The Prediction of Crack Growth in Bonded Joints under Cyclic-Fatigue Loading.

Part II: Analytical and Finite Element Studies

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Abstract

In Part 1 [1] the performance of adhesively-bonded joints under monotonic and cyclic-fatigue loading was investigated. The joints consisted of an epoxy-film adhesive which was employed to bond aluminium-alloy substrates. The effects of undertaking cyclic-fatigue tests in (a) a 'dry' environment of 55% relative humidity at 23°C, and (b) a 'wet' environment of immersion in distilled water at 28°C were studied. The basic fracture-mechanics data for these different joints in the two environments were measured, as well as the behaviour of single-lap joints. In the present paper, Part 2, a method for predicting the lifetime of adhesively-bonded joints and components has been investigated. This prediction method consists of three steps. Firstly, the fracture-mechanics data obtained under cyclic loading in the environment of interest have been modelled, resulting in an expression which relates the rate of crack growth per cycle, da/dN , to the maximum applied strain-energy release-rate, G_{max} , in a fatigue cycle. Secondly, this relationship is then combined with an analytical or a computational description of the variation of G_{max} with the crack length, a , and the maximum applied load per unit width, T_{max} , per cycle in the joint, or component. Thirdly, these data are combined and the resulting equation is integrated to give a prediction for the cyclic-fatigue lifetime of the bonded joint or component. The theoretical predictions from the above method, using different approaches to describe the variation of G_{max} with the crack length, a , and applied load, T_{max} , in the single-lap joint, have been compared and contrasted with each other, and compared with the cyclic-fatigue behaviour of the lap joints as ascertained from direct experimental measurements.

Keywords: C. Fracture mechanics; D. Fatigue; Durability; E. Joint Design; Life Prediction.

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Nomenclature

a	Crack length
a_o	Griffith (inherent) flaw size
a_f	Crack length at final failure
b	Width
c	Half the bonded lap length
CAE	Chromic-acid etch
D	Linear ('Region II') coefficient
D_f	Flexural rigidity of substrate
D_{fc}	Flexural rigidity of the bonded lap region
E_a	Young's modulus of adhesive
E_s	Young's modulus of substrate
G	Strain-energy release-rate
G_c	Adhesive fracture energy
G_{inf}	Strain-energy release-rate for an infinitely long joint
G_{max}	Maximum strain-energy release-rate applied in a fatigue cycle
G_{th}	Threshold strain-energy release-rate
G_I	Mode I component of the strain-energy release-rate
G_{II}	Mode II component of the strain-energy release-rate
G_{Imax}	Maximum mode I component of the strain-energy release-rate
GBD	Grit-blast and degrease
h	Substrate thickness
J	Contour integral
k	Bending moment factor
N	Number of cycles
N_f	Number of cycles to failure
n	Linear ('Region II') curve-fitting constant
n_1	Threshold ('Region I') curve-fitting constant
n_2	Fast fracture ('Region III') curve-fitting constant
PAA	Phosphoric-acid anodise
T	Load per unit width applied to the single-lap joint
T_f	Failure load per unit width for monotonic loading
T_{max}	Maximum load per unit width applied in a fatigue cycle
T_{min}	Minimum load per unit width applied in a fatigue cycle

T_{th}	Threshold value of the maximum load per unit width for the single-lap joints
TDCB	Tapered double-cantilever beam
t_a	Adhesive layer thickness
u_{max}	Maximum displacement
u_{min}	Minimum displacement
β	Slope of relationship between G_{max} and T_{max}^2
μ_a	Shear modulus of the adhesive
σ_f	Tensile strength of the single-lap joint
σ_{max}	Maximum peel stress
τ_{max}	Maximum shear stress
ν	Poisson's ratio

1. Introduction

In Part I [1] of the present work fracture-mechanics data were obtained using the tapered double-cantilever beam (TDCB) test geometry, with various surface pretreatments, tested in (a) a ‘dry’ environment of 55% relative humidity at 23°C, and (b) a ‘wet’ environment of immersion in distilled water at 28°C. These results will be used in the present paper to predict the cyclic-fatigue lifetime of adhesively-bonded single-lap joints when tested in both ‘dry’ and ‘wet’ environments.

The prediction method consists of three steps [2-6]. Firstly, the fracture-mechanics data obtained under cyclic loading in the environment of interest has been modelled [1,2,5,6] resulting in an expression which relates the rate of crack growth per cycle, da/dN , to the maximum applied strain-energy release-rate, G_{max} , in a fatigue cycle. Secondly, this relationship is then combined with an analytical or a computational description of the variation of G_{max} with the crack length, a , and applied load in the joint, or component. Obviously, these theoretical descriptions consider the detailed design aspects of the bonded joint, or bonded component. In the present work, the fatigue behaviour of single-lap joints will be predicted. Hence, a description of the variation of G_{max} with the length, a , of a propagating crack for a given maximum applied load per unit width, T_{max} , applied to the single-lap joint during the fatigue test needs to be ascertained; and various forms of this relationship are considered in the present paper. Thirdly, these relationships are combined, and the resulting equation is integrated to give a prediction for the cyclic-fatigue lifetime of the bonded joint or component. A basic theme behind this methodology is that the fracture-mechanics parameters are considered to be characteristic of the adhesive system and the test environment, and such material/test-environment ‘property’ data can be obtained in a relatively short time-scale but applied to different joint geometries tested over a relatively long time scale. (Since (a) the fracture-mechanics parameters are considered to be characteristic of the adhesive system and the test environment, and (b) the description of the variation of G_{max} with the length, a , of a propagating crack for a given maximum applied load per unit width, T_{max} , for any bonded joint or component design of interest (for example, whether a symmetrical or non-symmetrical single- (or double-) lap joint) can be ascertained from an analytical or numerical model.)

The theoretical predictions from various approaches to describing the relationship between G_{max} and the crack length, a , for a given maximum applied load per unit width, T_{max} , applied to the single-lap joint during the fatigue test are compared and contrasted with each other, and with the cyclic-fatigue behaviour of the single-lap joints which has been experimentally measured [1]. Also, in the present study, the effect of a ‘wet’ test environment has been considered when attempting to predict the fatigue lifetime of the bonded aluminium-alloy joints. Indeed, the combination of a ‘wet’

environment and cyclic-fatigue loading represents one of the most hostile environments for bonded joints.

2. Modelling the Fracture-mechanics Data

The lifetime prediction method uses the modified form of the Paris law [1-6] to describe the fracture-mechanics data, such that:

$$\frac{da}{dN} = DG_{\max}^n \left\{ \frac{1 - \left(\frac{G_{th}}{G_{\max}} \right)^{n_1}}{1 - \left(\frac{G_{\max}}{G_c} \right)^{n_2}} \right\} \quad (1)$$

where the values of the coefficients D , n , n_1 and n_2 are calculated from the experimental data [1].

The single-lap joints tested in Part 1 [1] did not contain any artificially-introduced cracks. However, the fracture-mechanics method assumes that there will be naturally-occurring cracks (termed ‘Griffith’ flaws) in the adhesive layer and that these will grow under the cyclic-fatigue loading to give fracture of the joint. Thus, the values of the initial ‘Griffith’ flaw size, a_o , are also needed in the predictive method and were calculated [1] using:

$$a_o = \frac{E_a G_c}{\pi \sigma_f^2} \quad (2)$$

It is possible to calculate two ‘Griffith’ flaw sizes from Equation (2), using a value of tensile strength, σ_f , either derived from tests on the bulk adhesive or from the single-lap joint tests. In the present study the bulk fracture stress was not available, hence the value of a_o was calculated using the value of σ_f from the single-lap joint tests. (However, it is most noteworthy that the results from the lifetime calculations described below are not significantly dependent upon the exact values employed for a_o [1-5]).

The values of the above parameters were determined [1] for the two test environments and for the various surface pretreatments which were employed. (Prior to bonding, the substrates were pretreated [1] using either a grit-blast and degrease (GBD) treatment, a chromic acid etch (CAE), or a phosphoric acid anodise (PAA) process.) The values of the above parameters are given in Table 1.

3. Descriptions of the Variation of G_{max} with a and T_{max} for the Single-Lap Joint

3.1 Introduction

Equation (1) may be re-arranged and integrated to give the number of cycles, N_f , to failure, such that:

$$N_f = \int_{a_o}^{a_f} \frac{1}{DG_{max}^n} \frac{1 - \left(\frac{G_{max}}{G_c} \right)^{n_2}}{1 - \left(\frac{G_{th}}{G_{max}} \right)^{n_1}} da \quad (3)$$

where a_o and a_f are the initial and final crack lengths. As noted above, the term G_{max} as a function of the crack length, a , and maximum load, T_{max} , applied in a fatigue cycle for the single-lap joint needs to be substituted into Equation (3), and the combined expression then integrated.

There are a number of published analytical and numerical descriptions for the maximum strain-energy release-rate, G_{max} , in a single-lap joint loaded in tension as a function of the crack length, a , for a given maximum applied load per unit width, T_{max} . These solutions are considered below and are then implemented in the predictive method via Equation (3).

3.2 Analytical Solutions

3.2.1 The Kinloch-Osiyemi (KO) Model

Kinloch and Osiyemi [2] have considered the transverse tensile stresses, or peel stresses, which act at the end of the single-lap joint, i.e. they have considered mode I (tensile) failure to be the operative failure mode. They derived an description for the variation of the maximum applied energy release rate, G_{max} , for a single-lap joint with applied load per unit width, T_{max} , and crack length, a , where:

$$G_{max} = \frac{12}{E_s h^3} \left(\frac{T_{max} (h + t_a)}{2} \right)^2 \left(\frac{1}{(1 + \lambda(c - a))^2} \right) \quad (4)$$

and $\lambda = \sqrt{\frac{T_{max}}{D_f}}$ (where T_{max} is the maximum load per unit width applied to the joint),

$D_f = \frac{E_s h^3}{12(1 - \nu^2)}$ is the flexural rigidity of a single arm, h is the substrate thickness, t_a is the thickness of the adhesive layer and $2c$ is the lap length.

The integration limit a_o in Equation (3) is taken to be the ‘Griffith’ flaw size (see Equation (2)) and the final crack length, a_f , may be found by rearrangement of Equation (4) to give:

$$a_f = c - \frac{1}{\lambda} \left[\left(\frac{3(T_{\max}[h + t_a])^2}{E_s h^3 G_c} \right)^{1/2} - 1 \right] \quad (5)$$

since rapid fracture of the joint will occur when $G_{\max} = G_c$. It should be noted that in the above analysis cracks are assumed to grow from both ends of the specimen [1], thus $a_f \leq c$. When $T_{\max}$ approaches the threshold value, T_{th} , then the value of a_f calculated using Equation (5) may be greater than half the lap length. Obviously, in these cases, a_f is set to be equal to the value of c .

3.2.2 The Fernlund, Papini, McCammond and Spelt (FPMS) Model

The approach taken by Fernlund, Papini, McCammond and Spelt [7,8] for obtaining the energy release-rate in a single-lap joint is based on the J-integral method for large deformations, together with large-deformation beam theory. The contributions to the energy release-rate are assumed to arise from the axial strain, created by the load applied to the joint, and from the induced bending moments, caused by rotation of the substrates.

The contribution to the maximum strain-energy release-rate, G_{inf} , from the axial strain for a joint with infinitely long arms is given by:

$$G_{inf} = \frac{T_{\max}^2}{4E_s h} \quad (6)$$

where E_s is the modulus of the substrate. The total strain-energy release-rate, $G_{\max}$, including the contribution from the induced bending moments, is given by:

$$G_{\max} = G_{inf} \left(1 + \frac{3}{4} \frac{1 - \tanh^2(\lambda_1 c)}{[1/\sqrt{8} + \tanh(\lambda_1 c)]^2} \right) \quad (7)$$

where $\lambda_1 = \sqrt{\frac{T_{\max}}{D_{fc}}}$, and the flexural rigidity, D_{fc} for the lap region is given by:

$$D_{fc} = \frac{E_s (2h)^3}{12(1-\nu^2)} \quad (8)$$

The ratio of the mode I (tensile) to the mode II (in-plane shear) component of the total strain-energy release-rate may be obtained from the relationship:

$$\frac{G_{II}}{G_I} = \frac{4}{3} + \frac{4\sqrt{2}}{3} \tanh(\lambda_1 c) + \frac{2}{3} \tanh^2(\lambda_1 c) \quad (9)$$

It should be noted that no account is taken of the adhesive layer, as in general $h \gg t_a$.

For a crack propagating a distance a , with a ligament length therefore of $(c-a)$ from both ends of the lap, the maximum value of the strain-energy release-rate, G_{max} , obtained from Equation (7) will be given by:

$$G_{max} = G_{inf} \left(1 + \frac{3}{4} \frac{1 - \tanh^2(\lambda_1 (c-a))}{[1/\sqrt{8} + \tanh(\lambda_1 (c-a))]^2} \right) \quad (10)$$

and Equation (10) can now be substituted into Equation (3), as for the Kinloch and Osiyemi model, and the combined equation integrated to obtain the predicted lifetime.

3.2.3 The Krenk and Hu (KH) Model

In work first published by Krenk [9], and then later by Hu [10], the maximum applied strain-energy release-rate, G_{max} , is given via the solution:

$$G_{max} = \frac{t_a}{2} \left(\frac{\sigma_{max}^2}{E_a} + \frac{\tau_{max}^2}{\mu_a} \right) \quad (11)$$

where E_a is the Young's modulus of the adhesive, μ_a is the shear modulus of the adhesive, and σ_{max} and τ_{max} are the maximum peel stress and the maximum shear stress, respectively, in the single-lap joint for an applied load per unit width of T_{max} .

Thus, both Krenk and Hu consider the mode I (tensile) and mode II (in-plane shear) contributions. However, they employed somewhat different analyses to derive the values of σ_{max} and

τ_{max} in the single-lap joint. Krenk [9] assumed a beam on an elastic foundation model to calculate the peel stresses and then deduced the shear stresses from equilibrium conditions in the two bonded substrates. Whereas, Hu [10] used the Cherepanov-Rice J-integral method.

The main difference between the analyses outlined by Krenk and Hu, is that Hu takes into account the reduction of the bending moment due to the rotation of the substrates. It is well documented that this needs to be included into any analysis of the single-lap joint and therefore the method proposed by Hu is considered to be a more accurate model of the stress distribution in the single-lap joint. The values of σ_{max} and τ_{max} , for a corresponding ligament length of $(c-a)$, are given by:

$$\sigma_{max} = \lambda_{\sigma}^2 M_o \frac{\sinh(2\lambda_{\sigma}(c-a)) - \sin(2\lambda_{\sigma}(c-a))}{\sinh(2\lambda_{\sigma}(c-a)) + \sin(2\lambda_{\sigma}(c-a))} + \lambda_{\sigma} V_o \frac{\cosh(2\lambda_{\sigma}(c-a)) + \cos(2\lambda_{\sigma}(c-a))}{\sinh(2\lambda_{\sigma}(c-a)) + \sin(2\lambda_{\sigma}(c-a))} \quad (12)$$

$$\tau_{max} = -\frac{1}{8}\lambda_{\tau}\left(T_{max} + \frac{6}{h}M_o\right)\coth(\lambda_{\tau}(c-a)) - \frac{3}{8(c-a)}\left(T_{max} - \frac{2M_o}{h}\right) \quad (13)$$

where: $\lambda_{\sigma}^4 = \frac{E_a}{2t_a D_f}$, $\lambda_{\tau}^2 = \frac{8}{ht_a} \frac{\mu_a(1-\nu^2)}{E_s}$

and: $M_o = \frac{1}{2}kT_{max}h$ and $V_o = kT_{max}\sqrt{3\frac{T_{max}}{hE_s}}$ (14)

and k is the Goland and Reissner [11] bending moment factor, calculated from the work of Zhao et al. [12]:

$$k = [1 + \lambda(c-a)]^{-1} \quad \text{where} \quad \lambda = \sqrt{\frac{T_{max}}{D_f}}$$

In the Hu analysis, the mode I and II components of G_{max} may be separated out, and the mode I contribution to the maximum applied strain-energy release-rate, G_{Imax} , is given by:

$$G_{I\max} = \frac{t_a}{2} \frac{\sigma_{\max}^2}{E_a} \quad (15)$$

Equation (11) or (15) may now substituted into Equation (3), and integrated.

3.3 Numerical Solutions

3.3.1 The Basic Model

It should be noted that the analytical models described above cannot distinguish between crack growth cohesively through the centre of the adhesive layer and crack growth at, or very close to, the adhesive/substrate interface. However, the finite-element analysis (FEA) approach can obviously model either a cohesive or an interfacial crack. Since, for both the single-lap, and for the TDCB specimens which used to obtain fracture-mechanics data, joints subjected to fatigue cycling both of these loci of failure were experimentally observed [1], both types of crack are now theoretically modelled using the FEA approach. For the single-lap joint, the exact locations of the cohesive and interfacial cracks are shown in Figure 1: ‘Case 1’ and ‘Case 2’, respectively. These crack locations were chosen since they are known to be the regions of high stress concentrations in the lap joints [13]; and the failure of single-lap joints was observed to occur at these locations [1].

The FEA approach adopted has been previously described in detail [4] and only a brief description is given here. The pre-processing of the model was undertaken using ‘PATRAN’ software, and the analysis was undertaken using ‘ABAQUS’ software. The J -integral method was used to calculate values of G_{max} as a function of the crack length, a , in the joint and the applied load per unit width, T_{max} . As previously [4] noted, we ensured that there was no domain dependency and that no further mesh refinement was necessary. The cracks were assumed to be sharp, and the crack faces were assumed to lie on top of one another in the undeformed configuration. Multi-point constraints were used to tie together all the crack-tip nodes. A typical model of the joint was composed of 1136 elements and 3692 nodes, giving 7384 degrees of freedom. A range of crack lengths and applied loads were used in the model. The assumed crack lengths, a , used in the model were between about 0.1 and 5 mm, and the applied loads per unit width, T_{max} , corresponded to between 40% and 100% of the monotonic failure loads that were measured experimentally.

3.3.2 Results

The FEA results revealed that, as expected, the total maximum strain-energy release-rate, G_{max} , for a hypothetical crack in a single-lap joint is a function of both the crack length, a , and the applied load, T_{max} . Thus, the mathematical description of G_{max} as a function of the crack length, a , and the applied load, T_{max} , to use in Equation (3) was formulated in two parts. Firstly, values of G_{max} were plotted against the values of T_{max}^2 for each crack length, for both cohesive and interfacial cracks, see Figures 2a and 2b respectively. In agreement with linear-elastic fracture-mechanics (LEFM) theory and previous work [3,4], linear relationships between G_{max} and T_{max}^2 were obtained from these modelling studies. Secondly, the gradient, β , (where $\beta = G_{max}/T_{max}^2$) of each line was then plotted against the crack length, a , see Figure 3.

The relationships between the gradient, β , and the crack length, a , for both cohesive and interfacial crack growth, may now be described by an empirical relationship of the form, as in previous work [4]:

$$\beta = \frac{G_{\max}}{T_{\max}^2} = C_1 a^4 + C_2 \quad (16)$$

where C_1 and C_2 are empirically-derived constants. As may be seen from the FEA results shown in Figure 3, a single relationship of the above form could be employed to give a very good fit to both the cohesive and interfacial crack FEA data. The best fit relationship is:

$$\beta = \frac{G_{\max}}{T_{\max}^2} = 14a^4 + 9 \times 10^{-9} \quad (17)$$

In order to predict the number of cycles, N_f , to failure for the single-lap joints, Equation (17) may now be substituted into Equation (3), and the resulting expression integrated between the limits of a_o and a_f in order to obtain the value of N_f for a given value of the maximum applied load, $T_{\max}$, per unit width applied in the fatigue cycle.

It should be noted that in the FEA modelling, we have chosen not to separate the mode I and mode II contributions to the total strain-energy release rate, $G_{\max}$. This is because in the FEA modelling the relative values of the mode I and mode II components are actually dependent upon the element size, close to the crack tip, for an interfacial crack, whereas the total strain-energy release-rate is independent of the crack-tip element-size [14]. This effect arises from the oscillatory nature of the stresses at an interface [15].

4. Results and Discussion

4.1 Introduction

The fracture-mechanics data from the tests conducted on the tapered double-cantilever beam (TDCB) specimens have been used to predict the lifetime of the single-lap joints when subjected to cyclic-fatigue loading, via combining Equation (3) with the results from the various analytical models and the finite element analyses and then integrating the final expression. These predictions have also been compared with the experimentally-measured lifetimes. The results are shown in Figures 4 to 6.

As noted previously, an important requirement of the modelling approach is that the loci of joint failure are the same for the TDCB specimens as for the single-lap joint test specimens whose fatigue lifetime is to be predicted, under the cyclic-fatigue tests performed in the ‘dry’ or ‘wet’ test environment. If this is the case, then the fracture-mechanics data generated using the TDCB specimens may be used with confidence to predict the cyclic-fatigue lifetimes of the (initially uncracked) single-lap joints. This was the case for all the various pretreatments and test conditions, except for the PAA-pretreated joints tested in the ‘wet’ environment. The reasons for the observed discrepancy with these joints were discussed in detail in Part I [1] and this prevented the prediction of the cyclic-fatigue lifetime of the single-lap joints where a PAA pretreatment had been used together with testing of the joints in the ‘wet’ environment.

Finally, it should be noted that, in comparison with metallic materials, the values of the exponent, n , in Equation (3) for polymeric adhesives are relatively high. This implies that, for adhesive joints, the rate of fatigue crack growth may rapidly increase for relatively small increases in the applied strain-energy release rate, G_{max} , and hence for relatively small increases in the value of T_{max} . Thus, it may be argued that predicting a lower limit, threshold load, T_{th} , (below which cyclic-fatigue crack growth will not be observed) is the appropriate design philosophy in the case of adhesively-bonded joints. Indeed, this has been the conventional fatigue criterion adopted by industrial design engineers for many years. Thus, the present work will especially discuss the importance of being able to predict the value of T_{th} .

4.2 Effect of the Description of the Variation of G_{max} with a and T_{max}

As may be seen from the results given in Figures 4 to 6, there are differences between the predictions when the various descriptions of the variation between G_{max} with respect to a and T_{max} are employed, and there are a number of noteworthy points.

Firstly, the predictions of T_{max} versus N_f for the single-lap joints which are derived via the FEA description and the analytical descriptions of FPMS and Krenk-Hu (KH), for the variation of G_{max} with a and T_{max} , are in very good agreement. This is clearly very encouraging and may be seen as good support for the accuracy of the present FEA modelling studies. Now, these approaches all include both the mode I and mode II contributions to the maximum applied strain-energy release-rate, G_{max} . (Hence, the contributions from both the peel and shear stresses acting on the single-lap joint are considered.)

Secondly, only in the FEA model was the path of the crack growth considered, although this had no significant effect on the predictions using the FEA description for the variation of G_{max} with a and T_{max} .

Thirdly, the fatigue data predicted via the Kinloch-Osiyemi (KO) and the Krenk-Hu mode I (KH-Mode I) descriptions of the variation of G_{max} with a and T_{max} are in poor agreement with each other and, generally, with the results from the FEA, FPMS and Krenk-Hu descriptions mentioned above. It will be recalled that in the Kinloch-Osiyemi (KO) and the Krenk-Hu mode I (KH-Mode I) solutions only a mode I contribution to the strain-energy release-rate, G_{max} , is considered.

Fourthly, comparing the predictions of T_{max} versus N_f for the single-lap joints using the various surface pretreatments and tested in the ‘dry’ and the ‘wet’ test environments, then the Kinloch-Osiyemi (KO) and Krenk-Hu mode I (KH-Mode I) models basically represent the lower and upper bounds, respectively, for the experimental results. The predictions which are derived from the FEA description and the analytical descriptions of FPMS and Krenk-Hu (KH), for the variation of G_{max} with a and T_{max} , vary from being in reasonable agreement with the experimental results (e.g. Figure 4b) to being in poor agreement with the experimental results (e.g. see Figures 5 and 6). Also, it may be noted that the agreement between the theoretical predictions and the experimental results is clearly poorer as one moves to higher values of T_{max} ; i.e. to lower values of N_f . A possible reason for this may be the inherent scatter in the experimental fracture-mechanics data in the linear region (i.e. ‘Region II’) of the graphs such as that shown in Figure 3 in Part I [1]. Hence, there exists some uncertainty in the values of the linear, ‘Region II’, constants, n , given in Table 1, and the values of these constants may affect the theoretical predictions when the fatigue tests on the lap joints are conducted at relatively high values of T_{max} . It may also be possible that there is a period of time needed to initiate the initial crack, or flaw, in the adhesive layer, or at (or near) the interface, under the cyclic-fatigue loading. Such a time period for crack initiation has been ignored in the present predictions [4,16] but in Part I [1] it was considered that such an initiation time would be relatively short and would not, therefore, account for the disagreement between the theoretical and experimental results shown in the present Figures 4 to 6. Further, it may be that the disagreement is due to the FEA, FPMS and Krenk-Hu (KH) descriptions all considering the total value of the strain-energy release-rate, G_{max} , since we have not attempted to divide this total value into the separate mode I and mode II contributions. However, apart from the uncertainties of undertaking this task for the FEA solution, as noted above, it has been shown [17,18] that for relatively tough adhesives, such as that used in the present work, the total strain-energy release-rate appears to be the critical parameter for fatigue crack propagation, rather than any individual component. Indeed, the present work has confirmed that taking an approach which only considers a mode I contribution (e.g. the KO and KH-mode I methods)

does not appear to improve the accuracy of the predictions. Finally, it might be that creep of the adhesive is occurring as one moves to higher values of T_{max} ; i.e. to lower values of N_f .

4.3 Predictions of the Threshold, T_{th} , Value

As discussed above, the present authors consider that it is of high importance to be able to predict accurately the value of T_{th} and such comparisons are drawn in Table 2. (As in Part 1 [1], the fatigue threshold values of the maximum applied load per unit width, T_{th} , are quoted at a value of $N_f = 10^7$ cycles, except for the CAE-pretreated joints tested in the ‘wet’ environment where a value of $N_f = 10^6$ cycles was used to avoid extrapolation.) As may be seen, the most consistently accurate approach is to employ the FEA description for the variation of G_{max} with a and T_{max} . This yields predictions for the threshold, T_{th} , values, below which no fatigue crack growth will be observed, which are typically within $30 \pm 10\%$ of the experimentally measured value of T_{th} . Thus, the agreement between theory and experiment is very good from the viewpoint of using the fracture-mechanics approach outlined here, and previously [2-5], for quantitative design studies.

5. Conclusions

The fracture-mechanics approach described in the present paper has been shown to provide a robust method for predicting the threshold values when undertaking cyclic-fatigue tests of adhesively-bonded lap joints. This prediction method consists of three steps. Firstly, the fracture-mechanics data obtained under cyclic loading in the environment of interest has been modelled, resulting in an expression which relates the rate of crack growth per cycle, da/dN , to the maximum applied strain-energy release-rate, G_{max} , in a fatigue cycle. Secondly, this relationship is then combined with an analytical or a computational description of the variation of G_{max} with the crack length, a , and the maximum applied load per unit width, T_{max} , in the joint, or component. Thirdly, these data are combined and the resulting equation is integrated to give a prediction for the cyclic-fatigue lifetime of the bonded joint or component. The theoretical predictions from the above method, using the different analytical or computational approaches to describe the variation of G_{max} with the crack length, a , and applied load, T_{max} , in the single-lap joint, have been compared and contrasted with each other, and compared with the cyclic-fatigue behaviour of the lap joints as ascertained from direct experimental measurements. In particular, the computational description, based upon FEA model, has been shown to be in good agreement with the equivalent analytical models. Furthermore, it has been shown to yield a predicted value for the threshold, T_{th} , value, below which no fatigue crack growth will be observed, which is typically within $30 \pm 10\%$ of the experimentally measured value of T_{th} for the lap joints based upon the different surface pretreatments which have been employed prior to bonding and tested in either ‘dry’ or ‘wet’ test conditions.

This agreement between theory and experiment is very good from the viewpoint of using the fracture-mechanics approach outlined here, and previously [2-5], for quantitative design studies. Since (a) the fracture-mechanics parameters are considered to be characteristic of the adhesive system and the test environment, and (b) the description of the variation of G_{max} with the length, a , of a propagating crack for a given maximum applied load per unit width, T_{max} , for any bonded joint or component design of interest (for example, whether a symmetrical or non-symmetrical single- (or double-) lap joint) can be ascertained from an analytical or numerical model.

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Tables

Table 1: Fatigue coefficients from Equation (1) extracted from the experimental fracture-mechanics data from tests conducted using the TDCB specimens [1].

	Gritblast and Degrease (GBD)		Chromic-Acid Etch (CAE)	Phosphoric-Acid Anodise (PAA)	
Coefficient	Dry	Wet	Dry and Wet	Dry	Wet
D ($m^2/N.cycle$)	1.61×10^{-23}	8.49×10^{-13}	1.51×10^{-26}	1.59×10^{-21}	1.41×10^{-13}
n	6.35	2.89	7.52	5.55	2.85
n_1	10	10	10	10	10
n_2	10	10	10	10	10
G_c (J/m^2)	600	600	1550	1650	1650
G_{th} (J/m^2)	125	25	200	200	50
a_0 (mm)	1.10	1.10	0.78	0.73	0.73

Table 2 Comparisons between the experimental and the predicted threshold, T_{th} , values for single-lap joints.

T_{th} (kN/m)	GBD		CAE		PAA
	Dry	Wet	Dry	Wet	Dry
Experimental	85	50	215	215	225
FEA	120	55	150	155	140
FPMS	135	55	160	165	160
Kinloch-Osiyemi	85	40	120	125	120
Krenk-Hu Mode I	225	65	270	275	230
Krenk-Hu	135	95	160	165	155
No of cycles at the threshold	10^7	10^7	10^7	10^6	10^7

Figures Caption

Figure 1: The loci of crack propagation modelled using the FEA method, for the single-lap joint specimens. Case 1 is for cohesive fracture in the adhesive layer and Case 2 is for interfacial failure along the adhesive/substrate interface.

Figure 2: Maximum strain-energy release-rate, G_{max} , versus the maximum applied load, T_{max}^2 , per unit width for single-lap joint. Data from the FEA model for (a) cohesive crack propagation and (b) interfacial crack propagation.

Figure 3: The gradient β ($=G_{max}/T_{max}^2$) versus crack length, a , in single-lap joints. The triangular points are the FEA data for cohesive crack propagation, the circular points are FEA data for interfacial crack propagation; and the solid line is the fitted relationship, Equation (17).

Figure 4: Variation of ratio, T_{max}/T_f , of the maximum applied load per unit width applied in the fatigue cycle to the monotonic failure load versus the logarithmic number of cycles, N_f , to failure for tests on single-lap joints. The experimental data and predictions are for specimens using the GBD pretreatment and conducted in the (a) ‘dry’ environment, and (b) ‘wet’ environment. (Monotonic failure load per unit width, T_f , of the single-lap joint using the GBD pretreatment is $T_f=274$ kN/m).

Figure 5: Variation of ratio, T_{max}/T_f , of the maximum applied load per unit width applied in the fatigue cycle to the monotonic failure load versus the logarithmic number of cycles, N_f , to failure for tests on single-lap joints. The experimental data and predictions are for specimens using the CAE pretreatment and conducted in the (a) ‘dry’ environment, and (b) ‘wet’ environment. (Monotonic failure load per unit width, T_f , of the single-lap joint using the CAE pretreatment is $T_f=538$ kN/m).

Figure 6: Variation of ratio, T_{max}/T_f , of the maximum applied load per unit width applied in a fatigue cycle to monotonic failure load versus the logarithmic number of cycles, N_f , to failure for tests on single-lap joints. The experimental data and predictions are for specimens using the PAA pretreatment and conducted in the ‘dry’ environment. (Monotonic failure load per unit width of the single-lap joint using the PAA pretreatment is $T_f=569$ kN/m).

Figure 1: The loci of crack propagation modelled using the FEA method, for the single-lap joint specimens. Case 1 is for cohesive fracture in the adhesive layer and Case 2 is for interfacial failure along the adhesive/substrate interface.

(a)

(b)

Figure 2: Maximum strain-energy release-rate, G_{max} , versus the maximum applied load, T_{max}^2 , per unit width for single-lap joint. Data from the FEA model for (a) cohesive crack propagation and (b) interfacial crack propagation.

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(a)

(b)

Figure 4: Variation of ratio, $T_{\max}/T_f$, of the maximum applied load per unit width applied in the fatigue cycle to the monotonic failure load versus the logarithmic number of cycles, N_f , to failure for tests on single-lap joints. The experimental data and predictions are for specimens using the GBD pretreatment and conducted in the (a) 'dry' environment, and (b) 'wet' environment. (Monotonic failure load per unit width, T_f , of the single-lap joint using the GBD pretreatment is $T_f=274$ kN/m).

(a)

(b)

Figure 5: Variation of ratio, T_{max}/T_f , of the maximum applied load per unit width applied in the fatigue cycle to the monotonic failure load versus the logarithmic number of cycles, N_f , to failure for tests on single-lap joints. The experimental data and predictions are for specimens using the CAE pretreatment and conducted in the (a) 'dry' environment, and (b) 'wet' environment. (Monotonic failure load per unit width, T_f , of the single-lap joint using the CAE pretreatment is $T_f=538$ kN/m).

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