CYCLIC-FATIGUE CRACK GROWTH IN COMPOSITE AND ADHESIVELY-BONDED STRUCTURES: THE FAA SLOW CRACK GROWTH APPROACH TO CERTIFICATION AND THE PROBLEM OF SIMILITUDE

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ABSTRACT: In 2009 the US Federal Aviation Administration (FAA) introduced a slow crack-growth approach for certifying composite and adhesively-bonded structures. This approach requires that the growth of a delamination or disbond is slow, stable and predictable under cyclic-fatigue loads. To predict growth in aircraft structures requires a methodology for translating laboratory crack-growth data to full-scale structures. Whilst this need not be a fracture-mechanics based approach, the present paper focuses on fracture-mechanics approaches since they have been widely adopted for this purpose for certifying aircraft structures. This approach uses the ‘similitude hypothesis’ combined with the concept of a crack-driving force (CDF) to link the results from laboratory tests to the cyclic-fatigue behaviour seen in full-scale aircraft tests. The present paper reveals that the range of the strain-energy release rates, ΔG , is not a valid crack-driving force. In contrast, in the present paper, a valid scheme is identified and proven to be appropriate.

Keywords: Similitude, adhesives, composites, fatigue crack growth, delamination.

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NOMENCLATURE

a	crack/disbond/delamination length
A	a constant in the Hartman-Schijve equation
ADCB	asymmetric double-cantilever beam
da/dN	rate of crack growth per cycle
C	constant in the Paris crack-growth equation
CDF	crack driving force
CFRP	carbon-fibre reinforced-plastic
D	constant in the Hartman-Schijve crack-growth equation
DCB	double-cantilever beam
G	strain-energy release-rate
G_I / G_{II}	ratio of the Mode I strain-energy release-rate to the Mode II strain-energy release-rate
G_{Ic}	critical value of the Mode I strain-energy release-rate under quasi-static loading
G_{max}	maximum value of the applied strain-energy release-rate in the fatigue cycle
G_{min}	minimum value of the applied strain-energy release-rate in the fatigue cycle
ΔG	range of the applied strain-energy release-rate in the fatigue cycle (= $G_{max} - G_{min}$)
$\Delta\sqrt{G}$	= $\sqrt{G_{max}} - \sqrt{G_{min}}$
$\Delta\sqrt{G_{th}}$	the value of $\Delta\sqrt{G}$ at a value of da/dN of 10^{-10} m/cycle
$\Delta\sqrt{G_{thr}}$	range of the fatigue threshold value of $\Delta\sqrt{G}$
Δk	crack driving force, see Equation (7)
K	stress-intensity factor
K_{max}	maximum value of the applied stress-intensity factor in the fatigue cycle
K_{min}	minimum value of the applied stress-intensity factor in the fatigue cycle
ΔK	range of the applied stress-intensity factor in the fatigue cycle, as defined below
ΔK	= $K_{max} - K_{min}$
ΔK_{thr}	range of the fatigue threshold value of the applied stress-intensity factor
m	exponent in the Paris crack-growth equation
n	exponent in the Hartman-Schijve crack-growth equation

N	number of fatigue cycles
P_{max}, P_{min}	maximum and minimum loads in a fatigue test
r	distance ahead of the crack tip
R	load ratio ($= \sigma_{min}/\sigma_{max}$)
R^2	the linear correlation coefficient
SERR	strain-energy release-rate
σ_{max}	the maximum value of the applied stress in a cycle
σ_{min}	the minimum value of the applied stress in a cycle

1 INTRODUCTION

It has been observed that in composites and adhesively-bonded structures delaminations and disbonds can arise naturally and grow, both in-service and in full-scale fatigue tests, under representative flight loads [1-3]. These observations, coupled with the introduction by the US Federal Aviation Administration (FAA) of a slow crack-growth approach to certifying composite and adhesively bonded structures [4], has led to a renewed interest [1, 5-13] in how to predict the growth of delaminations and disbonds under cyclic-fatigue loads. The precise wording used in the FAA Airworthiness Advisory Circular (AAC) 20-107B [4], which covers both composites and adhesively-bonded joints, is:

“The traditional slow growth approach may be appropriate for certain damage types found in composites if the growth rate can be shown to be slow, stable and predictable. Slow growth characterization should yield conservative and reliable results. As part of the slow growth approach, an inspection program should be developed consisting of the frequency, extent, and methods of inspection for inclusion in the maintenance plan.”

Whilst there are several possible ways in which this may be able to be achieved, the present paper will focus on the application of fracture-mechanics based approaches to predicting delamination and disbond growth in composite and adhesively-bonded structures.

The application of fracture-mechanics to the design, certification and sustainment of airframes requires a means for translating laboratory crack-growth data under cyclic-fatigue loading to the extent and rate of crack growth as observed in full-scale aircraft structures. This requires the ‘similitude hypothesis’ to be invoked. For the ‘similitude hypothesis’ to be applicable requires a valid crack-driving force (CDF). Whereas in metals it is common to use the range in the applied stress-intensity factor, ΔK , as the CDF, in composites and adhesively-bonded structures it is common to assume the range in the strain-energy release-rate (SERR), ΔG , to be the CDF. (Or variants thereof, for example, where the maximum value of the SERR, G_{max} , in the fatigue cycle is used instead of ΔG .) Thus, the rate of crack growth per cycle, da/dN , in composites and adhesive joints has often been expressed in the form of the Paris crack-growth equation:

$$da/dN = C\Delta G^m \quad (1)$$

or variants thereof, where C and m are experimentally-determined constants. However, several researchers [5, 6, 9, 11, 14] have questioned the validity of ΔG , as well as its common variants, as the valid term to use for the CDF for delamination and disbond growth.

Now, standards for measuring the relationship between da/dN and ΔG for composites and adhesives are currently being developed [7, 8]. The present paper reveals that if, as it would appear, that for composites and adhesively-bonded structures the term ΔG is not an appropriate CDF, then the ability to use da/dN versus ΔG data determined from laboratory test coupons to assess and certify composite and adhesively-bonded designs in accordance with the FAA AAC [4] is questionable. From this it follows that care should be taken to ensure that any Standard for fatigue tests on composites or adhesively-bonded joints that is developed can be used to relate da/dN to a valid CDF. If a valid CDF could be determined, and if this CDF could account for the change in delamination growth rates associated with different initial sizes of any delamination, it would raise the possibility that the results obtained from the Standards could be employed to enable the FAA slow crack-growth criteria to be used to certify composite and adhesively-bonded designs. Such an approach would not only enable a new design to be certified but it would also enable a damage-tolerance assessment of the effect delaminations and disbonds, found in-service aircraft, to be performed. This would enable inspection intervals to be determined. This aspect is particularly important to manage the fleet and for aircraft sustainment.

The present paper begins by discussing the successful methodology that has been adopted by researchers studying fatigue crack growth in metals, where a stress-intensity factor approach has been adopted. In this previous work on metals, a consideration of the effects of the R -ratio has been used to confirm the validity of using the stress-intensity factor approach as the CDF in invoking the ‘similitude hypothesis’. The paper then examines the equivalent approach in polymeric-matrix fibre-composite materials and adhesively-bonded joints, where a SERR has to be adopted. A CDF that is based upon the parameter $\Delta\sqrt{G}$ ($=\sqrt{G_{max}} - \sqrt{G_{min}}$) is discussed in detail and it is shown that this parameter appears to meet all the requirements that were discussed above. Finally, the effects of (a) changing the R -ratio, (b) the variability seen in the growth of both ‘long cracks’ and ‘short cracks’, and (c) fatigue

cracks that grow from small naturally-occurring material discontinuities are all successfully modelled using a form of the Hartman-Schijve crack-growth equation, which employs the term $\Delta\sqrt{G}$ as the CDF. Finally, an example is discussed which demonstrates the potential for the Hartman-Schijve approach to be employed to use long-crack delamination growth data to predict the fatigue growth of smaller, naturally-occurring, delaminations.

2. THE ‘SIMILITUDE HYPOTHESIS’ FOR METALS

2.1 Introduction

The ‘similitude hypothesis’, also referred to in [15] as the ‘similarity principle’, plays a central role in aircraft design, certification and sustainment. For metals the starting point for the basic ‘similitude hypothesis’ can be expressed [16] as:

“Two different cracks growing in two specimens of the identical material with the same thickness and the same CDF, and with the same value of K_{max} , will grow at the same value of da/dN .”

For metals, the CDF is generally taken to be the range in the stress-intensity factor $\Delta K (= K_{max} - K_{min})$, where K_{max} is the maximum value of K in the fatigue cycle and K_{min} is the minimum value of K in the cycle.

At this stage it should be noted that, for a constant ΔK , the mean stress increases as the R -ratio increases. As such, for a constant ΔK the value of da/dN should be faster for higher R -ratio (or load ratio) cyclic-fatigue tests. The following section explores this aspect in the case of fatigue crack-growth in metals.

2.2. Effect of R -ratio

The effect of the $R (= \sigma_{min}/\sigma_{max})$ ratio, where σ_{min} is the minimum value of the applied stress in a cycle and σ_{max} is the maximum value of the applied stress in a cycle, is shown in Figures 1 to 4. Figure 1, from [17], presents da/dN versus ΔK data for solution-treated over-aged (STOA) Ti-6Al-4V at values of $R = 0.1, 0.5$ and 0.8 . Figure 2, from [18], presents the da/dN versus ΔK curves for a 12% Cr steam-turbine blade steel tested at $R = 0.1, 0.5$ and 0.8 . Figure

3, from [19], presents the da/dN versus ΔK curves for a AA2324-T359 aluminium alloy tested at values of $R = -1, 0.3, 0.5$ and 0.7 . Figure 4, from [20], presents the da/dN versus ΔK curves for a AA7050-T7451 tested at values of $R = 0.1$ and 0.7 . The common feature of each of these plots is that for a given CDF (i.e. a given value of ΔK) an increase in the R -ratio increases the mean stress and thereby increases the crack growth rate, da/dN , see [21]. Another way of saying this is that as the R -ratio increases then the da/dN versus ΔK curves move to the left. Thus, from Figures 1 to 4, the basic ‘similitude hypothesis’ stated above in Section 2.1 needs the following corollary:

“For two different cracks growing in two specimens of the identical materials with the same thickness and the same CDF (taken to be ΔK), then the crack in the specimen subjected to a higher R -ratio will grow at a faster da/dN value.”

Figure 1 Crack growth in STOA Ti-6Al-4V, from [17].

Figure 2 Crack growth in a 12% Cr steam-turbine blade steel tested at $R = 0.05, 0.5$ and 0.8 , from [18].

Figure 3 Crack growth in AA 2324-T359 tested at $R = -1, 0.1, 0.3, 0.5$ and 0.7 , from [19].

Figure 4 Crack growth in AA7050-T7451 tested at $R = 0.1$ and 0.7 , from [20].

3 IMPLICATIONS FOR FATIGUE CRACK-GROWTH IN COMPOSITES AND ADHESIVELY-BONDED STRUCTURES

3.1 Introduction

Now, there are several reasons why the SERR, G , approach, and not the stress-intensity factor approach, has invariably been used when employing a fracture-mechanics approach to investigate the failure of polymeric-matrix fibre-composite materials and adhesively-bonded joints, see for example [5, 7, 8, 22-28]. There is the difficulty of calculating the stress-intensity factor around the crack tip in anisotropic and inhomogeneous materials and bonded joints. Also, in any case, a dominant, singular stress-field is not observed. Finally, the equivalence between the SERR and the stress-intensity factor approach is very unclear for such composite materials and bonded joints.

Thus, considering the analysis of cyclic-fatigue test data obtained from laboratory test coupons, at first sight the most obvious and corresponding parameter against which to plot

da/dN is the range of applied strain-energy release-rate, ΔG , such that the term ΔG is the CDF and:

$$\Delta G = G_{max} - G_{min} \quad (2)$$

where G_{max} and the G_{min} are the maximum and minimum values of the applied strain-energy release-rate in a fatigue cycle, respectively. However, it is noteworthy that, for composites and structural adhesives, many authors have selected the parameter, G_{max} , rather than ΔG , to employ when analysing the fatigue behaviour, see [5-8, 12, 24-29]. This is on the grounds that, during the unloading part of the fatigue cycle, the debonded surfaces typically come into contact which results in facial interference of the adhesive surfaces; and the belief that this may give an artificially high value of G_{min} [28]. However, if the term G_{max} is used as the CDF then no attention is paid to the minimum value of the SERR, G_{min} , in the fatigue cycle. Indeed, authors [30] have shown that there are instances when taking G_{max} as the CDF may lead, very misleadingly, to an apparent strong dependence of the measured fatigue behaviour upon the R -ratio employed.

The question as to the level of crack-closure associated with fatigue tests performed at high values of R is complex. At this stage it should be recalled that, for both delamination growth in composite materials and crack growth in metals, when the crack/delamination is large the crack-closure effects tend to be less at high R -ratios [31]. For metals, ASTM 647-13a explains that the da/dN versus ΔK curves are crack-length dependent and that for small naturally-occurring cracks that, regardless of the R -ratio, there is little, if any, crack closure. As such, for metals, R -ratio effects are a ‘long-crack’ phenomena. The situation with respect to composites and adhesively-bonded structures is that there is minimal data available on the growth of delaminations or disbonds that have arisen from small naturally-occurring material discontinuities. However, as previously discussed, it has been observed that in composites and adhesively-bonded structures that delaminations and disbonds can arise naturally and grow, both in-service and in full-scale fatigue tests, under representative flight loads [1-3]. This is similar to the findings in metal aircraft [32] and suggests a similar behaviour: i.e. that for delaminations and disbonds that arise from small naturally-occurring material discontinuities there will be little, if any, R -ratio effects. However, the data to support this hypothesis is lacking. It does, however, highlight the need for tests in which delaminations and disbonds are allowed to initiate and grow naturally. This topic, i.e. the effect of initial delamination length on crack growth, will be discussed later in the present paper.

Thus, for the equivalent basic ‘similitude hypothesis’ to that stated in Section 2.1 for metals, we have now for delaminations in composites and disbonds in adhesive joints the statement that:

“Two different cracks growing under the same Mode of loading in two specimens of the identical material with the same thickness and the same CDF (taken to be ΔG), and the same value of G_{max} , will grow at the same value of da/dN .”

Now, one way of testing whether the above equivalent basic ‘similitude hypothesis’ is valid, or not, is to examine the effect of the R -ratio on this proposed hypothesis. Indeed, authors [5] have already suggested that the above approach of simply replacing ΔK by ΔG , or by G_{max} , may lead, somewhat misleadingly, to an apparent strong, and often incorrect, dependence of the measured fatigue behaviour upon the R -ratio employed. This is discussed in detail in the next Section.

3.2 Effect of R -ratio

3.2.1 Introduction

As commented above, there is already evidence that the above approach of simply replacing ΔK by ΔG , or by G_{max} , may lead, somewhat misleadingly, to an apparent strong, and often incorrect, dependence of the measured fatigue behaviour upon the R -ratio employed. Now, from the stated corollary to the ‘similitude hypothesis’ and the effect of the R -ratio, see Section 2.2, the equivalent corollary for composites and adhesive joints to that for metals would be:

“For two different cracks growing under the same Mode of loading in two specimens of the identical materials with the same thickness and the same CDF (taken to be ΔG), then the crack in the specimen subjected to a higher R -ratio will grow at a faster value of da/dN .”

In the next Section we examine whether this is theoretically likely to be a correct statement.

3.2.2 Theoretical Considerations

Consider Figure 5 which presents, schematically, two cyclic-fatigue tests, which we will term ‘Test 1’ and ‘Test 2’, with the same CDF of ΔG . Assume that in ‘Test 1’ we loads P_{1max} and P_{1min} and that in ‘Test 2’ we have loads P_{2max} and P_{2min} , where P_{1max} is greater than P_{2max} . The tests are such that for a given crack length, a , the values of the CDF (equal to ΔG) are the same in both ‘Test 1’ and ‘Test 2’.

Figure 5 Schematic diagram of two tests.

Now, taking ΔG to be the CDF, then $\Delta G (= G_{max} - G_{min})$ is proportional to $P_{max}^2 - P_{min}^2$. As such, assuming that the CDF is equal to ΔG , and since, at crack length a , the CDF is the same in both tests, we have:

$$P_{1max}^2 - P_{1min}^2 = P_{2max}^2 - P_{2min}^2 \quad (3)$$

As a result we see that:

$$1 - R_1^2 = (P_{2max}/P_{1max})^2 (1 - R_2^2) \quad (4)$$

Since P_{2max} is less than P_{1max} then:

$$1 - R_1^2 < (1 - R_2^2) \quad (5)$$

so that:

$$R_1 > R_2 \quad (6)$$

Thus, if the two tests have the same ΔG and ‘Test 1’ has a maximum load greater than ‘Test 2’, then the R -ratio associated with ‘Test 1’ will be greater than the R -ratio associated with ‘Test 2’.

Now, the question that arises is do the experimental results from a test, such as that depicted in ‘Test 1’, confirm that this test does indeed has a higher value of da/dN for a given value of ΔG compared to a test such as depicted in ‘Test 2’. Since, ‘Test 1’ gives rise to a higher R -ratio and so it should exhibit a more rapid rate of fatigue crack-growth for a given value of the CDF ($=\Delta G$). As was indeed shown to be the case for the metal data given in Figures 1 to 4 when the CDF was taken to be ΔK .

3.2.3 Experimental Results

To answer this question consider the example given in [33]. The experimentally-determined da/dN versus ΔG relationships presented in Figure 5b in [5], for Mode I double-cantilever beam (DCB) tests on an unidirectional carbon-fibre reinforced-plastic (CFRP) laminate, are reproduced in Figure 6. Recall that for a valid CDF we require that as the R -ratio increases the da/dN versus ΔG curves should move to the left; and this means that for a given ΔG the value of da/dN should increase as the R -ratio increases. In contrast, in Figure 6 the experimental da/dN versus ΔG curves actually move to the right with increasing R -ratio. That is, for a given ΔG , the value of da/dN is actually observed to decrease as the value of the R -ratio increases. This behaviour is also seen in the experimentally-determined da/dN versus ΔG curves presented in [10, 34, 35], see Figures 7-9. In these figures the results shown are from Mode I DCB fatigue tests, apart from the data shown in Figure 7 where fatigue results [10] from asymmetric double-cantilever beam (ADCB) specimens consisting of aluminium-alloy substrates bonded using a single-part, heat-cured, toughened-epoxy adhesive are given. In these ADCB tests the ratio of the Mode Mix, G_I/G_{II} , was approximately five and the crack propagated through the adhesive layer but close to an adhesive/substrate interface.

Thus, the statement which defines the ‘similitude hypothesis’ and the effect of the R -ratio, as given above in Section 3.2.1, has been shown to be invalid when the CDF is taken to ΔG , or G_{max} as discussed previously. Hence, in the next section we consider how to resolve this major anomaly.

Figure 6 Plot of da/dN versus ΔG_I given in [33] for delamination growth in double-cantilever beam (DCB) tests using an unidirectional CFRP laminate.

Figure 7 Plot of da/dN versus ΔG data given in [10] for disbond growth in asymmetric double-cantilever beam (ADCB) specimens consisting of aluminium-alloy substrates bonded using a single-part, heat-cured, toughened-epoxy adhesive.

Figure 8 Plot of da/dN versus ΔG_I data given in [34] for delamination growth in DCB tests using an interleaved CFRP laminate.

Figure 9 Plot of da/dN versus ΔG_I data given in [35] for disbond growth in DCB specimens consisting of CFRP substrates bonded using a single-part epoxy-paste adhesive.

4 RESOLVING THE ‘SIMPLITUDE’ HYPOTHESIS ANOMALY

References [5, 6] have suggested that this anomaly vanishes if $\Delta\sqrt{G}$ ($=\Delta\sqrt{G_{max}} - \Delta\sqrt{G_{min}}$) is used as the CDF. Whereas [5, 6] only presented a few examples, this is now confirmed in Figures 10-13 where the data previously shown in Figures 6-9 have been replotted but now taking the CDF to be $\Delta\sqrt{G}$. In these examples, it may indeed be seen that for two different cracks growing in two specimens of the identical materials with the same thickness and the same CDF (taken now to be $\Delta\sqrt{G}$), then the crack in the specimen subjected to a higher R -ratio grows at a faster da/dN value. Or put another way, for the specimen subjected to the higher R -ratio then a higher value of da/dN is recorded for a given value of the CDF ($\Delta\sqrt{G}$). Thus, the anomaly found when the value of the CDF was taken as ΔG , or G_{max} , is resolved when the CDF is taken to be $\Delta\sqrt{G}$.

Figure 10 Plot of da/dN versus $\Delta\sqrt{G_I}$ (from Figure 6) for delamination growth in DCB tests using a unidirectional CFRP laminate.

Figure 11 Plot of da/dN versus $\Delta\sqrt{G}$ (from Figure 7) for disbond growth in ADCB specimens consisting of aluminium-alloy substrates bonded using a single-part, heat-cured, toughened-epoxy adhesive.

Figure 12 Plot of da/dN versus $\Delta\sqrt{G_I}$ (from Figure 8) for delamination growth in DCB tests using an interleaved CFRP laminate.

Figure 13 Plot of da/dN versus $\Delta\sqrt{G_I}$ (from Figure 9) for disbond growth in DCB specimens consisting of CFRP substrates bonded using a single-part epoxy-paste adhesive.

5 $\Delta\sqrt{G}$ AND THE HARTMAN-SCHIJVE REPRESENTATION

5.1 Introduction

An important feature that is needed if a fracture-mechanics based methodology is to be used for aircraft design and sustainment is an ability to account (a) for delaminations and disbonds that arise and grow in-service aircraft from small naturally-occurring material discontinuities, (b) the variability seen the growth histories for delaminations and disbonds that grow from naturally-occurring material discontinuities in operational aircraft, and (c) for the R -ratio effects that are associated with such small naturally-occurring material discontinuities. In metallic airframes this is done by collapsing the various R -ratio dependent da/dN versus ΔK curves onto a single ‘master curve’. There are several ways in which this master curve can be determined. The recent reviews of the field of fatigue crack-growth and damage-tolerance [32, 36] has revealed that, in metals, the effects of (a) changing the R -ratio, (b) the variability seen in the growth of both ‘long cracks’ and ‘short cracks’, and (c) fatigue cracks that grow from small naturally-occurring material discontinuities may all be modelled using a form of the Hartman-Schijve crack-growth equation, which as explained in [32] is a variant of the NASGRO Equation [37]. These equations are based on the premise that the value of da/dN is not governed simply by the value in ΔK seen in a cycle but rather by the amount that ΔK is greater than its threshold value [38].

The Hartman-Schijve approach aims to give a unique and linear representation for the fatigue relationships that have been experimentally obtained. In principle, such a unique, i.e. ‘master’, linear representation of the fatigue data should account for R -ratio, variability in crack growth histories and ‘short-crack’ effects. The Hartman-Schijve approach should, therefore, be of great assistance to a designer who wishes to predict the rate of crack growth in an adhesively-bonded or a composite component, as well as to fleet operators who wish to assess the effect of disbonds and/or delaminations found in operational aircraft on the remaining life of the structure; and thereby determine the appropriate maintenance actions. The Hartman-Schijve equation used in [1, 8, 11, 20, 24, 25, 32, 36, 39-44] is basically an empirical equation that for metals takes the form:

$$da/dN = D(\Delta k)^n \quad (7)$$

where D and n are constants, and Δk is taken to be:

$$\Delta k = \frac{(\Delta K - \Delta K_{thr})}{\sqrt{\{1 - K_{max}/A\}}} \quad (8)$$

and, as explained in [20, 32, 36], the values of A and ΔK_{thr} are chosen to ensure that Equation (7) captures the entire crack growth history. This approach has the advantage that it can represent the growth of both long and small, naturally-occurring, cracks and can capture the variability seen in crack-growth histories as observed in both constant amplitude and operational flight-load spectra [24, 32] as well as the coalescence of small cracks and their subsequent growth.

As demonstrated above to resolve the ‘similitude hypothesis’ anomaly, and to accurately describe the cyclic-fatigue behaviour of adhesive joints and polymeric fibre-composites, the term $\Delta\sqrt{G}$ should be employed as the CDF, which is equivalent to ΔK as used for metals. Thus, the form of the Hartman-Schijve equation now becomes:

$$da/dN = D \left[\frac{\Delta\sqrt{G} - \Delta\sqrt{G_{thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}} \right]^n \quad (9)$$

where D , n and A are constants and, as for metals, the value of A is generally chosen to so as to capture the entire da/dN history [1, 11, 24, 25]. The term $\Delta\sqrt{G_{thr}}$ represents the fatigue threshold. For composites and structural adhesives, it is often found from experimental tests that a clearly defined threshold value exists, below which little fatigue crack growth occurs [11, 26]. In this case the value of the threshold, $\Delta\sqrt{G_{thr}}$, is taken to be the experimentally-determined value, see [11, 24]. If this is not the case, then the concepts described in the ASTM Standard [45], which are widely used by the metals community, may be employed. This Standard defines a threshold value which, in the above terminology, may be taken to be the value of $\Delta\sqrt{G}$ at a value of da/dN of 10^{-10} m/cycle. This is termed $\Delta\sqrt{G_{th}}$ and hence, by re-arrangement of Equation (9), the value of $\Delta\sqrt{G_{thr}}$ is given by:

$$\Delta\sqrt{G_{thr}} = \Delta\sqrt{G_{th}} - \sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}} \left[\frac{10^{-10}}{D} \right]^{1/n} \quad (10)$$

with the experimental data having G with units of J/m^2 and da/dN with units of m/cycle.

The Hartman-Schijve equation, i.e. Equation (9), is able to both represent and collapse disbond and delamination data onto a single curve and account for the variability in the crack-growth data, as is shown in [1, 8, 11, 25, 41-44]. In general the value of n is approximately 2, see [1, 11, 41-44]. The ability to represent the variability seen in delamination growth is a major advance, since delamination growth in composites generally exhibits a high degree of scatter [1, 12, 27]. Further, the ability to represent the scatter associated with delaminations and disbonds that grow from naturally-occurring material discontinuities in operational aircraft is essential both for certifying new designs and also for fleet-management purposes.

The ability of Equation (9) to collapse the delamination data shown in Figures 6-9 onto a single, linear curve with a value of n of approximately two, is shown in Figures 14 to 17. The values $\Delta\sqrt{G_{thr}}$ and A used in Figures 14-17 are given in Table 1.

Figure 14 Hartman-Schijve representation of delamination growth in DCB tests using an unidirectional CFRP laminate. (See Figures 6 and 10.)

Figure 15 Hartman-Schijve representation of disbond growth in ADCB specimens consisting of aluminium-alloy substrates bonded using a single-part, heat-cured, toughened-epoxy adhesive. (See Figures 7 and 11.)

Figure 16 Hartman-Schijve representation of delamination growth in DCB tests using an interleaved CFRP laminate. (See Figures 8 and 12.)

Figure 17 Hartman-Schijve representation of disbond growth in DCB specimens consisting of CFRP substrates bonded using a single-part epoxy-paste adhesive. (See Figures 9 and 13.)

Table 1. Values $\Delta\sqrt{G_{thr}}$ and A used in Figures 14-17.

	$\Delta\sqrt{G_{thr}}$ ($\sqrt{\text{J/m}^2}$)	A (J/m^2)
Figure 14, [33]		
R = 0.2	5.75	190
R = 0.5	4.49	190
R = 0.7	3.45	190
Figure 15, [10]		
R = 0.1	10.0	3350
R = 0.1	8.90	3350
R = 0.5	9.40	3350
R = 0.5	5.74	3350
Figure 16, [34]		
R = 0.1	5.59	160*
R = 0.5	7.54	160*
Figure 17, [35]		
R = 0.1	5.30	250
R = 0.5	9.60	250

* Value given in [34] for G_c .

5.2 Effect of delamination length on growth

One of the key lessons learnt when assessing the growth of cracks in (metallic) operational aircraft is the need to use a da/dN versus ΔK curve that differs from that obtained using tests described in the main body of the ASTM fatigue crack-growth standard E647-13a [32, 45-48]. This is due to the fatigue crack-growth being dependent of the length of the initial crack, see Appendix X3 in [45]. Indeed, as explained in [45] using a da/dN versus ΔK curve that has been determined using tests described in the main body of the ASTM standard can lead to very non-conservative (i.e. unsafe) estimates for the fatigue life of the structure.

In this context a recent paper [49] has shown that for the delamination growth in composites the da/dN versus ΔG curves can also be a function of the initial damage size. To illustrate this effect in [49] test data associated with delamination growth in CFRP specimens at an $R= 0.5$ was presented. The DCB test specimens consisted of 32-ply unidirectional CFRP laminates with a nominal cured thickness of 5 mm. Various lengths of initial delaminations were tested

and the resultant da/dN versus $\Delta\sqrt{G_I}$ curves, which show a strong dependency on the size of the initial delamination which was 4.1, 12.7 or 51.3 mm, are given in Figure 18.

Figure 18 Plot of da/dN versus $\Delta\sqrt{G_I}$ for delamination growth in DCB tests using an unidirectional CFRP laminate for three different initial sizes of delamination [49].

Since [49] did not give values of G_{Ic} , it is only possible to plot da/dN as a function of $(\Delta\sqrt{G_I} - \Delta\sqrt{G_{Ithr}})$. Nevertheless, as can be seen in Figure 19, assuming that the crack growth is a function of the amount that the CDF exceeds the threshold value, this plot does collapse the disparate curves onto a single, linear ‘master’ curve. The values of $\Delta\sqrt{G_{Ithr}}$ used in Figures 19 are given in Table 2. Here it should be noted that as the length of the initial delamination reduces then so does the value of $\Delta\sqrt{G_{Ithr}}$. This mirrors the observations seen for crack growth in metals and suggests that for small sub-millimetre initial defects the threshold will be quite small. This finding is consistent with fleet experience and the results of full-scale tests [1-3] where such delaminations and/or disbonds that have arisen in-service have indeed grown. It also raises the potential for the Hartman-Schijve approach to be employed to use long-crack delamination growth data to predict the growth of smaller, naturally-occurring, delaminations, as has been reported recently in [24].

At this point it should be noted that, whereas the present paper has focused on the Hartman-Schijve approach which is a variant of the NASGRO Equation, other approaches, such as that proposed in [6, 21, 49] which relate da/dN to the change in strain-energy per cycle, also look promising.

Figure 19 Plot of da/dN versus $(\Delta\sqrt{G_I} - \Delta\sqrt{G_{thr}})$ for delamination growth in DCB tests using an unidirectional CFRP laminate for three different initial sizes of delamination. (See Figure 18.)

Table 2. Values of $\Delta\sqrt{G_{Ithr}}$ used in Figure 19.

Delamination length (mm)	$\Delta\sqrt{G_{Ithr}}$ ($\sqrt{\text{J/m}^2}$)
4.1	5.78
12.7	6.32
51.3	9.22

6. CONCLUSIONS

For the US Federal Aviation Administration (FAA) slow crack-growth approach to certifying composite and adhesively-bonded structures to be viable, a means is required for translating laboratory coupon test data on delamination and disbond growth to real delamination and disbond growth as seen in full-scale aircraft structures. This, in turn, requires invoking a ‘similarity hypothesis’ and determining a valid crack driving force (CDF) together with Standards for determining the relationship between the CDF and the rate of fatigue crack growth, da/dN . Unfortunately, the present paper suggests that neither the term ΔG , nor the term G_{max} , can be employed as a valid CDF. It thus follows that for composites and adhesively-bonded structures the ability to use da/dN versus ΔG data determined from laboratory test coupons to assess and certify designs in accordance with the aforementioned FAA approach is questionable. From this it follows that care should be taken to ensure that any test standard developed can also be used to relate da/dN to a CDF that can be used for the FAA approach to certify composite and adhesively-bonded designs, and thereby enable a damage-tolerance evaluation of delaminations and disbonds to be performed for damage found in-service aircraft.

Fortunately, we have also seen that, unlike ΔG , or G_{max} , the term $\Delta\sqrt{G}$ appears to fulfil the requirements to be a valid CDF in that for a fixed $\Delta\sqrt{G}$ then increasing the R -ratio increases the value of da/dN . We have also shown that the Hartman-Schrive variant of the NASGRO equation, which is commonly used for assessing crack growth in metals, with the terms ΔK

and ΔK_{th} replaced by $\Delta\sqrt{G}$ and $\Delta\sqrt{G_{thr}}$, respectively, shows promise for modelling the delamination and disbond growth in composite and adhesively-bonded structures. Of particular interest is that the Hartman-Schrive equation appears to collapse the experimentally-measured data onto a single, linear ('master') curve regardless of the value of the R -ratio. It also appears to have the potential to collapse the delamination growth curves obtained for different initial delamination lengths onto a single 'master' curve. Thus, the approach outlined in the present paper has validated a sound 'similarity hypothesis'. Hence, a valid term for the CDF, and a scheme which meets the FAA requirements for translating laboratory coupon test data on delamination and disbond growth to real delamination and disbond growth as seen in full-scale aircraft structures, has been identified.

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