The Residual Stress Intensity Factors for Cold-Worked Cracked Holes: a Technical Note

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Abstract

Cold—working of riveted holes reduces the stress intensity factor associated with cracks that may develop at the hole boundary, by creating a compressive residual stress field. The residual stress field is determined using the finite element method and the reduction of the stress intensity factor for different values of the interpenetration is evaluated with the weight function method, in the case of an infinite plate made from an elastic perfectly plastic material, and having a hole with two symmetrical cracks. Once the weight function of the structure is known, further calculation of the stress intensity factors for different loadings like remote uniform stress, point load that simulates the action of the rivet, etc., can be performed without difficulty.

1. Introduction

Problems related with ageing aircraft may be reduced by enhancing the fatigue performance of aeronautical structures, especially in critical zones, acting as stress raisers, such as access and riveted holes. Fastener hole fatigue strength may be enhanced by creating compressive residual circumferential stresses around the hole. This technique – cold-work – has been used in the aeronautical industry for the past thirty years to delay fatigue damage and retard crack propagation. Research has been concentrated mainly on modelling the residual stress field using analytical or numerical two-dimensional (2D) or three-dimensional (3D) methods [1–5], on the experimental measurement of the residual stress field [6,7], on the experimental characterization of the cold-worked hole behaviour in fatigue [8,10], and on the stress intensity factor calibration for cracks that may develop after cold work [2], [11,12]. Subtopics considered include the consideration of thickness effects [5,13], the consideration of eventual pre-existence of cracks of various sizes before hole expansion is carried out [8], the possible re-cold-working of already cold-worked holes [14], and the stress analysis of neighbouring cold-worked holes [15].

The compressive circumferential residual stress field around the rivet holes is created by applying pressure on the hole surface by means of a mandrel. Once the pressure is removed, the desired residual compressive stress field is achieved. According to Leon [16], the main benefits associated with the improvement of the fatigue life are the reduction of unscheduled maintenance, increasing the time between inspection intervals, reduction of maintenance costs and improvement of aircraft readiness.

Two cold-working processes are normally used in the aeronautical industry [16,17]: the split sleeve process, using a solid tapered mandrel and a lubricated split sleeve, and the split mandrel process, using a lubricated, hollow and longitudinally slotted tapered mandrel. In service conditions, cracks may initiate and grow from the surface of the hole. However, due to

the compressive residual stress, there will be a minimum value of remote tensile stress required to open the crack. Furthermore, once the cracks are open, the respective stress intensity factor, K , will be smaller than the one obtained in the absence of cold-working. Therefore, the cold-working process retards crack growth, increasing the fatigue life of the structure. Since the reduction of the stress intensity factor is a function of the residual stresses, it is important to relate the magnitude of the residual stress field with the expansion of the mandrel (or with the pressure applied to the rivet hole) when designing a riveted connection.

The objectives of this paper are to determine the residual stress field and to characterize the effect of cold-work by means of a residual stress intensity factor associated with the residual compressive stress field. The residual stress intensity factor shows the reduction of the stress intensity factor of the cracked structure when the hole is cold worked, compared with the case when no cold work is applied. For this purpose, the weight function method is used together with the expressions of the residual stress field in order to determine the residual stress intensity factor, as a function of the interpenetration and crack length, for a given material and geometry. Finite element calculations are also performed to assess the values of the residual stress intensity factor determined with the weight function method.

3. The structure under investigation

In this study, infinite plate with a central hole of radius $R = 10$ mm with pressure acting upon the hole to create a residual stress field is studied (Fig. 1). Two symmetrical cracks develop from the hole boundary (Fig. 2). Different crack lengths, up to $a/R = 1.2$ are considered.

The residual stress field was determined for different values of the radial interpenetration, defined as:

$$i = \frac{\bar{D} - D}{D} [\%] \quad (1)$$

where $\bar{D}$ is the diameter of the mandrel and D – the initial diameter of the hole. Five different values of the interpenetrations, namely $i = 1\%$, 2% , 4% , 6% and 8% are considered. For each value of i , the residual stress intensity factor is determined for the studied crack lengths.

The material is considered as elastic-perfectly plastic, and the mechanical properties are presented in Table 1.

3. Determination of the residual stress field

In order to determine the residual stress field for each value of the interpenetration i , the finite element method is used. A finite element model is created using the code ABAQUS [18]. The mesh shown in Figure 3 consists of 10,000 plane stress quadrilateral four noded elements. Figure 3,a) shows the model of a quarter of the plate, whose dimensions are 10 times greater than the hole radius, in order to model an infinite plate. Figures 3,b), c) and d) show details of the mesh near the hole.

For determining the residual stress field, a non-linear geometric procedure is used. In the first step, the radial interpenetration is applied to an elastic-plastic material. In the second step, the radial interpenetration was removed by setting free the nodal displacements at the hole boundary. In this second step, the material is still considered to be elastic-plastic, so that reverse plasticity might be modelled. At the end of the second step, the residual stress field has already been created. A similar procedure was used by Pavier et al. [19] for obtaining the residual stress field in a finite plate with hole. Numerical values of the circumferential stress (the one that opens/closes the cracks emanating from the hole) are extracted and polynomial interpolations are used for obtaining expressions for the residual stress field.

In order to check the stress intensity factor values obtained through weight function, a third step follows, in which the material constitutive law is modified to perfectly elastic; a small crack was opened by setting free the correspondent nodal displacements; and the J integral is calculated for 20 different paths. Further steps follow, in which consecutive increasing crack length are considered. At the end, each analysis provides 20 J – integral estimates for different crack lengths. From the J – integral values, the residual stress intensity factor was computed for each crack length and for the considered radial interpenetration. The process was repeated for the all the considered values of radial interpenetration.

4. Brief description of the weight function algorithm

A very efficient method for determining the stress intensity factor is the *weight function method*, introduced by Bueckner [20]. In order to use it, it is necessary to know a complete solution (the stress intensity factor and the displacements of the crack faces) for a crack problem for one loading system. Using these results, one may obtain the solution for the stress intensity factor for the same crack configuration with any other loading.

Rice [21] showed that, if the stress intensity factor $K_{Ir}(a)$, and the displacement field $u_{Ir}(x,a)$ for a cracked body under a symmetrical loading (called *the reference case*) are known, the Mode I weight function can be determined, in the co-ordinate system shown in fig. 4, from:

$$h(x,a) = \frac{E'}{K_{Ir}} \frac{\partial u_{Ir}(x,a)}{\partial a} \quad (2)$$

where $E' = E$ for the case of plane stress, and $E' = E(1 - \nu^2)$ for plane strain.

Once the weight functions are determined for a given geometry, then the stress intensity factor for any other loading system applied to the same cracked body can be calculated by:

$$K = \int_0^a \sigma(x) \cdot h(x, a) dx = \frac{E'}{K_{Ir}} \int_0^a \sigma(x) \cdot \frac{\partial u_{Ir}(x, a)}{\partial a} dx. \quad (3)$$

In equation (3), $\sigma(x)$ are the stresses on the crack line that appear in the uncracked body due to the loading for which the stress intensity factor is calculated.

Values for the stress intensity factor for different structures and loadings can be found in several stress intensity factor handbooks [22-24], but very seldom accompanied by expressions of the crack face displacements. In order to be able to apply the weight function technique in this case, several approaches were proposed. The approach used in this paper is the one of Petroski and Achenbach [25]. They use the well-known expression of the displacement around the crack tip in an infinite cracked plate:

$$u_y(x, a) = \frac{4K}{E'} \left(\frac{a-x}{2\pi} \right)^{1/2}. \quad (4)$$

in which $K = \sigma(\pi a)^{1/2}$. Starting from this expression, they propose for the crack face displacements a series expansion having the first term in the form given by (4), and the other terms tend to zero while approaching the crack tip:

$$u(x, a) = \sum_n C_n \left(\frac{a}{2} \right)^{\frac{1}{2}-n} (a-x)^{\frac{1}{2}+n}. \quad (5)$$

From this series expansion, they used only the first two terms, written in the form:

$$u_{Ir}(a, x) = \frac{\sigma_0}{E' \sqrt{2}} \left[4F\left(\frac{a}{L}\right) a^{1/2} (a-x)^{1/2} + G\left(\frac{a}{L}\right) a^{-1/2} (a-x)^{3/2} \right], \quad (6)$$

where $F(a/L)$ and $G(a/L)$ are functions of the crack length and characteristic dimension.

The function $F(a/L) = K/[\sigma(\pi a)^{1/2}]$ can be calculated from the solutions for the stress intensity factor taken from handbooks and $G(a/L)$ is obtained from equation (3) written for the reference case $K = K_{Ir}$ (self-consistency). In this case, one obtains:

$$K_{Ir}^2 = E' \int_0^a \sigma_r(x) \cdot \frac{\partial u_{Ir}(x, a)}{\partial a} dx, \quad (7)$$

$\sigma_r(x)$ being the crack line stress in the reference case. Introducing (6) in (7), integrating with respect to a and using the known values of the reference stress intensity factor, one obtains an equation in which $G(a/L)$ is the only unknown. Solving this equation it yields that:

$$G\left(\frac{a}{L}\right) = \frac{[I_1(a) - 4F(a/L)\sqrt{a} \cdot I_2(a)]\sqrt{a}}{I_3(a)}, \quad (8)$$

with:

$$I_1(a) = \pi\sqrt{2}\sigma_0 \int_0^a F^2\left(\frac{a}{L}\right) \cdot a da, \quad (9)$$

$$I_2(a) = \int_0^a \sigma_r(x) \cdot (a-x)^{1/2} dx, \quad (10)$$

$$I_3(a) = \int_0^a \sigma_r(x) (a-x)^{3/2} dx. \quad (11)$$

Once the weight function is known, then the stress intensity factor can be determined from equation (7) for any other loading case $\sigma(x)$, as:

$$K = \frac{E'}{K_{Ir}} \int_0^a \sigma(x) \frac{\partial u_{Ir}(x, a)}{\partial a} dx = \frac{\sigma_0}{K_{Ir} \sqrt{2}} \int_0^a \sigma(x) \frac{\partial}{\partial a} \left[4F\left(\frac{a}{L}\right) a^{1/2} (a-x)^{1/2} + \right. \\ \left. + G\left(\frac{a}{L}\right) a^{-1/2} (a-x)^{3/2} \right] dx \quad (12)$$

5. Weight function procedure

In order to apply the weight function method, a reference case must be chosen. Since the weight function is independent of the loading, any loading case is suitable for obtaining it. That

is why one should choose a very simple loading case, with known results from the literature. For this work, the loading case of remote tensile stress was considered suitable. The reference stress intensity factor can be found in [22] and is given in Table 2 in the usual nondimensional form

$$F(a/R) = K_{Ir} / \sigma \sqrt{\pi a} .$$

The values given in [22] are obtained considering the crack length a' measured from the centre of the hole, and not as in the system of axes from fig. 3, in which the weight function equations are expressed. That is why the values are recalculated, taking into account the new crack length a which is $a = a' - R$ (see Fig. 2).

In order to use these values in the weight function equations, a polynomial fit should be found. The following result is obtained:

$$F\left(\frac{a}{R}\right) = 3.39 - 6.47\left(\frac{a}{R}\right) + 11.025\left(\frac{a}{R}\right)^2 - 9.964\left(\frac{a}{R}\right)^3 + 3.543\left(\frac{a}{R}\right)^4 \quad (13)$$

For applying equation (2) to determine the weight function, the expression of the crack face displacements should be derived following relation (6). The coefficient $G(a/R)$ is calculated according equations (8–11) in which the expression $\sigma_r(x)$ of the stress distribution on the crack line is given by the well known equation from theory of elasticity [26], written in the co-ordinate system from Fig. 1 as:

$$\sigma_r(x) = \frac{\sigma}{2} \left[2 + \left(\frac{R}{x+R} \right)^2 + 3 \left(\frac{R}{x+R} \right)^4 \right] \quad (14)$$

After determining the displacement variation in the reference case, the stress intensity factor can be calculated for the loading consisting of residual stress, using equation (12). A MAPLE worksheet capable of automating the calculation of K values was created and thus, a parametric study of the residual stress intensity factor for different values of the crack length and initial pressure is easily performed.

It should be mentioned that similar calculations were made by Grandt and Kullgren [27] that presented values of the stress intensity factor for a complex loading consisting of residual stress and remote uniform stress. Since their work involved a finite plate, a comparison between the results is not possible, although the residual stress intensity factor in their work can be easily obtained.

5. Results and discussion

The variation of the circumferential residual stress, obtained by the finite element procedure described above is shown in Figure 5, for all the values of interpenetration considered. An interpolation procedure was used for determining up to four degree polynomial expressions of the residual stress field, suitable for using in equation (12) in order to determine the residual stress intensity factor. One can notice that the resulting stress must be fit by two or even three different polynomial expressions, corresponding to the different trends of the curves. Consequently, the integral in equation (12) will be decomposed into two or three integrals.

The values of the residual stress intensity factor obtained with the weight function technique are shown in Table 3 for the considered values of the interpenetration. In Figure 6, the obtained results are plotted for comparison together with the results obtained by finite element method, using the J integral. From this figure, one can notice that the agreement between the weight function results and those obtained by finite element is excellent.

Conclusions

The residual stress intensity factor, meaning the reduction of the stress intensity factor due to cold work process was determined in this paper, using the residual stress values obtained

through finite element method. An approach of the weight function method was used, in which reference values of the stress intensity factor for remote uniform traction and an approximate expression of the crack face displacement were involved. The results obtained through the weight function method were checked by finite element calculations. The agreement between the results obtained by these two methods was excellent, validating the weight function approach.

The residual stress intensity factor that was calculated in this paper shows the amount with which the stress intensity factor may be reduced by performing a cold work process at the rivet holes before the structure enters in service. These results can be superimposed on the values of the stress intensity factor for different loadings encountered in industry (as remote stress or point or distributed load on the hole surface that models the action of a rivet) in order to calculate the values of the stress intensity factor for complex loadings.

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CAPTIONS FOR TABLES AND FIGURES

Table 1: Mechanical properties for elastic-perfectly plastic material

Table 2: Reference stress intensity factor for two symmetrical cracks

Table 3: Residual stress intensity factors for different interpenetrations

Figure 1: Cold worked hole in an infinite plate

Figure 2: Two symmetrical cracks emanating from the hole

Figure 3: Details of the finite element mesh

Figure 4: Co-ordinate system for the weight function equations

Figure 5: Variation of the residual stress for different values of interpenetration

Figure 6: Variation of the residual stress intensity factor for different values of interpenetration. Comparison between weight function and finite element results

Table 1

Young modulus E [GPa]	Poisson ratio ν	Yield stress σ_y [MPa]
71,4	0.3	285

Table 2

a'/R	$F(a'/R)$ [21]	a/R	$F(a/R)$
1.01	0.3256	0.01	3.2722
1.02	0.4514	0.02	3.2236
1.04	0.6082	0.04	3.1012
1.06	0.7104	0.06	2.9859
1.08	0.7843	0.08	2.8817
1.1	0.84	0.1	2.7860
1.15	0.9322	0.15	2.5811
1.2	0.9851	0.2	2.4130
1.25	1.0168	0.25	2.2736
1.3	1.0358	0.3	2.1562
1.4	1.0536	0.4	1.9711
1.5	1.0582	0.5	1.8329
1.6	1.0571	0.6	1.7262
1.8	1.0495	0.8	1.5743
2	1.0409	1	1.4721
2.2	1.0336	1.2	1.3995
2.5	1.0252	1.5	1.3235

Table 3

a/R	Residual K [MPa $\sqrt{\text{m}}$]
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	Interpenetration				
	1%	2%	4%	6%	8%
0.01	– 3.02	– 5.65	– 5.64	– 5.64	– 5.65
0.06	– 5.97	– 13.44	– 13.58	– 13.57	– 13.56
0.11	– 6.35	– 16.28	– 18.09	– 18.06	– 18.04
0.16	– 6.03	– 17.50	– 21.75	– 21.73	– 21.70
0.21	– 5.04	– 17.60	– 23.88	– 24.67	– 24.66
0.26	– 3.94	– 17.33	– 24.88	– 27.16	– 27.47
0.31	– 3.10	– 16.51	– 25.00	– 28.12	– 29.60
0.36	– 2.50	– 15.40	– 24.64	– 28.36	– 30.47
0.41	– 2.09	– 14.15	– 23.99	– 28.17	– 30.73
0.46	– 1.80	– 12.79	– 23.10	– 27.67	– 30.58
0.51	– 1.59	– 11.34	– 22.04	– 26.92	– 30.12
0.56	– 1.43	– 9.82	– 20.83	– 25.98	– 29.42
0.61	– 1.30	– 8.28	– 19.50	– 24.87	– 28.53
0.66	– 1.20	– 7.20	– 18.08	– 23.64	– 27.47
0.71	– 1.11	– 6.37	– 16.58	– 22.31	– 26.29
0.76	– 1.02	– 5.69	– 15.03	– 20.89	– 24.99
0.81	– 0.94	– 5.11	– 13.44	– 19.41	– 23.61
0.86	– 0.85	– 4.61	– 11.83	– 17.87	– 22.15
0.91	– 0.76	– 4.16	– 10.28	– 16.30	– 20.65
0.96	– 0.68	– 3.78	– 9.16	– 14.69	– 19.11
1.01	– 0.60	– 3.44	– 8.25	– 13.07	– 17.54
1.06	– 0.53	– 3.15	– 7.49	– 11.53	– 15.98
1.11	– 0.47	– 2.91	– 6.86	– 10.39	– 14.44
1.16	– 0.43	– 2.71	– 6.33	– 9.46	– 12.93

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6