TECHNICAL NOTE

Quantitative assessment of the influence of surface roughness on soil stiffness

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The nature of soil stiffness at small strains remains poorly understood. The relationship between soil stiffness (e.g. shear stiffness, G_0) and isotropic confining pressure (p') can be described using a power function with exponent (b), that is, $G_0 = A (p'/p_r)^b$, where A is a constant and p_r is an arbitrary reference pressure. Experimentally determined values of b are usually around 0.5 and these are higher than the value of 0.33 that can be analytically determined using Hertzian theory. Hertzian theory considers contact between two smooth, elastic spheres; however, in reality, inter-particle contacts in soil are complex with particle shape and surface roughness affecting the interaction. Thus Hertzian theory is not directly applicable to predict real soil stiffness. It has, however, provided a useful basis to develop an analytical framework to consider the influence of particle surface roughness on small-strain soil stiffness. Here, earlier contributions using this framework are extended and improved by paying particular attention to roughness and the tangential contact stiffness. Stiffness values calculated using the newly derived analytical expressions were compared with the results of bender element tests on samples of borosilicate glass beads (ballotini) whose surface roughness was quantified using an optical interferometer. The analytical expression captures the experimentally observed sensitivity of the small-strain shear modulus to surface roughness.

KEYWORDS: discrete-element modelling; elasticity; friction; laboratory tests; stiffness

INTRODUCTION

In the case of soil under isotropic loading, the relationship between the soil shear modulus at small strains (G_0) and the isotropic confining pressure (p') is generally believed to follow a power function having a coefficient of exponent (b), that is, $G_0 = A (p'/p_r)^b$, where p_r is an arbitrary reference pressure. McDowell & Bolton (2001) highlighted that the analytical estimate of $b = 0.33$, which can be obtained using Hertzian theory for spheres (Hertz, 1882), is smaller than that usually obtained from experiments, where $b \approx 0.5$. Goddard (1990) showed that particle geometry plays a role: a value of $b = 0.5$ can be analytically expected by considering contacts to be conical instead of spherical. The surface asperities that exist on the rough surface of real sand grains may also affect the b value.

Experimental research that quantitatively relates particle roughness to soil stiffness has rarely been reported due to the difficulty in accurately measuring roughness (Otsubo *et al.*, 2014). Santamarina & Cascante (1998) conducted resonant column tests using rough (rusty) and smooth steel spheres. They found greater wave velocity in the smooth spheres, which is in agreement with the earlier findings of Duffy & Mindlin (1956). Sharifipour & Dano (2006) also found similar results when smooth and rough (corroded by hydrofluoric acid) ballotini were compared. The magnitude of the surface roughness was not quantified in either of those papers.

Yimsiri & Soga (2000) presented a useful approach to quantify the influence of roughness on small strain stiffness based upon contact mechanics for rough surfaces (Greenwood & Tripp, 1967; Johnson, 1985) and a micromechanics-based constitutive model (Chang & Liao, 1994). This model has the disadvantage of giving a physically unfeasible negative Poisson ratio for apparently reasonable ratios of normal stiffness to tangential stiffness. In their model, Yimsiri & Soga (2000) assumed that the tangential contact stiffness is not influenced by surface roughness. Recent tribology research has shown that the surface roughness reduces both the normal and tangential contact stiffness (e.g. Gonzalez-Valadez *et al.*, 2010). The current contribution demonstrates that inclusion of this more recent research finding enables a refinement of the expressions proposed by Yimsiri & Soga to establish a more accurate analytical framework.

This contribution first revisits the analytical study presented by Yimsiri & Soga (2000) and demonstrates how recent tribological research can be used to modify the expression for tangential contact stiffness in developing their model. In the second part of the paper, the results of wave velocities measured in bender element tests on isotropically loaded ballotini samples, whose roughness was quantified using optical interferometry, are presented to validate the newly derived analytical expressions that relate overall (macro-scale) stiffness to the contact stiffness parameters.

THEORETICAL DERIVATION OF SHEAR MODULUS FOR SMOOTH ELASTIC CONTACTS

Hertz (1882) developed expressions to describe contact between smooth elastic surfaces. Hertzian theory has been used as a basis to explain the relationship between soil shear

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modulus and confining pressure (e.g. McDowell & Bolton, 2001). According to Hertzian theory (Johnson, 1985) the normal contact stiffness (K_N) between two identical smooth spheres, is given by

$$K_N = \frac{2G_p}{1-\nu_p}a \quad (1)$$

$$a = \left[\frac{3r(1-\nu_p)}{8G_p} \right]^{1/3} F_N^{1/3} \quad (2)$$

where G_p represents particle shear modulus; ν_p is the particle Poisson ratio; a is the circular (smooth) contact area radius; r denotes the radius of the identical contacting spheres; and F_N is the normal inter-particle contact force. Mindlin (1949) described the tangential contact stiffness (K_T) between smooth spheres using Hertzian theory. This model was extended to general cases which consider various loading histories by Mindlin & Deresiewicz (1953), who give the following expression of the tangential contact stiffness for virgin (initial) inter-particle tangential loading, F_T

$$K_T = \frac{4G_p}{2-\nu_p}a \left(1 - \frac{F_T}{\mu F_N} \right)^{1/3} \quad (3)$$

where μ is the coefficient of inter-particle friction. Equations (1) and (3) lead to the following expression for the contact stiffness ratio (R_K) for smooth contacts

$$R_K = \frac{K_T}{K_N} = \frac{2(1-\nu_p)}{2-\nu_p} \left(1 - \frac{F_T}{\mu F_N} \right)^{1/3} \quad (4)$$

Chang & Liao (1994) used a micromechanics-based model to relate the shear modulus (G_0) of an assembly of randomly packed identical spheres to K_N and K_T . Using kinematic and static hypotheses which assume uniform strain and uniform stress, respectively, expressions for upper and lower bound estimates of the elastic modulus were proposed

$$G_{0, \text{Kinematic}} = \frac{2Nr^2K_N}{3V} \cdot \frac{2+3R_K}{5} \quad (5)$$

$$G_{0, \text{Static}} = \frac{2Nr^2K_N}{3V} \left(\frac{5R_K}{3+2R_K} \right) \quad (6)$$

where N is the total number of particle contacts in the sample of volume V . The ratio N/V can be obtained from the particle radius (r), the sample void ratio (e) and the mean coordination number (N_C) as expressed in Yimsiri & Soga (2000) as follows

$$\frac{N}{V} = \frac{3N_C}{8\pi r^3(1+e)} \quad (7)$$

THEORETICAL DERIVATION OF SHEAR MODULUS FOR ROUGH ELASTIC CONTACTS

Influence of surface roughness on normal contact stiffness

Greenwood *et al.* (1984) and Johnson (1985) proposed a non-dimensional roughness parameter (α) to extend Hertzian theory to rough contacts

$$\alpha = \frac{S_q}{\delta_N} \quad (8)$$

where S_q is the root mean square (RMS) roughness; and δ_N denotes overlap of contacting spheres as used in Hertzian theory. The RMS roughness is defined as (Thomas, 1982)

$$S_q = \sqrt{\frac{1}{n} \sum_{i=1}^n (Z_i^2)} \quad (9)$$

where n is the number of measured data points; and Z_i is the elevation of data point i relative to the reference surface.

When two rough surfaces having S_{q1} and S_{q2} are considered, S_q in equation (8) can be replaced by a combined roughness, that is, $S_q^2 = S_{q1}^2 + S_{q2}^2$ (Greenwood *et al.*, 1984; Johnson, 1985). Yimsiri & Soga (2000) used α to relate the radius of circular contact area between two rough surfaces (a^{Rough}) to the smooth equivalent (a^{Smooth}) as follows

$$a^{\text{Rough}} = \left(\frac{-2.8}{\alpha + 2} + 2.4 \right) a^{\text{Smooth}} \quad (10)$$

At an extremely large normal load, α approaches zero and $a^{\text{Rough}} \rightarrow a^{\text{Smooth}}$. Assuming that Hertzian theory of $r \delta_N = 2a^2$ is still applicable to rough contacts, the overlap of rough spheres can be analysed as

$$\delta_N^{\text{Rough}} = \frac{2}{r} \left[\left(\frac{-2.8}{\alpha + 2} + 2.4 \right) a^{\text{Smooth}} \right]^2 \quad (11)$$

Yimsiri & Soga (2000) derived the normal contact stiffness for rough contacts by differentiating F_N with respect to δ_N

$$K_N^{\text{Rough}} = \frac{dF_N}{d\delta_N^{\text{Rough}}} \quad (12)$$

Influence of surface roughness on tangential contact stiffness

The effect of surface roughness on the tangential contact stiffness is complex. Yimsiri & Soga (2000) referred to an experimental study by O'Connor & Johnson (1963) and assumed that K_T^{Rough} equals K_T^{Smooth} . However, this assumption results in the Poisson ratio of the assembly becoming negative when $K_T^{\text{Rough}} > K_N^{\text{Rough}}$ (i.e. $R_K^{\text{Rough}} > 1$) according to the following equations proposed by Chang & Liao (1994)

$$\nu_{s, \text{Kinematic}} = \frac{1 - R_K}{4 + R_K} \quad (13)$$

$$\nu_{s, \text{Static}} = \frac{1 - R_K}{2 + 3R_K} \quad (14)$$

where $\nu_{s, \text{Kinematic}}$ and $\nu_{s, \text{Static}}$ are the Poisson ratios obtained using the kinematic and static assumptions. To overcome this drawback, it is essential to select an appropriate value for K_T^{Rough} . Knowing R_K and K_N^{Rough} , K_T^{Rough} can be obtained using equation (4). The influence of the surface roughness on R_K has been reported in recent tribology research; Campañá *et al.* (2011) and Medina *et al.* (2013) assumed the same R_K for both smooth and rough contacts. In contrast, a lower R_K for rough contacts was reported by Gonzalez-Valadez *et al.* (2010), whose ultrasound tests showed that $R_K^{\text{Rough}} < R_K^{\text{Smooth}}$, and R_K^{Rough} increases as the normal contact force increases. Here it is assumed that $R_K^{\text{Rough}} = R_K^{\text{Smooth}}$.

The coefficient of inter-particle friction, μ , for rough contacts is needed to calculate equation (4). Cavarretta *et al.* (2010) and Senetakis *et al.* (2013) obtained the inter-particle friction by shearing one particle over another. Cavarretta *et al.* (2010) observed a higher friction for rough contacts than smooth ones. Note that this type of experiment is non-trivial and very challenging to interpret. In contrast, plastic theory predicts lower friction coefficient with larger

Fig. 1. (a) Inclined contact planes at asperities between rough-rough surfaces and (b) smooth-smooth surfaces

roughness due to yielding of asperities (Chang *et al.*, 1988; Kogut & Etsion, 2004; Chang & Zhang, 2005).

Rough contacts can be modelled as a system of multiple micro-contacts, each being a smooth spherical surface. Referring to Fig. 1, the inter-particle forces of F_N and F_T can be decomposed into normal ($f_{N,i}$) and tangential contact forces ($f_{T,i}$) that act on an individual micro-contact i . The magnitude of $f_{T,i}/f_{N,i}$ depends upon the micro-contact orientation. Summing this ratio over all the micro-contacts, gives

$$\frac{F_T}{\mu F_N} \cong \sum_i \frac{f_{T,i}}{\mu f_{N,i}} \quad (15)$$

Thus, equation (4) can be applied to rough contacts using $K_N^{\text{Rough}} = K_N^{\text{Smooth}}$. The resultant expressions for K_T^{Rough} are given in Table 1. Substitution of K_N^{Rough} and K_T^{Rough} into equations (5) and (6) gives the shear modulus of the assembly.

EXPERIMENTS

Tested materials

The material tested comprised borosilicate ballottini spheres with diameters between 2.4 mm and 2.7 mm (shear modulus, $G_p = 25$ GPa, specific gravity = 2.23, particle Poisson ratio, $\nu_p = 0.2$). Typical microscope images and optical interferometry surface topographies of these particles are shown in Fig. 2. The rough ballottini were made by milling the smooth ballottini as described by Cavarretta *et al.* (2012). Forty surface roughness measurements were conducted on each material using a Fogale Microsurf 3D (Fogale, 2005). The effects of surface curvature were considered in the roughness measurements, and Fig. 2 summarises the roughness values as measured and after flattening using a built-in motif analysis function available in the Fogale software (Fogale, 2005).

Cubical cell apparatus and sample preparation

A cubical cell apparatus was used, whereby pressures were applied to a cubical sample using flexible air-filled cushions (Ko & Scott, 1967; Sadek & Lings, 2007). The cubical samples ($100 \times 100 \times 100 \text{ mm}^3$) were prepared using a pluviation device that maintains a constant drop height

(Camenen *et al.*, 2013). The measured void ratios were 0.632 and 0.679 and the measured relative densities were 42% ($e_{\min} = 0.557$ and $e_{\max} = 0.698$) and 47% ($e_{\min} = 0.585$ and $e_{\max} = 0.746$), for the smooth and rough ballottini samples, respectively. Note that the size of the tested materials exceeds the maximum recommended particle size for which this test is applied (up to 2.00 mm in diameter; JGS 0161 (JGS, 2009)). A vacuum confinement of 50 kPa was applied while the sample was gently moved into the cubical cell apparatus (O'Donovan *et al.*, 2014).

Bender element testing

Bender element testing was initially developed by Shirley (1978) and Shirley & Hampton (1978). Bender/extender (BE) elements which are able to generate shear waves (S waves) and compression waves (P waves) were used in this research (Lings & Greening, 2001). Details of the installation of the bender elements using the cubical cell apparatus are described by O'Donovan *et al.* (2014). The bender elements were inserted into the faces of the cubical sample, while it was still subject to vacuum confinement of about 50 kPa; then the vacuum confinement was systematically reduced as the cushion pressure was increased, initially to an isotropic cell pressure of 50 kPa. Bender element tests were carried out at discrete confining pressures (50, 100, 200, 300, 400 and 500 kPa) both during loading and unloading. After increasing the confining pressure to the next level, a pause of at least 1 h was applied to allow for creep of the sample.

At each confining pressure a sinusoidal wave with a frequency of 15 kHz and 270° of phase delay was transmitted. The high frequency chosen should minimise the near-field effects in the received signal (Arroyo *et al.*, 2003). The importance of choosing a sensible method to identify the wave arrival has been discussed extensively (e.g. Yamashita *et al.*, 2009). This research uses a peak to peak method in which the time delay between the peaks of the transmitted and received waves is considered to be the travel time.

Test results

A typical series of the received S-wave voltages in one direction for smooth and rough samples at various confining pressures is illustrated in Fig. 3. The vertical axis gives transmitted and received voltages normalised by their maximum values; the relevant test confining pressure is indicated on each voltage trace. Arrows show the first and second peaks in received waves. As the confining pressure increases, the first peaks of the received waves appear earlier, indicating higher velocities. Comparing Figs 3(a) and 3(b) the differences in response are due to the combined effects of differences in surface stiffness and differences in sample void ratio.

Table 1. Summary of contact model presented by Yimsiri & Soga (2000) and a suggested modification

Model	Normal contact stiffness, K_N	Tangential contact stiffness*, K_T
Hertz – Mindlin & Deresiewicz (1953)	$K_N^{\text{Smooth}} = \frac{2G_p}{1-\nu_p} \left[\frac{3r(1-\nu_p)}{8G_p} \right]^{1/3} F_N^{1/3}$	$K_T^{\text{Smooth}} = \frac{2(1-\nu_p)}{2-\nu_p} K_N^{\text{Smooth}} \left(1 - \frac{F_T}{\mu F_N} \right)^{1/3}$
Yimsiri & Soga (2000) Modified expression	$K_N^{\text{Rough}} = \frac{dF_N}{d\delta_N^{\text{Rough}}}$	$K_T^{\text{Rough}} = \frac{2(1-\nu_p)}{2-\nu_p} K_N^{\text{Rough}} \left(1 - \frac{F_T}{\mu F_N} \right)^{1/3}$

*Note: Tangential contact stiffness is for a virgin tangential load.

Fig. 2. Microscope images and surface topographies of tested materials: (a) smooth ballotini; (b) rough ballotini

Fig. 3. S-wave response in (a) smooth assembly and (b) rough assembly in XY direction at various mean confining pressures (arrows indicate the first and second peaks in received waves)

The relationships between the elastic moduli and the elastic wave velocities are assumed to be applicable here, that is

$$M_0 = \rho V_P^2 \quad (16)$$

$$G_0 = \rho V_S^2 \quad (17)$$

where M_0 and G_0 are the constrained and shear moduli, respectively; ρ is the sample bulk density; V_P and V_S are the compression and shear wave velocities, respectively. The Poisson ratio of the sample (ν_s) can be calculated by assuming applicability of elastic theory for homogeneous and isotropic materials (Kumar & Madhusudhan, 2010).

$$\nu_s = \frac{M_0 - 2G_0}{2(M_0 - G_0)} \quad (18)$$

The calculated moduli include the effects of soil density. A correction factor based on a void ratio function of the form proposed by Hardin & Richart (1963)

$$F(e) = \frac{(B - e)^2}{1 + e} \quad (19)$$

was applied to G_0 for both smooth and rough assemblies. Regression analyses were used to fit functions through the experimental data of $V_s - p'$ and $e - p'$ to interpolate values of V_s and e at additional values p' . Best surface fitting through the larger interpolated dataset showed that B is approximately 2.9 and that this value is equally valid for both materials. A value of 2.17, derived for rounded sand particles (Hardin, 1965), has previously been used by Kuwano & Jardine (2002) and Yang & Gu (2013) for data on glass ballotini.

The normalised shear modulus $G_0/F(e)$ in XY (X wave propagation direction, Y wave polarisation) and YX (Y wave propagation direction, X wave polarisation) directions are plotted against the isotropic confining pressure in Fig. 4. Here, only data for the loading case are presented. As the confining pressure increases the difference between smooth and rough samples gradually reduces, as reported in the

Fig. 4. Pressure dependency of shear stiffness in isotropic loading for smooth and rough ballotini samples based on shear wave velocity measurements of waves propagated and polarised in the horizontal plane XY of the cubical sample (note that b is the power coefficient of stiffness–pressure relation, and S_q is the root mean square of roughness)

analytical study by Yimsiri & Soga (2000). The power coefficients for the smooth ballotini sample ranged from 0.35 to 0.37, while those for the rough ballotini sample ranged from 0.53 to 0.66. Note that, with the exception of one measurement point at low confinement pressure that could have affected the quality of the contacts, there is very good agreement between the measurements in both directions for both smooth and rough samples.

DISCUSSION AND COMPARISON BETWEEN ANALYSIS AND EXPERIMENTS

In order to use experimental data to validate the newly derived analytical expressions of stiffness, a number of particle-scale parameters were needed. Referring to equations (4)–(7), the normal and tangential contact forces (F_N and F_T), the void ratio (e) and the mean coordination number (N_C) were obtained from DEM simulations which considered similar cubical samples (O'Donovan, 2013) and similar particle size distributions. These data gave $0.0665 \leq F_T/F_N \leq 0.0687$, $0.697 \geq e \geq 0.677$ and $5.38 \leq N_C \leq 5.63$ as p' increased from 0.1 MPa to 1 MPa. The friction coefficient for the ballotini (μ) was taken as 0.0805 based on Cavarretta *et al.* (2012). Referring to Fig. 5 there is a good agreement between the experimental data and the analytical predictions using the static assumption. The kinematic assumption overestimates the shear modulus in both cases; however, it does capture the experimental trend, that is, the rough particles are softer than the smooth particles and the difference in stiffness between the rough and the smooth materials decreases with increasing p' .

The evolution of the Poisson ratio (ν_s) at different confining pressures is compared in Fig. 6. The analytical values derived from equations (13) and (14) gave lower estimates for ν_s over the range of examined confining pressures when compared with the experiments. However, the analytical expression for ν_s does not depend on the surface roughness. The static hypothesis was again in better agreement with the experimental results for smooth particles. It is interesting that the experimental value for rough particles decreased as the confining pressure increased, while the opposite trend was observed for the smooth

Fig. 5. Comparison between analytical model and experimental results for relationship between shear modulus and isotropic confining pressure

Fig. 6. Evolution of Poisson ratio at various isotropic confining pressures

particles. Similar experimental results were reported by Sharifipour & Dano (2006) where smooth and rough (corroded) ballotini were compared. It is worth mentioning that Suwal & Kuwano (2013) compared the Poisson ratio obtained in static and dynamic tests and found that the dynamic tests gave a larger value.

CONCLUSIONS

This contribution has revisited the analytical model proposed by Yimsiri & Soga (2000) that relates elastic stiffness of an assembly of particles to particle-scale parameters. Drawing on recent experimental research, the model was extended to include a reduction in the inter-particle tangential stiffness with surface roughness. Incorporation of this feature results in more realistic values of shear modulus and Poisson ratio; in particular, the negative Poisson ratio values which were obtained when the original model was used with (plausible) contact stiffness ratios exceeding 1 are now avoided. To validate the new

model, bender element tests on smooth and artificially roughened ballotini were performed in a cubical cell. The particle surface roughnesses were quantified using an optical interferometer, to enable direct comparison with the modified analytical expression. Additional particle-scale data needed for the analytical expression were obtained from an equivalent DEM simulation. The estimates of small-strain shear modulus obtained using the new analytical model were in good agreement with the experimental data when the static hypothesis was used, while the expression derived using the kinematic hypothesis was qualitatively similar. Both the analytical model and the experimental data show that increasing particle surface roughness reduces the shear modulus at small strains, and the magnitude of this reduction reduces with increasing isotropic confining pressure. The analytical and experimental data both indicate that the power coefficient (b) increases with surface roughness. The analytical expression for Poisson ratio does not consider surface roughness, and the expression from the static hypothesis gave a better match to the experimental data than that obtained using the kinematic hypothesis.

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NOTATION

A	constant
a	circular contact area radius
a^{Rough}	circular contact area radius (rough contacts)
a^{Smooth}	circular contact area radius (smooth contacts)
B	constant
b	power function exponent
e	sample void ratio
e_{max}	maximum void ratio
e_{min}	minimum void ratio
$F_{(e)}$	void ratio correction
F_N	normal inter-particle contact force
F_T	tangential inter-particle contact force
$f_{N,i}$	normal contact force on micro-contact i
$f_{T,i}$	tangential contact force on micro-contact i
G_0	shear stiffness (soil)
G_p	particle shear modulus
K_N	normal contact stiffness
K_N^{Rough}	normal contact stiffness for rough contacts
K_T	tangential contact stiffness
K_T^{Smooth}	tangential contact stiffness – smooth contacts
K_T^{Rough}	tangential contact stiffness – rough contacts
M_0	constrained modulus (soil)
N	total number of particle contacts in sample of volume V
N_C	mean coordination number
p'	isotropic confining pressure
p_r	arbitrary reference pressure
r	particle radius
R_K	contact stiffness ratio
R_K^{Rough}	contact stiffness ratio (rough contacts)
R_K^{Smooth}	contact stiffness ratio (smooth contacts)
S_q, S_{q1}, S_{q2}	root mean square (RMS) roughness
V_p	compression wave velocity
V_s	shear wave velocity
Z_i	elevation of data point i relative to the reference surface
α	non-dimensional roughness parameter
δ_N	overlap of contacting spheres
δ_N^{Rough}	overlap of rough contacting spheres
ν_p	particle Poisson ratio

ν_s	sample Poisson ratio
$\nu_{s, \text{Kinematic}}$	sample Poisson ratio obtained using kinematic hypothesis
$\nu_{s, \text{Static}}$	sample Poisson ratio obtained using static hypothesis
μ	coefficient of inter-particle friction
ρ	sample bulk density

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