- Q filter in the migration. The instability for each frequency component is independent of data and is affected only by migration models. The stabilization problem may be treated separately from the wavefield-extrapolation scheme. The proposed strategy is to construct supersedent of attenuation coefficients, based on given velocity and Q models, before performing wavefield extrapolation in the space-frequency domain. This stabilized algorithm for inverse- Q filtered migration is applicable to subsurface media with vertical and lateral variations in velocity and Q functions. It produces a seismic image with enhanced resolution and corrected timing, comparable to an ideal image without the earth attenuation effect.

INTRODUCTION

In seismic migration, surface-recorded seismic waves are back-propagated through the subsurface medium. This is an inverse process of wave propagation, as is inverse- Q filtering, in which the earth attenuation effects including energy absorption and velocity dispersion are compensated for. Thus, a migration process also should account for subsurface attenuation effects. By performing inverse- Q filtering and migration processes simultaneously, one can restore the high frequencies to balance the spectrum of the seismic image and correct the phases and the associated timings of reflections. This procedure is referred to as inverse- Q filtered migration.

In a viscoacoustic medium, seismic wave amplitudes and arrival times are changed because of energy absorption and velocity dispersion, respectively. Consequently, migration by wavefield extrapolation without compensating for these earth attenuation effects produces a result with a diffused image and incorrectly positioned reflectors. Including an inverse- Q filter in migration, however, raises the problem of numerical instability. Because the amplitudes of high-frequency components grow rapidly in wavefield extrapolation, numerical round-off errors tend to be amplified drastically with increasing depth (Dai and West, 1994; Mittet et al., 1995; Cui and He, 2004). This causes a huge amount of undesirable noise in the result, even if the input data set is free of observed noise (Wang, 2002, 2006).

The improvement of an inverse- Q filter over seismic resolution is equal to three times the bandwidth increment plus two times the signal-to-noise ratio (S/N) increment (Wang, 2003). Only when the sum of these two factors is positive can the seismic resolution be enhanced. Therefore, one of the focus points in an inverse- Q filtered migration procedure is stabilization to ensure that the resultant S/N is maximized.

Zhang and Wapenaar (2002) suggest limiting the number of extrapolation steps and limiting the maximum angle of migration dip to derive a conditionally stable extrapolation operator. However, that does not solve the problem because of the limitation on migration depth and dip angle. The first stabilized algorithm for migration with inverse- Q filtering, presented by Wang and Guo (2004a), is implemented in the wavenumber-frequency domain. It applies only to a subsurface medium with a vertically variable velocity function and vertically variable Q model. Nevertheless, this preliminary investigation may offer some insight into stabilization issues in the extrapolation operators of wavefield downward continuation.

This paper proposes a strategy to implement inverse- Q filtered migration for subsurface media with both vertical and lateral variations in the velocity and Q models. A 1D wave equation, variable in the z -direction, is used at each lateral location of the model to derive a "supersedent" to supersede attenuation coefficients to stabilize the migration process. This 1D equation-based extrapolation operator has a maximum value of compensation among a series of compo-

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nents in the migration sequence, so constructed attenuation coefficients will stabilize not only the 1D equation but also other components successively applied in the migration.

The stabilized attenuation coefficients or, equivalently, frequency-dependent complex-valued wavenumbers, are calculated only once and are applied to each component in the migration sequence. The scheme is illustrated with a space-frequency domain finite-difference implementation following Gazdag and Sguazzero (1984), but it is modified by using a complex velocity in the derivation. The effectiveness is demonstrated with a synthetic data set.

In general, we always can extract this 1D wave equation from a high-dimensional wave equation, even if we implement the migration in a sequence consisting of a 1D wave-equation-based, phase-shift-type step, and we always can construct the supersedent of attenuation coefficients in advance. Therefore, the proposed strategy is a general solution to stabilize a wave-equation-based, inverse- Q filtered migration.

STABILIZATION OF AN INVERSE- Q FILTERED MIGRATION

Inverse- Q filtered migration

Start the derivation with a one-way wave equation in the Fourier transform domain:

$$\frac{\partial U(k_x, z, \omega)}{\partial z} = ik_z U(k_x, z, \omega), \quad (1)$$

here U is the plane wave of angular frequency ω , k_x is the horizontal wavenumber, and k_z is the vertical wavenumber. The solution to wave equation 1 is (Gazdag, 1978)

$$U(k_x, z + \Delta z, \omega) = U(k_x, z, \omega) \exp[ik_z \Delta z], \quad (2)$$

where Δz is the step length of wavefield downward extrapolation. The vertical wavenumber k_z is defined by

$$k_z = \frac{\omega}{\bar{v}} \sqrt{1 - \frac{k_x^2}{k^2}}, \quad (3)$$

where $\bar{v} \equiv \bar{v}(z)$ is half of the constant wave propagation speed (Loewenthal et al., 1976) and $k = \omega/\bar{v}$ is the wavenumber. To account for the earth attenuation effect, the real-valued velocity $\bar{v}$ is replaced by a frequency-dependent, complex-valued velocity $\bar{c}$, defined as (Kolsky, 1956; Futterman, 1962; Kjartansson, 1979)

$$\frac{1}{\bar{c}} = \left(1 - \frac{i}{2Q}\right) \frac{1}{\bar{v}} \left(\frac{\omega}{\omega_h}\right)^{-\gamma}, \quad (4)$$

where $\bar{c} \equiv \bar{c}(z, \omega)$, $Q \equiv Q(z)$ is the earth quality factor function, $\gamma \equiv \gamma(z) = (\pi Q(z))^{-1}$, and ω_h is a reference frequency with $\omega < \omega_h$ (Wang and Guo, 2004b). The vertical wavenumber in equation 3 becomes

$$k_z = \left(1 - \frac{i}{2Q}\right) \frac{\omega}{\bar{v}} \left(\frac{\omega}{\omega_h}\right)^{-\gamma} \sqrt{1 - \frac{k_x^2}{k^2}}. \quad (5)$$

For the migration procedure incorporating the inverse- Q filter, the challenge is how to create a stabilized migration operator for each extrapolation step.

Wang and Guo (2004a) demonstrate the inverse- Q filtered migration algorithm described above, which is applicable to subsurface

models with vertical variations in velocity and Q function. From this preliminary investigation, one can draw the following two conclusions regarding the stabilization of migration with inverse- Q filtering. First, to stabilize the problem, one must evaluate the earth attenuation effect accumulated from the recording surface down to the current depth, not just the effect within the current extrapolation step. Second, the stabilization procedure for each frequency ω is independent of data and is affected only by the velocity and Q models. Therefore, one can implement the stabilization treatment separately from any typical migration scheme for the wavefield extrapolation.

Stabilization

To derive the stabilized extrapolation operator for migration with lateral variations in velocity and Q functions, we use a 1D equation, variable only in the z -direction, as follows:

$$\frac{\partial U(z, \omega)}{\partial z} = i \frac{\omega}{\bar{c}} U(z, \omega). \quad (6)$$

Its phase-shift solution is

$$U(z + \Delta z, \omega) = U(z, \omega) \exp\left[i \frac{\omega}{\bar{c}} \Delta z\right]. \quad (7)$$

The wave equation for migration can be divided into a series of extrapolation equations, applied to the seismic data successively. Among this series of equations, the 1D wave equation 6 is a key equation that also gives the maximum value of amplitude compensation, as shown in the next section in the x - ω -domain finite-difference migration method. If we can stabilize its solution 7 — that is, stabilize the calculation of the frequency-dependent, complex-valued wavenumber $k \equiv \omega/\bar{c}$ — we also can stabilize the other extrapolation equations in the series.

The extrapolation operator accumulated from the recording surface down to the current extrapolation step n is

$$\begin{aligned} \exp\left[i \sum_{\ell=1}^n \frac{\omega}{\bar{c}(z^{(\ell)}, \omega)} \Delta z\right] &\approx \exp\left[\sum_{\ell=1}^n \frac{\omega}{2\bar{v}(z^{(\ell)})Q(z^{(\ell)})} \Delta z\right] \\ &\times \exp\left[i \sum_{\ell=1}^n \frac{\omega}{\bar{v}(z^{(\ell)})} \left(\frac{\omega}{\omega_h}\right)^{-\gamma} \Delta z\right], \end{aligned} \quad (8)$$

where $\bar{v}(z^{(\ell)}) \equiv \bar{v}(\ell \Delta z)$ and $Q(z^{(\ell)}) \equiv Q(\ell \Delta z)$ are the migration velocity and Q values at the ℓ th layer. The first exponential factor on the right-hand side is the amplitude operator, in which the following approximation,

$$\frac{1}{2Q} \left(\frac{\omega}{\omega_h}\right)^{-\gamma} \approx \frac{1}{2Q},$$

is made because $(\omega/\omega_h)^{-\gamma} \rightarrow 1 \ll 2Q$ when $\gamma \equiv (\pi Q)^{-1} \rightarrow 0$.

The second exponential factor in equation 8 is the phase operator, in which the factor $(\omega/\omega_h)^{-\gamma}$ with a minor discrepancy from the unity modifies the real wavenumber $\omega/\bar{v}$ to correct the phase distortion. For stabilization, according to the investigation in Wang (2006) for inverse- Q filtering, we consider only the amplitude operator in the extrapolation operator because the phase operator is unconditionally stable.

To stabilize the extrapolation, we define an amplitude-attenuation operator as

$$W(n, \omega) = \exp \left[- \sum_{\ell=1}^n \frac{\omega}{2\bar{v}(z^{(\ell)})Q(z^{(\ell)})} \Delta z \right] \quad (9)$$

and calculate a stabilized amplitude-compensation operator by

$$W^{-1}(n, \omega) = \frac{W(n, \omega) + \sigma^2}{W^2(n, \omega) + \sigma^2}, \quad (10)$$

where σ^2 is a stabilization factor. Wang (2006) derives an empirical relationship between the stabilization factor σ^2 and a user-specified gain limit to explicitly control the amplitude gain in the inverse-Q filter.

This stabilization scheme also has a factor σ^2 in the numerator, proposed first by Wang (2004), so this scheme does not suppress high frequencies in the input data (Wang, 2006). Note also that this stabilization procedure differs from that in Wang and Guo (2004a) by stabilizing only the amplitude operator and keeping the phase operator unaffected.

We now define a stabilized attenuation coefficient $\alpha_{st}(z^{(\ell)}, \omega)$ as

$$\exp \left[\sum_{\ell=1}^n \alpha_{st}(z^{(\ell)}, \omega) \Delta z \right] = W^{-1}(n, \omega). \quad (11)$$

This coefficient is a supersedent to the actual attenuation coefficient and will be used for amplitude compensation within the ℓ th step extrapolation. For the first step $\ell = 1$, the stabilized attenuation coefficient is

$$\alpha_{st}(z^{(1)}, \omega) = \frac{\omega}{2\bar{v}(z^{(1)})Q(z^{(1)})}. \quad (12)$$

For the current n th layer, it is given by

$$\alpha_{st}(z^{(n)}, \omega) = \frac{1}{\Delta z} \ln[W^{-1}(n, \omega)] - \sum_{\ell=0}^{n-1} \alpha_{st}(z^{(\ell)}, \omega). \quad (13)$$

Note that we generate an operator for the current layer by constructing an operator from the surface to the current layer and subtracting the previous steps. Therefore, the operator depends on the full subsurface above the current layer.

Stabilizing the extrapolation operator means generating a stabilized complex wavenumber $\omega/\bar{c}$. Given the stabilized attenuation coefficients above, the complex wavenumber in equations 6 and 7 is stabilized also as

$$\frac{\omega}{\bar{c}(z^{(n)}, \omega)} = \frac{\omega}{\bar{v}(z^{(n)})} \left(\frac{\omega}{\omega_h} \right)^{-\gamma} - i\alpha_{st}(z^{(n)}, \omega). \quad (14)$$

For wavefield extrapolation at depth $z^{(n)}$ with lateral variations in velocity and Q functions, we first construct the frequency-dependent, stabilized attenuation function $\alpha_{st}(z^{(n)}, \omega)$ separately at each x -position because one often assumes that the migration model has smooth lateral variations in velocity and Q functions. Once this laterally variant, 2D slice in the x - ω space is built, we can build a stabilized extrapolation operator at depth $z^{(n)}$ in the space-frequency domain.

THE FINITE-DIFFERENCE EXTRAPOLATOR

Using the stabilization scheme presented in the previous section, we find in this section an example of a stabilized finite-difference migration method. The derivation closely follows Gazdag and Sguazzero (1984) by replacing a real velocity with the frequency-dependent, complex-valued velocity $\bar{c}$.

The square root of the Helmholtz operator for the viscoacoustic wave equation, or the vertical wavenumber, may be approximated to the second order by a continued fraction as

$$k_z = \sqrt{\frac{\omega^2}{\bar{c}^2} - k_x^2} \approx \frac{\omega}{\bar{c}} \sqrt{1 - \frac{k_x^2}{|k|^2}} \approx \frac{\omega}{\bar{c}} \left(1 - \frac{\frac{s_2 k_x^2}{|k|^2}}{1 - \frac{s_1 k_x^2}{|k|^2}} \right), \quad (15)$$

where $\bar{c} \equiv \bar{c}(x, z, \omega)$ in this case and

$$|k|^2 = \left(\frac{\omega}{\bar{c}} \right)^* \left(\frac{\omega}{\bar{c}} \right), \quad (16)$$

for stabilization. In the nonlinear approximation 15, s_1 and s_2 are constants that have been chosen to maximize the accuracy of the approximation. For example, the approximation with $s_1 = 0.37637$ and $s_2 = 0.47824$ is accurate for $\theta \equiv \sin^{-1}(k_x/|k|)$ less than about 65° (Lee and Suh, 1985; Halpern and Trefethen, 1988). The one-way wave equation 1 becomes

$$\frac{\partial U(k_x, z, \omega)}{\partial z} = i \frac{\omega}{\bar{c}} \left(1 - \frac{\frac{s_2 k_x^2}{|k|^2}}{1 - \frac{s_1 k_x^2}{|k|^2}} \right) U(k_x, z, \omega). \quad (17)$$

This extrapolation equation is solved by splitting it into two extrapolations (Gazdag and Sguazzero, 1984):

$$\frac{\partial U(k_x, z, \omega)}{\partial z} = i \frac{\omega}{\bar{c}} U(k_x, z, \omega) \quad (18)$$

and

$$\frac{\partial U(k_x, z, \omega)}{\partial z} = i \frac{\omega}{\bar{c}} \left(\frac{-\frac{s_2 k_x^2}{|k|^2}}{1 - \frac{s_1 k_x^2}{|k|^2}} \right) U(k_x, z, \omega). \quad (19)$$

These two extrapolations may be applied alternately in small Δz steps.

Extrapolation equation 18 can be presented in the x - ω domain as equation 6 with a phase-shift solution 7. From this is derived stabilization formula 14 for $\omega/\bar{c}$ in the previous section. Once we calculate the stabilized wavenumber $\omega/\bar{c}$, we apply it to both migration steps 18 and 19. Comparing equations 18 and 19, we see that they differ in a factor that has an absolute magnitude less than one. Thus, stabilization formula 14 stabilizes both extrapolation equations 18 and 19.

To express equation 19 in the x - ω domain, we multiply it by the denominator of its right-hand side; and then apply an inverse Fourier transformation with respect to k_x to both sides. The result is

$$\left(1 + \frac{s_1}{|k|^2} \frac{\partial^2}{\partial x^2}\right) \frac{\partial}{\partial z} U(x, z, \omega) = i \frac{\omega}{\bar{c}} \frac{s_2}{|k|^2} \frac{\partial^2}{\partial x^2} U(x, z, \omega). \quad (20)$$

This is solved numerically by a finite-difference scheme. If Δx is the grid spacing and Δz is the extrapolation step, the wavefield at grid point (m, n) is denoted by $U_m^{(n)} = U(m\Delta x, n\Delta z, \omega)$. Then equation 20 is discretized by using the following approximations:

$$U_m^{(n+1/2)} \approx \frac{1}{2}(U_m^{(n+1)} + U_m^{(n)}), \quad (21)$$

$$\frac{\partial}{\partial z} U_m^{(n+1/2)} \approx \frac{1}{\Delta z}(U_m^{(n+1)} - U_m^{(n)}), \quad (22)$$

and

$$\frac{\partial^2}{\partial x^2} \approx \frac{\delta_{xx}}{1 + \eta \Delta x^2 \delta_{xx}}, \quad (23)$$

where $\delta_{xx} = (1, -2, 1)/\Delta x^2$ is the second-order finite-difference operator and η is an adjustable constant included to increase the accuracy of the second-order spatial derivative. Following Claerbout (1985), we set the value $\eta = 1/4 - 1/\pi^2 \approx 0.15$, instead of $\eta = 1/12$ in the Taylor expansion, to restore accuracy artificially at the higher wavenumbers up to the Nyquist component at the cost of diminished accuracy for the lower wavenumbers. The finite-difference formula of equation 20 is

$$\begin{aligned} U_m^{(n+1)} + (a - ib)(U_{m-1}^{(n+1)} - 2U_m^{(n+1)} + U_{m+1}^{(n+1)}) \\ = U_m^{(n)} + (a + ib)(U_{m-1}^{(n)} - 2U_m^{(n)} + U_{m+1}^{(n)}), \end{aligned} \quad (24)$$

where

Figure 1. (a) Velocity model and (b) associated attenuation model for synthetic seismic modeling.

$$a = \frac{s_1}{|k|^2 \Delta x^2} + \eta \quad \text{and} \quad b = \left(\frac{\omega}{\bar{c}}\right) \frac{s_2 \Delta z}{2|k|^2 \Delta x^2} \quad (25)$$

with $\omega/\bar{c}$ evaluated by the stabilized formula 14.

Equation 24, for all values of m , represents a system of linear equations. This system is solved for each ω value; then an image is formed by applying the imaging condition.

MIGRATION EXAMPLES

Figure 1 shows the Marmousi velocity model and the corresponding attenuation ($1/Q$) model designed in this paper. Figure 2 displays two zero-offset sections of synthetic seismic data, where Figure 2a is generated by using a finite-difference viscoacoustic wave-equation method, based on the velocity and attenuation models in Figure 1, and Figure 2b is generated by using the wave equation without attenuation. The finite-difference modeling used here is a frequency-domain implementation that incorporates the effects of attenuation

Figure 2. (a) Synthetic seismic profile based on the velocity and Q models shown in Figure 1. (b) Synthetic seismic profile based on the velocity model in Figure 1a but without considering the attenuation effect.

through the complex velocity definition (Pratt, 1990). Comparing these two synthetic seismic sections we can see the amplitude absorption and phase distortion in the section including the earth attenuation effect. We now use these synthetic seismic sections to demonstrate the effectiveness of the proposed inverse- Q filtered migration algorithm.

Figure 3a and b compares the two migration results of the synthetic data set shown in Figure 2a, where Figure 3a uses a conventional migration without the inverse- Q filter and Figure 3b uses the proposed migration with the inverse- Q filtering algorithm. The inverse- Q filtered migration result in Figure 3b shows that the amplitudes of reflections have been strengthened and the resolution has been enhanced. Mittet et al. (1995) demonstrate that even when migrating with a Q model deviating by 10% from the correct one, the images still would be better focused and of higher quality than when no compensation was performed.

Figure 3. (a) Migration result of the synthetic data set in Figure 2a, using conventional migration without the inverse- Q filter. (b) Migration result of Figure 2a, using the proposed inverse- Q filtered migration method. (c) Migration result of the synthetic data set of Figure 2b. In this case, neither the synthetic data nor the migration considers the earth attenuation effect.

To verify the result of proposed inverse- Q filtered migration further, a conventional migration without Q compensation (Figure 3c) is performed on the synthetic data set without the earth's attenuation effect (Figure 2b). When we take into account the attenuation effect in the migration of a realistic synthetic data set, we produce the image (Figure 3b) comparable to an ideal image as if there were no attenuation effect (Figure 3c). All the migration results in Figure 3 are plotted to the same scale.

Figure 4 compares migrated seismic traces, selected arbitrarily at a distance of 750 m, from the three migration results shown in Figure 3. The top trace (corresponding to Figure 3a) is the conventional migration result without inverse- Q filtering. The middle trace, with the boosted amplitude of later wavelets, is the result of inverse- Q filtered migration (Figure 3b) and is very close to the ideal situation of pure acoustic media (Figure 3c).

In comparison with the top trace without inverse- Q filtering, the inverse- Q filtered migration has moved wavelets in the middle trace leftward and has made them match the interface depth in the true model (the bottom trace). For example, the weak peak at 2500 m has been shifted to 2475 m and now matches the ideal trace. The amplitude of this wavelet at 2475 m is not as high as the ideal wavelet because the stabilization scheme automatically limits compensation when a high-frequency component is attenuated completely from the original data. In this way, the stabilization scheme avoids boosting the ambient noise.

Figure 4. (a) Comparison of migrated seismic traces, selected arbitrarily at a distance of 750 m from the three migration results shown in Figure 3. The three traces, from top to bottom, correspond to Figure 3a–c, respectively. (b) Comparison of spectra. The dashed line is the amplitude spectrum of Figure 3a, the solid line is the amplitude spectrum of Figure 3b, and the dotted line is the amplitude spectrum of Figure 3c. The solid and dotted lines are close to each other, and the discrepancy at a high wavenumber (35–40 km^{-1}) can be attributed to the stabilization.

Figure 4b compares the amplitude spectra of three migration results. The dashed line represents the amplitude spectrum of migration without inverse- Q filtering (Figure 3a). The solid line shows the amplitude spectrum of inverse- Q filtered migration (Figure 3b) and is close to the dotted line, the amplitude spectrum of Figure 3c. The discrepancy between the solid and dotted lines at high wavenumbers ($35\text{--}40\text{ km}^{-1}$) can be attributed to the stabilization. Each of these three curves is an averaged spectrum, averaging the amplitude spectra of individual traces over the whole seismic section.

For real data applications, a reliable Q model with lateral variation is needed. Zhang and Ulrych (2002) suggest estimating Q from common-midpoint (CMP) gathers. Wang (2004) proposes a Q analysis method based on the seismic stack section.

CONCLUSIONS

Seismic migration incorporating inverse- Q filtering, or inverse- Q filtered migration, performs amplitude compensation and phase correction simultaneously during the migration process. A stabilized scheme needs to evaluate the attenuation effect accumulated from the recording surface down to the current extrapolation depth, not just the effect within the current step. Because stabilization for each frequency is independent of data and is affected only by the velocity and Q models, it is treated separately from the wavefield extrapolation. The essential element of the proposed strategy is to construct stabilized attenuation coefficients, based on the given velocity and Q models, before performing wavefield extrapolation in the space-frequency domain.

The aim of stabilization is to make inverse- Q filtered migration work for media with lateral variations in velocity and attenuation models and to produce a seismic image with true amplitudes and correct timings of reflections. Applications of the method to synthetic data examples demonstrate that the inverse- Q filtered migration image is comparable to an ideal image without the earth attenuation effect. This method could well be robust in real seismic data application if a reliable Q model is used. The same stabilization scheme is applicable to the prestack domain for migration with inverse- Q filtering.

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