Latching-Declutching Control of Wave Energy Converters using Derivative-Free Optimization

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Abstract—We consider predictive control of a wave energy converter (WEC) that can switch between three modes, namely power generation, declutched with no power generation or latched with zero velocity. We propose a formulation that turns the optimal control problem into a small dimensional discrete optimization problem, where the only decision variables are bounds on the latching time and power-take-off time, while the objective function is computed from the trajectory of a hybrid system with linear dynamics in each sample interval. The optimization problem is solved using a novel derivative-free algorithm that exploits the quantization of the decision variables in order to reduce the number of function evaluations. Two closed-loop formulations are also studied within a receding horizon implementation: the first one uses past wave information and can double the energy generation compared to the uncontrolled case, while the second formulation uses predictions of future waves and is able to result in a further increase in energy generation. The benefits of co-designing the physical system and controller is compared to the sequential approach of first optimizing the physical system without control, followed by controller design.

Index Terms—latching control, declutching control, wave energy, coordinate-search, derivative-free optimization, co-design

I. INTRODUCTION

THE energy absorption of wave energy converters (WECs) can be significantly improved by various control strategies [1]–[3]. We focus on combining two particular classes of control strategies, namely latching and declutching control. Both control strategies are based on altering the dynamics of the WEC: latching control (LC) locks the device from time to time so that it does not move, while declutching control (DC) bypasses the power-take-off (PTO) mechanism for certain time intervals. LC and DC are easy to implement, hence are of great interest to practitioners [3]. For monochromatic waves and linear dynamics, LC is optimal when the device resonance period is shorter than the wave period and DC is optimal when the device resonance period is longer than the wave period [4]. We demonstrate that latching and declutching control can be combined, which we call latching-declutching control (LDC), and modeled as a hybrid state timed automaton. LDC therefore aims to inherit the properties of both LC and DC.

Two approaches for LC are described in [5]: the first one is based on a semi-analytical solution, which can only be applied to regular waves, and the second one uses optimal control by modeling the latching action as a large damping (called

‘weak’ latching). In [6] optimal control theory was used for DC and the problem was solved using an indirect approach, i.e. where the optimality conditions are first derived before being discretized. In [7], where LDC is referred to as latched-operating-declutched control, ‘weak’ modelling is used; the binary constraints on the control inputs are relaxed and the system modeled as a continuous-variable controlled system and ‘weak latching’ is used in the model. The optimal control problem is solved using an indirect approach, similar to [6]. The optimal control problem arising from our formulation will instead be modeled as a two-dimensional optimization problem, which is more suitable for real-time implementation.

The control method in this paper is based on a derivative-free coordinate-search (CS) optimization algorithm [8]. This is because the cost function in the optimal control problem is discontinuous, in general, and the decision variables are discretized, due to the sampled-data nature of the computer-controlled system. We also utilize the quantized nature of the control variable, extending our one-dimensional coordinate search algorithm in [9] to the higher dimensional case. The algorithm allows reduces the number of function evaluations compared to the non-quantized version. The proposed algorithm is then applied to the LDC problem in a receding horizon control scheme. That is, at each sampling instance, we solve an optimization problem over a certain time horizon and implement only the first part of the computed best control action. We will also show that it is beneficial to co-design the physical parameters together with the control strategy.

The paper is organized as follows: In Section II, an uncontrolled WEC model will be described, followed with the model for LDC and the definition of the optimal control problem. LC and DC will also be discussed as two special cases of LDC. We propose two closed-loop formulations in Section III, namely one that uses past data while the other relies on future predictions. In Section IV we will demonstrate that the objective function of the optimal control problem can be discontinuous, which motivates the use of derivative-free optimization algorithms. For a computer-controlled device, the control action can only be implemented at each sampling instance, hence the optimization problem has discrete variables. We therefore adapt a derivative-free coordinate search method to the optimization of a function over a discretization of the Euclidean space. Section V will show that it is beneficial to co-design the system with the controller in consideration, rather than optimally tuning the system followed with designing the control strategy. We also present simulation results for LDC, LC and DC. Our new coordinate-search method is compared to other optimization algorithms in Section VI, where it is shown

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Fig. 1. 1-DOF heaving buoy WEC.

that the proposed algorithm outperforms others in terms of the number of function evaluations. Section VII presents some conclusions and outlines possibilities for further research.

II. MODEL AND PROBLEM FORMULATION

A. Latching-Declutching Control

The model of WECs used here is general for a type of WEC known as a point absorber; see Figure 1. Similar models are widely used in WEC control studies, due to simplicity and sufficient accuracy for wave energy control applications [5], [7], [10]. Here we only consider the 1 degree-of-freedom (DOF) case for the sake of simplicity, but the concept and the algorithm presented can easily be generalized to multi-DOF systems or other type of WECs, such as line absorbers. In the time domain, the model is given by:

$$(M + \mu_\infty)\ddot{\zeta}(t) + \int_0^t K(t - \tau)\dot{\zeta}(\tau)d\tau + B_{pto}\dot{\zeta}(t) + (k_s + \rho g S)\zeta(t) = F_e(t), \quad (1)$$

where ζ is the displacement of the WEC buoy from the equilibrium, M is the mass of the buoy, μ_∞ is the infinite frequency added mass, $K(\cdot)$ is the impulse response kernel and can be calculated using time-domain simulations via software, such as WAMIT and ACHIL3, ρ is the water density, g is the gravity constant, S is the submerged cross-section area of the buoy and $F_e(\cdot)$ is the excitation force due to the incident waves. B_{pto} and k_s are the power-take-off (PTO) damping coefficient and the spring constant of the PTO mechanism, respectively. The PTO mechanism converts mechanical energy to electrical energy, e.g. via a linear alternator or hydraulic system and rotary generator. The details of the PTO are abstracted away here, but see [2] for an introduction.

The system (1) is a differential-integral equation. The integral term causes difficulty in analysis, but can be approximated by a finite-dimensional state-space model [11]–[13]. The approximated term is the damping-like part of the radiation force (the convolution integral in (1)), which is $F_{rad}(t) := \int_0^t K(t - \tau)\dot{\zeta}(\tau)d\tau$. The approximating state-space system is of the form

$$\dot{q}(t) = A_r q(t) + B_r \dot{\zeta}(t), \quad F_{rad}(t) = C_r q(t) + D_r \dot{\zeta}(t) \quad (2)$$

where $q(t) \in \mathbb{R}^{n_r}$ is the approximation state and (A_r, B_r, C_r, D_r) are the approximating coefficient matrices, which can be calculated via the impulse response to state-space method of [13]. The state of the approximated model

Fig. 2. State transition diagram.

is defined as $p(t) := [\zeta(t) \ v(t) \ q^\top(t)]^\top$, where the velocity $v(t) := \dot{\zeta}(t)$. Note that F_e is the convolution of wave elevation and the impulse response of the device to the elevation [11]. Therefore, the excitation force F_e can also be calculated in a similar way.

The LDC system can be modeled as a timed hybrid automaton, as depicted in Figure 2. The system has three modes: a PTO-active mode (power is being generated), latched mode (velocity is kept at zero) and declutched mode (device is free to move and no power is generated). We define two binary functions δ_ℓ and δ_a , where $(\delta_\ell(t), \delta_a(t)) := (1, 0)$ in the declutched mode, $(\delta_\ell(t), \delta_a(t)) := (1, 1)$ in the PTO-active mode and $(\delta_\ell(t), \delta_a(t)) := (0, 1)$ in the latched mode. In each mode, the WEC can therefore be modeled as a linear system that satisfies

$$\begin{aligned} \dot{p}(t) &= g(p(t), \delta_\ell(t), \delta_a(t), F_e(t)) \\ &:= \begin{bmatrix} \delta_\ell(t)v(t) \\ \frac{\delta_\ell(t)}{M + \mu_\infty} (F_e(t) - (k + \rho g S)\zeta(t) \\ \quad - (B_{pto}\delta_a(t) + D_r)v(t) - C_r q(t)) \\ A_r q(t) + \delta_\ell(t)B_r v(t) \end{bmatrix}. \end{aligned} \quad (3)$$

The variable Ξ_ℓ indicates how much time has gone by since the system switched to the latched mode, whereas Ξ_a indicates how much time has gone by since the system was in the PTO-active mode and the velocity was zero; note that the timers are always increasing inside each mode. The control inputs/manipulated/decision variables are ℓ and a .

Starting in the PTO-active mode, the device can either be latched or declutched for some time to alter the dynamics. To minimize risk of damage, we restrict latching to only happen when $v(t) = 0$. Likewise, the device can leave the declutched mode and enter the PTO-active mode only when $v(t) = 0$. Let a half-period be defined as the time from when $v(t)$ becomes 0 to the next time when $v(t)$ becomes 0 (hence, if the latched mode is involved, the time when latching starts is the beginning of a half-period). We use a to denote how long the system should be outside the declutched mode and use ℓ to denote how long the system should be in the latched mode within a half-period. The rules for the mode transitions are as follows:

- The device leaves the declutched mode only if the velocity is zero. When $v(t) = 0$, $\Xi_\ell(t)$ and $\Xi_a(t)$ are reset. If $\ell(t) = 0$ and $a(t) > 0$, then the system switches to the

Fig. 3. Energy absorbed in kWh against latching time $\hat{\ell}$ and PTO-active time $\hat{a}$ under a JONSWAP excitation force with peak period $T_p = 10$ s over 2000 s. The initial conditions are $p(0) = 0$, $\delta_\ell(0) = 1$, $\delta_a(0) = 0$, $\Xi_\ell(t) = 0$ and $\Xi_a(0) = 0$. Other parameters are in the Appendix.

PTO-active mode. If $\ell(t) > 0$, then the system switches to the latched mode, regardless of the value of $a(t)$;

- The device leaves the latched mode if and only if $\Xi_\ell(t) > \ell(t)$. If $\Xi_a(t) \leq a(t)$, then the system switches to the PTO-active mode; otherwise the system switches to the declutched mode; it follows that the switch from latched mode to PTO-active mode is possible only if $a(t) > \ell(t)$. Note that Ξ_a is not reset when the system switches from the latched mode to the PTO-active mode;
- The system switches from the PTO-active mode to the declutched mode if $\Xi_a(t) > a(t)$. When $v(t) = 0$, $\Xi_\ell(t)$ and $\Xi_a(t)$ are reset. The device will switch from the PTO-active mode to the latched mode if $v(t) = 0$, $\ell(t) > 0$ and $\Xi_a(t) \leq a(t)$;

The state vector of the hybrid system is defined as

$$z(t) := [p^\top(t) \ \delta_\ell(t) \ \delta_a(t) \ \Xi_\ell(t) \ \Xi_a(t)]^\top.$$

Full state feedback is assumed here, but an estimator can be designed by treating the switched system as a linear time-varying system or differential inclusion, as in [14].

The aim is to compute functions ℓ and a such that the energy generated is maximized. Instead of solving this challenging infinite-dimensional problem, we consider what would happen if ℓ and a were to be constant over a short time interval $[t_0, t_f]$. This results in the following optimal control problem with only two decision variables:

$$\max_{(\hat{\ell}, \hat{a})} J(\hat{\ell}, \hat{a}, z(t_0), F_e, t_0, t_f) := \int_{t_0}^{t_f} B_{pto} \delta_a(s) v^2(s) ds \quad (4a)$$

$$\text{s.t. } \ell(s) = \hat{\ell} \in [0, \bar{\ell}], \ a(s) = \hat{a} \in [0, \bar{a}], \ \forall s \in [t_0, t_f], \quad (4b)$$

where $B_{pto} \delta_a(s) v^2(s)$ is the instantaneous power absorbed by the PTO and the functions (δ_a, v) satisfy the hybrid dynamic rules and equations defined above with initial state $z(t_0)$ and excitation force profile $F_e|_{[t_0, t_f]}$. The maximum allowed values for latching time $\bar{\ell}$ and PTO-active time $\bar{a}$ are given by $\bar{\ell}$ and $\bar{a}$, respectively.

Figure 3 shows the contours of the cost function $(\hat{\ell}, \hat{a}) \mapsto J(\hat{\ell}, \hat{a}, z(0), F_e, 0, 2000)$ for a Joint North Sea Wave Project (JONSWAP) spectrum-generated excitation force, which is often used to simulate irregular wave conditions [5].

As will be discussed in Section III, (4) will be solved repeatedly in a closed-loop receding horizon scheme with a

sample period $h < t_f - t_0$; the solution of (4) is implemented over the interval $[t_0, t_0 + h]$ before solving (4) again, but with a shifted time interval $[t_0 + h, t_f + h]$ and new initial state $z(t_0 + h)$, etc. The actual ℓ and a trajectories that are applied to the WEC therefore change every h seconds, despite constraining ℓ and a to be constant when solving (4). This implementation can therefore be considered to be a predictive controller with the simplest possible implementation of move blocking [15], namely one block per control input.

B. Two Special Cases

1) *Latching Control (LC)*: An LC system contains only two modes, namely the PTO-active mode and latched mode. The LDC model above contains LC as a special case if $\delta_a(0) = 1$ and if we constrain $a(t) = \infty$ so that $\delta_a(t) = 1$ for all t . The optimal control problem (4) for LC therefore reduces to an optimization problem with only one decision variable: $\max_{\hat{\ell} \in [0, \bar{\ell}]} J(\hat{\ell}, \infty, z(t_0), F_e, t_0, t_f)$. A more detailed discussion of this formulation can be found in [16].

2) *Declutching Control (DC)*: In DC the system is never latched, so that the two modes are the PTO-active mode and the declutched mode. The LDC model above contains DC as a special case if $\delta_\ell(0) = 1$ and if we constrain $\ell(t) = 0$ so that $\delta_\ell(t) = 1$ for all t . The optimal control problem (4) for DC therefore reduces to an optimization problem with only one decision variable: $\max_{\hat{a} \in [0, \bar{a}]} J(0, \hat{a}, z(t_0), F_e, t_0, t_f)$. A detailed discussion of this formulation can be found in [9].

III. SAMPLED-DATA RECEDING HORIZON CONTROL

In computer-controlled applications the system is often discretized in time with piecewise-constant inputs. At each sampling instant the state estimate, along with some other information, is fed to the controller (in this case an optimization solver) to obtain a control action.

The hybrid system and optimal control problem is therefore now discretized with a sufficiently small sample period $h > 0$ and mode changes occur only at each sample time $t \in h\mathbb{Z}$. The velocity could go to zero in-between sample instants, hence the condition $v(t) = 0$ in the transition diagram of Figure 2 is now replaced with the weaker condition of checking whether the velocity has changed sign, i.e. $v(t-h)v(t) < 0 \Rightarrow v(\tau) = 0$ for some $\tau \in [t-h, t]$. Because the WEC model is linear in-between sample instants, the solution to (3) can be discretized exactly so that

$$p(\tau) = [\zeta(\tau) \ v(\tau) \ q^\top(\tau)]^\top = p(t) + \hat{g}(p, \delta_\ell, \delta_a, F_e; t, \tau) \quad (5)$$

for all $\tau \in [t, t+h)$, $t \in h\mathbb{Z}$, where

$$\hat{g}(p, \delta_\ell, \delta_a, F_e; t, \tau) := \int_t^\tau g(p(s), \delta_\ell(t), \delta_a(t), F_e(s)) ds.$$

Algorithm 1 implicitly defines the state transition function $\phi(\cdot)$ for the hybrid system $z(t+h) = \phi(z(t), v(t-h), \ell(t), a(t))$.

Algorithm 1 can also be used to compute the sampled-data cost $J_h(\hat{\ell}, \hat{a}, z(t_0), F_e, t_0, t_f) := c(t_f)$ if $c(t_0) := 0$, given $z(t_0)$ and $(\ell(t), a(t)) = (\hat{\ell}, \hat{a})$ for all $t \in \{t_0, t_0+h, \dots, t_f-h\}$.

Algorithm 1 Sampled-data State and Cost Update

Require: State $z(t)$, velocity $v(t-h)$, inputs $(\ell(t), a(t))$ and cost $c(t)$, $t \in h\mathbb{Z}$

Ensure: State $z(t+h) = \phi(z(t), v(t-h), \ell(t), a(t))$ and updated cost $c(t+h) = c(t) + \int_t^{t+h} B_{pto} \delta_a(t) v^2(s) ds$, $t+h \in h\mathbb{Z}$

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1: switch  $(\delta_\ell(t), \delta_a(t))$  do
2:   case (1, 1) ▷ in PTO-active mode
3:     if  $v(t-h)v(t) < 0$  then
4:       Reset timers  $\Xi_\ell(t) := 0, \Xi_a(t) := 0$ .
5:       if  $\ell(t) > 0$  then
6:         Change mode  $(\delta_\ell(t), \delta_a(t)) := (0, 1)$ 
7:       else if  $\Xi_a(t) > a(t)$  then
8:         Change mode  $(\delta_\ell(t), \delta_a(t)) := (1, 0)$ 
9:     case (0, 1) ▷ in latched mode
10:      if  $\Xi_\ell(t) > \ell(t)$  then
11:        if  $\Xi_a(t) > a(t)$  then
12:          Change mode  $(\delta_\ell(t), \delta_a(t)) := (1, 0)$ 
13:        else
14:          Change mode  $(\delta_\ell(t), \delta_a(t)) := (1, 1)$ 
15:      case (1, 0) ▷ in declutched mode
16:        if  $v(t-h)v(t) < 0$  then
17:          Reset timers  $\Xi_\ell(t) := 0, \Xi_a(t) := 0$ .
18:          if  $\ell(t) > 0$  then
19:            Change mode  $(\delta_\ell(t), \delta_a(t)) := (0, 1)$ 
20:          else if  $a(t) > 0$  then
21:            Change mode  $(\delta_\ell(t), \delta_a(t)) := (1, 1)$ 
22:       $p(t+h) := p(t) + \hat{g}(p, \delta_\ell, \delta_a, F_e; t, t+h)$ 
23:       $\Xi_\ell(t+h) := \Xi_\ell(t) + h$  and  $\Xi_a(t+h) := \Xi_a(t) + h$ 
24:       $(\delta_\ell(t+h), \delta_a(t+h)) := (\delta_\ell(t), \delta_a(t))$ 
25:       $c(t+h) := c(t) + B_{pto} \delta_a(t) \int_t^{t+h} v(s) ds$ 

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$h\}$, with $t_0, t_f \in h\mathbb{Z}$. It follows that (4) can be replaced by the sampled-data optimal control problem

$$\max_{(\hat{\ell}, \hat{a}) \in S(\bar{\ell}) \times S(\bar{a})} J_h(\hat{\ell}, \hat{a}, z(t_0), F_e, t_0, t_f), \quad (6)$$

where $S(b) := (h\mathbb{Z}) \cap [0, b]$. Note that, rather unusually, the decision variables in (6) take on values from a finite subset of the reals instead of an uncountable domain, due to the fact that mode transitions are made only at each sample instant. A new derivative-free optimization algorithm for solving this problem is given below in Section IV.

We now define two formulations: one that uses recorded wave data and one that uses the predictions of incoming waves.

Because the peak period of ocean waves is stable over a short interval [7], [17], the optimal latching time and PTO-active time for previous waves can be considered a good estimate of the optimal variables for incoming waves. The past data problem at time $t \in h\mathbb{Z}$ is defined as

$$\max_{(\hat{\ell}, \hat{a}) \in S(\bar{\ell}) \times S(\bar{a})} J_h(\hat{\ell}, \hat{a}, z(t-T), F_e, t-T, t), \quad (7)$$

where $T \in h\mathbb{N}$ is the horizon length and $[\ell_p^*, a_p^*](z, F_e, t)$ denotes the solution to (7).

On the other hand, a good estimate for the future excitation force might be available. A future data problem, which uses

the current state and predicted force, is defined as

$$\max_{(\hat{\ell}, \hat{a}) \in S(\bar{\ell}) \times S(\bar{a})} J_h(\hat{\ell}, \hat{a}, z(t), F_e, t, t+T), \quad (8)$$

where $[\ell_f^*, a_f^*](z, F_e, t)$ denotes the solution to (8).

Note the differences between the initial states, starting times and final times of (7) and (8). Given knowledge of the state $z(t-T)$ and excitation force F_e over the time interval $[t-T, t]$, the assumption is that the solution to the past data problem (7) would be approximately equal to the solution to the future data problem (8) at time t if the horizon length T is ‘sufficiently’ short. Section V contains numerical results to show when this is valid.

As is standard in predictive control, we propose that problem (7) or (8) is solved at each sample instant and that the solution is implemented in a receding horizon fashion, i.e.

$$[\ell, a](t) = [\ell_\bullet^*, a_\bullet^*](z, F_e, kh), \forall t \in [kh, kh+h), k \in \mathbb{Z} \quad (9a)$$

and the state of the closed-loop system satisfies

$$z(t+h) = \phi(z(t), v(t-h), \ell_\bullet^*(z, F_e, t), a_\bullet^*(z, F_e, t)), \quad (9b)$$

where $\bullet = p$ for the past data problem (7) and $\bullet = f$ for the future data problem (8). Note that the receding horizon policy is a time-varying state feedback law.

For the LC and DC special cases, the implementation is similar and details can be found in [16] and [9], respectively.

IV. DERIVATIVE-FREE OPTIMIZATION

Because of the switching dynamics, the cost function J is non-smooth and discontinuous, in general, even for the continuous-time case with $h = 0$. Furthermore, the decision variables in (6) are discrete if $h > 0$. Hence, derivative-free optimization algorithms are suitable candidates for solving the optimal control problems defined above.

The discontinuity in the cost function is usually caused by sudden changes in the velocity trajectory. Figures 4(a) and 4(b) demonstrate such a situation for LC. Figure 4(a) shows that when $\hat{\ell}$ increases from 6.2 s to 6.3 s, there is a sudden change in the velocity trajectory. Consequently, in Figure 4(b) the cost function is discontinuous between 6.2 s and 6.3 s.

This feature also applies to DC. Figure 4(c) shows that when $\hat{a}$ changes from 4 s to 4.1 s, the trajectory changes instantly, resulting in a discontinuity in the objective function, as can be seen in Figure 4(d).

Clearly, if the optimal control problems for LC and DC could have discontinuous cost functions, then the same difficulty could arise for the more general case of LDC. Gradient-based algorithms are therefore likely to encounter problems when computing the solution to (6).

Algorithm 2 is a new, derivative-free algorithm, which modifies the well-known coordinate-search (CS) method [8] to a function defined over a discretization of the Euclidean space. A key feature of the algorithm is that the objective function is evaluated at most only once for each point in the domain and that there are at most $2n$ function evaluations at each iteration, where n is the number of decision variables. Since the function evaluation takes up most of the computation

Fig. 4. Cost function is discontinuous, in general: (a) and (b) are for latching control, while (c) and (d) are for declutching control.

time, these properties are particularly useful in our application, where $n = 2$ for LDC and $n = 1$ for LC and DC.

Algorithm 2 computes a (suitably-defined) local maximum of a function $f : \mathcal{X} \rightarrow \mathbb{R}$, where $\mathcal{X}$ is a finite set of real vectors in $h\mathbb{Z}^n$ and $h \in \mathbb{R}_{>0}$. In the algorithm description, $D_{\oplus} := \{e_1, \dots, e_n, -e_1, \dots, -e_n\}$ is a maximal positive basis [8] for $\mathbb{R}^n$, where e_i is the unit vector with the i^{th} element equal to 1, $\lceil \cdot \rceil$ is the ceiling operator, $\lfloor \cdot \rfloor$ is the floor operator and the superscript (k) denotes the k^{th} iterate. Note that the step size $\alpha^{(k)} \in \mathbb{N}$ will increase if a better iterate is found and decrease otherwise. A local maximum is found if the step size does not change and $\alpha^{(k)} = 1$.

Algorithm 2 returns locally optimal points. However, with suitable step size update parameters (e.g. see the Appendix), the solver is able to find the global maximum in most practical cases, especially if the algorithm is warm started with the solution from the previous sample instant. Hence, at each sample time $t \in h\mathbb{Z}$, the initial guess $x^{(0)} := (\ell(t-h), a(t-h))$ and the solution $x^* := (\ell(t), a(t))$ so that (ℓ, a) satisfies the receding horizon control law (9a).

V. CYBER-PHYSICAL CO-DESIGN AND SIMULATIONS

Traditionally, the physical system is designed without considering the control strategy; the uncontrolled system is tuned to its optimal performance against certain working conditions. Following this, the controller is designed to maximize the system's performance. Co-design takes a different approach and designs the physical system and controller (cyber system) at the same time. In other words, the optimal physical parameters are a function of the control parameters and vice versa.

For simplicity, we will only consider the PTO damping B_{pto} as the physical design parameter. Figure 5 is the contour plot of

Algorithm 2 Derivative-free coordinate-search (CS) with discretized decision variables

Require: $f : \mathcal{X} \rightarrow \mathbb{R}$: objective function with finite domain $\mathcal{X} \subset h\mathbb{Z}^n \subset \mathbb{R}^n$, where sampling distance $h \in \mathbb{R}_{>0}$; $x^{(0)} \in \mathcal{X}$: initial guess; $\alpha^{(0)} \in \mathbb{N}$: initial step size; $\gamma > 1$, $\beta \in (0, 1)$: step size update parameters;

Ensure: x^* is a local maximizer of f , i.e. $f(x^*) \geq f(x)$ for all $x \in \{x^* + d \in \mathcal{X} \mid d \in hD_{\oplus}\}$.

1: Define

$$\hat{f}(x) := \begin{cases} f(x) & \text{if } x \in \mathcal{X} \\ -\infty & \text{if } x \notin \mathcal{X} \end{cases}$$

2: Set k to 0 and $y^{(0)} := f(x^{(0)})$

3: Initialize history set $H^{(0)} := \{(x^{(0)}, y^{(0)})\}$

4: **repeat**

5: Compute poll set $P^{(k)} := \{x^{(k)} + \alpha^{(k)}d \mid d \in hD_{\oplus}\}$

6: Evaluate $\hat{f}$ at points in $P^{(k)}$ not yet visited to form

$$E^{(k)} := \left\{ (x, \hat{f}(x)) \mid x \in P^{(k)} \text{ and } \nexists y : (x, y) \in H^{(k)} \right\}$$

7: Update history set $H^{(k+1)} := H^{(k)} \cup E^{(k)}$

8: **if** $\{(x, y) \in E^{(k)} \mid y > y^{(k)}\}$ is non-empty **then**

9: Choose any maximizing pair

$$(x^{(k+1)}, y^{(k+1)}) \in \arg \max_{(x, y) \in E^{(k)}} y$$

10: $\alpha^{(k+1)} := \lceil \gamma \alpha^{(k)} \rceil$

11: **else**

12: $x^{(k+1)} := x^{(k)}$ and $y^{(k+1)} := y^{(k)}$

13: $\alpha^{(k+1)} := \max\{1, \lfloor \beta \alpha^{(k)} \rfloor\}$

14: Increment k by 1

15: **until** $\alpha^{(k)} = \alpha^{(k-1)}$

16: $x^* := x^{(k)}$

Fig. 5. Power contours without control subjected to JONSWAP waves.

the PTO with no controller. Figure 5 suggests the optimal value for B_{pto} should be about 58–87 kN/m for a significant range of wave periods. Thus, if we followed the standard procedure, we would take B_{pto} around 78 kN/m for maximum PTO.

A. Latching-Declutching Control

Figure 6 is a power contour plot of the PTO of an LDC system. Compared to the uncontrolled case, the contours change significantly; the dependence on B_{pto} is much weaker. Therefore, one could design B_{pto} to meet some other criteria, rather than only considering power generation, e.g. one may wish to build a small WEC, which is cheaper to construct and can be placed closer together in a wave farm array.

Fig. 6. Power contours with LDC subjected to JONSWAP waves.

(a) Time domain result for future data formulation ($\delta_\ell(t) = 0$ when $v(t) = 0$, otherwise $\delta_\ell(t) = 1$).

(b) Energy absorption.

Fig. 7. Results for LDC subjected to a JONSWAP wave with $T_p = 7$ s, $B_{pto} = 900$ kNs/m.

Figure 7(a) shows the time domain results for the future data formulation for LDC (assuming perfect prediction). Figure 7(b) shows the total energy absorbed by both the future and past data formulations. Both formulations result in a 3-fold increase in energy generated. It is worth noticing that, at around 100 s in Figure 7(b), the past data formulation outperformed the future data formulation for a short interval. This is because the wave's characteristics changed significantly at around this time.

B. Latching Control

Figure 8 is the contour plot of the power take-off of an LC device for different wave periods and B_{pto} . Compared to the uncontrolled case, the maximum occurs at a peak period of about $T_p = 15$ s and the PTO bandwidth is also increased.

Figure 9 shows that by co-design one can achieve about 10% improvement for waves with long peak periods compared to first optimizing the physical parameters without control. Figure 9 plots the ratio of the power in Figure 8 to the power at the same T_p , but with $B_{pto} = 78$ kNs/m as obtained from Figure 5. Via co-design one might instead choose $B_{pto} =$

Fig. 8. Power contours with latching control subjected to JONSWAP waves.

Fig. 9. Ratio of power generated with LC to PTO with LC and a fixed $B_{pto} = 78$ kNs/m. The gray area indicates where the ratio is above 1.

(a) Time domain results for future data formulation. (b) Power and energy absorption.

Fig. 10. Results for LC subjected to a JONSWAP wave with $T_p = 10$ s ($B_{pto} = 300$ kNs/m).

59 kNs/m, which maximizes the PTO in Figure 8 and is smaller than the previously-obtained 78 kNs/m.

Figure 10(a) shows the time domain result for the future data formulation. Figure 10(b) shows the accumulated energy absorption for both future and past data formulations. The past data formulation gives 5 times more energy absorption than the uncontrolled case. The future data formulation, assuming accurate predictions, will give another 2-fold increase over the past data formulation.

C. Declutching Control

Figure 11 shows the power take-off under DC for different wave periods and B_{pto} . The range for B_{pto} is now increased to 1 MN/m to show the power-take-off for very large B_{pto} . Figure 11 reveals that, unlike the uncontrolled or LC cases, more power can be generated with larger values of B_{pto} .

Figure 12 shows the ratio of the power generated by DC compared to the uncontrolled system. Note that there is little improvement for long peak periods. However, for short peak

Fig. 11. Power contours with declutching control subjected to JONSWAP waves.

Fig. 12. Ratio of power generated with DC to PTO without control.

Fig. 13. Ratio of power generated with DC to power generated with DC and a fixed $B_{pto} = 78$ kNs/m. The black dotted line shows where the ratio is 1, the light gray area shows where the ratio is above 1.

periods, declutching control with $B_{pto} = 900$ kNs/m is able to achieve at least a 27% improvement over an uncontrolled WEC, with a maximum of 170% improvement at about $T_p = 6$ s. Therefore, after co-design we conclude that a larger B_{pto} will give better performance under declutching control, hence we choose $B_{pto} = 900$ kNs/m, which can be achieved by large devices, such as SEAREV [6].

Figure 13 quantifies the benefits of co-design for DC, compared to first optimizing the system without any control. For waves with $T_p < 6.2$ s, the controlled system with $B_{pto} = 78$ kNs/m performs better. However, normally WECs operate in environments with $T_p > 6$ s [5]. For incident waves with longer peak periods, note that as B_{pto} increases, the PTO ratio increases and achieves a maximum of 4.5 at $B_{pto} = 900$ kNs/m with $T_p = 11$ s. Compared to LC, the effect of co-design for DC is more significant.

Time domain results are shown in Figure 14(a). Figure 14(b) shows the accumulated energy absorption. The past data formulation gives about twice more energy than the uncontrolled case. The future data formulation gives another 16% increase over the past data formulation.

(a) Time domain results for future data formulation (excitation force is scaled).

(b) Energy absorption.

Fig. 14. Results for DC subjected to a JONSWAP wave with $T_p = 6$ s, $B_{pto} = 900$ kNs/m.

(a) Algorithm 2 for LDC.

(b) Algorithm 2 for DC.

(c) Simulated Annealing for LDC.

(d) Monotonic search for DC.

Fig. 15. Number of function evaluations taken by different algorithms.

VI. COMPARISON WITH OTHER OPTIMIZATION METHODS

We compare Algorithm 2 to other optimization algorithms when subjected to a JONSWAP wave. Figure 15(a) shows that when Algorithm 2 is applied to LDC the average number of function evaluations is 11.63, with a maximum of 33 evaluations. When Algorithm 2 is applied to DC, the average number of function evaluations per problem is 3.2, with the maximum number of function evaluations being 8 and minimum number of evaluations being 3; this is shown in Figure 15(b). All results are for the future data problem.

Simulated annealing (SA) is a widely-used derivative-free global optimization method [18]. Analogous to physical annealing processes, which are stochastic, SA has the chance to converge to the global maximum. As shown in Figure 15(c), the number of function evaluations for SA (see Appendix for parameters) is significantly larger than that of Algorithm 2.

For the two special cases of LC and DC, both optimization problems are one-dimensional. Since the decision variable is discrete, it is natural to start the search from 0 and increase the decision variable in each iteration by h until the objective function starts to decrease. Figure 15(d) shows the result when this monotonic search is applied to DC. The maximum number of function evaluations is 12 and the average is 8.6. Compared to the result for Algorithm 2, shown in Figure 15(b), both the maximum and average number of function evaluation are larger. Moreover, monotonic search is easily trapped in a local maximum, whereas Algorithm 2 almost always converges to the global maximum in practice.

VII. CONCLUSIONS AND FUTURE WORK

We proposed a computationally tractable formulation that unifies latching, declutching and latching-declutching control of a WEC using a hybrid timed automaton model. To solve the resulting optimal control problems, we also proposed a derivative free optimization algorithm that utilizes the quantized nature of the decision variables. The simulation results

show the controller's ability to increase the power-take-off of the WEC for different incoming irregular waves. The online closed-loop control formulation enables the algorithm to deal with changing sea conditions. Moreover, the computational burden of our derivative-free optimization algorithm could be suitable for real-time implementation in the near future.

Co-design can significantly increase the performance and robustness of a WEC, compared to first optimizing the physical system, followed by controller design. This is because the controller alters the closed-loop behaviour of a WEC, both qualitatively and quantitatively. Not only does the controller allow one to increase the power take-off for a given peak period, but the controller also allows one to increase the range of wave periods for which one can generate more energy, thereby extending the set of profitable operating conditions of the device. Optimal co-design could become a key methodology in determining the viability of generating power from waves. Co-design is especially effective when applied to declutching control and reveals an important feature of latching-declutching control, namely the bandwidth depends weakly on the PTO damping.

Future research could consider the latching and PTO-active time to be time-varying over the horizon in the control problem, rather than constraining them to be constant, hence the optimization problem will be larger and more challenging to solve. It will be interesting to determine how much more power may be generated by solving this generalized problem.

The incorporation of state constraints, e.g. due to excursion limits, would also be an important problem for further work. One could extend Algorithm 2 to deal with more general constraints using, for example, a penalty function approach.

One could also investigate how accurate wave predictions methods need to be in order for the future data formulation to perform sufficiently better than the past data formulation.

APPENDIX

For simulations, the parameters for the WEC model are $n_r = 5$, $M = 707\text{ t}$, $\mu_\infty = 244\text{ t}$, $g = 9.81\text{ m/s}^2$, $\rho = 1000\text{ kg/m}^3$, $S = 78.5\text{ m}^2$, $k_s = 240\text{ kN/m}$. The computed approximating matrices are

$$A_r = \begin{bmatrix} -0.5540 & 0.7775 & 0.4751 & 0.1593 & 0.1100 \\ -0.7775 & -0.0110 & -0.0534 & -0.0256 & -0.0168 \\ -0.4751 & -0.0534 & -0.5005 & -0.4424 & -0.2536 \\ 0.1593 & 0.0256 & 0.4424 & -0.8235 & -3.0958 \\ -0.1100 & -0.0168 & -0.2536 & 3.0958 & -0.5834 \end{bmatrix},$$

$$B_r = [-137.2169 \quad -17.4945 \quad -50.1384 \quad 20.0158 \quad -13.4921]^\top,$$

$$C_r = [-137.2169 \quad 17.4945 \quad 50.1384 \quad 20.0158 \quad 13.4921],$$

$$D_r = 302680.$$

The horizon length $T = 30\text{ s}$, sample period $h = 0.1\text{ s}$ and upper limits $\bar{\ell} = \bar{a} = 10\text{ s}$. The parameters for Algorithm 2 are $\alpha^{(0)} = 2$, $\beta = 0.5$, $\gamma = 1.5$. The initial temperature for simulated annealing is 10, the cooling rate is 2 and the initial random walk radius is 1.

All irregular waves were generated from a JONSWAP spectrum with wave elevation $H_s = 2\text{ m}$. In the simulations, different peak periods T_p were used for different control strategies, since LC typically performs better when T_p is large and DC performs better when T_p is small.

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