Non-adiabatic pressure loss boundary condition for modelling turbocharger turbine pulsating flow

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Abstract

This paper presents a simplified methodology of pulse flow turbine modelling, as an alternative over the meanline integrated methodology outlined in previous work, in order to make its application to engine cycle simulation codes much more straight forward. This is enabled through the development of a *bespoke non-adiabatic pressure loss boundary* to represent the turbine rotor. In this paper, turbocharger turbine pulse flow performance predictions are presented along with a comparison of computation duration against the previously established integrated meanline method. Plots of prediction deviation indicate that the mass flow rate and actual power predictions from both methods are highly comparable and are reasonably close to experimental data. However, the new boundary condition required significantly lower computational time and rotor geometrical inputs. In addition, the pressure

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wave propagation in this simplified unsteady turbine model at different pulse frequencies has also been found to be in agreement with data from the literature, thereby supporting the confidence in its ability to simulate the wave action encountered in turbine pulse flow operation.

Keywords: Turbocharger, Turbine, One-dimensional, Non-adiabatic pressure loss, Unsteady flow, Modelling

1. Introduction

Turbocharging is regarded as one of the key elements in the success of downsized internal combustion engine systems, an effective strategy towards CO₂ emissions reduction. These days, a turbocharger is no longer restricted to its conventional application, but in various other usages such as turbo-compounding [1], electrically assisted [2, 3] and steam turbocharging [4], where the turbine will often operate at more extreme conditions. In all cases, the process of engine-turbocharger matching during the development stage plays a significant role towards achieving the best possible system performance, in terms of minimizing fuel consumption while maintaining good transient response. In current industry practice, engine modelling does not consider the full unsteady analysis of the turbocharger turbine, but instead treats it as a quasi-steady device. While this traditional approach can provide adequate simulations of the engines steady state engine performance, its deficiencies become apparent when attempting to accurately predict transient response [5], especially so when the desire is to predict the benefit of turbocharger technologies such as twin scroll turbines. Numerous unsteady turbine models have been developed over the years, yet none of these models

19 have been widely implemented into commercial one-dimensional engine cycle
20 simulation codes, mainly due to the associated complexity.

21 *1.1. Background study*

22 Commercial one-dimensional engine cycle simulation software tools model
23 the turbocharger turbine by following a quasi-steady approach (turbine inlet
24 and outlet are considered as the points where the flow conditions are experi-
25 mentally measured). However, due to the reciprocating nature of the engine,
26 the flow entering the turbine is of a pulsating nature, the form of which varies
27 depending on engine speed, displacement, number of cylinders, etc. Over the
28 last fifty years, several researchers have investigated this phenomenon, show-
29 ing that during each engine cycle the instantaneous mass flow measured at
30 the turbine inlet does not follow the steady-state characteristic, but instead
31 forms an unsteady hysteresis loop, particularly at lower pulse flow frequencies
32 [6–17]. The unsteady characteristic was also found (experimentally [16] and
33 analytically [18]) to vary in accordance with turbine inlet pulse frequency.
34 However, detailed experimental flow field investigations of the turbine [19]
35 suggest that the rotor itself does operate in quasi-steady manner (compared
36 to the turbine stage as a whole), and this has been analytically supported
37 by Strouhal number analyses [16, 20] and 3D CFD [21–23]. This leaves the
38 turbine volute as the primary source of the observed unsteadiness.

39 The major deficiency of a quasi-steady turbine model is therefore the
40 lack of a spatial dimension (or dimensions), and the consequent inability to
41 account for the flow dynamics taking place within the turbine volute during
42 pulsating flow operation. Ref. [23] further highlighted the deficiencies of a
43 quasi-steady turbine model where the temperature dependent heat transfer

44 and mechanical losses do not vary according to on-engine conditions but are
45 restricted to the measured “apparent” steady-state characteristic [24]. Thus
46 modelling improvements have been obtained by incorporating, as a first step,
47 the volume of the volute in a 0D or “filling-and-emptying” approach [25], and
48 taking it a step further by explicitly modelling the volute as a one-dimensional
49 form to additionally capture pressure wave action effects [23, 26–33].

50 In order to model the quasi-steady and unsteady characteristics of the tur-
51 bine rotor and volute respectively, an adiabatic pressure loss (APL) bound-
52 ary and a series of 1D tapered (non-constant cross-sectional area) pipe ducts
53 have previously been considered [30, 34]. Once calibrated for steady flow
54 conditions, this type of unsteady turbine model has shown its ability to cap-
55 ture the wave action phenomena under pulsating flow operation, reflected in
56 the swallowing capacity hysteresis prediction and the correct variation trend
57 against growing pulsating flow frequency [18]. Although this model showed
58 satisfactory unsteady mass flow prediction, prediction of instantaneous tur-
59 bine power was less convincing. In an attempt to improve power prediction
60 [35], a turbine meanline model was integrated into the 1D model. From
61 the established flow state, the meanline model makes a performance predic-
62 tion using a number of empirical loss relations, which are in turn based on
63 knowledge of the turbine geometry and operating parameters. Validation at
64 different pulse flow frequencies suggested good potential for the integrated
65 1D-meanline turbine model [36, 37], but this new methodology, although
66 innovative, has a few potential downsides.

67 Firstly, the meanline model required detailed rotor geometrical inputs,
68 e.g., rotor blade angle and tip-to-shroud clearance (this will be discussed fur-

ther in Section 4). However, rotor geometrical information is rarely accessible especially during engine-turbocharger matching. Secondly, the instantaneous turbine power calculation was performed from the already established pulsating turbine instantaneous flow field, sampled into a complete pulse cycle such that the phase synchronization of the instantaneous velocity components extracted at different locations in the turbine volute can be accomplished (e.g., the gas flow velocity taken at the volute tongue for instantaneous rotor upstream tangential velocity calculation via the free-swirl relation). Even though this approach showed good results in cold-flow testing data validation, such a model may fall short during transient engine modelling in the absence of continuous complete pulse cycles under constant frequency. Lastly, the pressure loss boundary used in this methodology to establish the pulsating turbine flow field was considered to be adiabatic¹ [38]. This is not the case in real operating conditions, it makes the downstream flow enthalpy prediction inaccurate and it prevents the model from being used in those operating conditions in which the downstream gas flow is re-used (e.g., presence of turbo-compounding systems, second-stage turbocharging, etc.).

In this paper, the development of a *non-adiabatic pressure loss* (NAPL) boundary as a turbine rotor boundary will be presented. The formulation details will first be presented in Section 2, followed by turbine steady-state performance validation in Section 3. The pulse flow turbine performance predictions will be compared with the literature unsteady experimental data [39] and with the meanline integration prediction method [37] (its formula-

¹The boundary was originally formulated to model adiabatic flow devices, e.g., valve, throttle, gauze, etc., but not a turbine rotor.

tion will also be briefly discussed in Section 2.5) at four pulsatile frequencies of three operating speeds in Section 4. In addition to flow prediction performance, the computational time for this NAPL rotor boundary will also be discussed in Section 5.

1.2. Experimental results

The experimental data used in this paper for model validation was generated in the cold-flow turbine test facility available at Imperial College London. The test facility features a low-inertia, high-speed permanent magnet eddy-current dynamometer as the loading device [39] and a rotary disc pulse generator to generate the pulse flow at different frequencies [9]. The turbocharger turbine studied here is of a single-entry mixed-flow turbine type in which the rotor was designed by [15] and coupled to a commercial nozzle-less turbine volute. The steady and pulsating flow turbine performance has been experimentally evaluated at six operating speeds from 27.0 to 53.7 rps/ $\sqrt{K}$ [39]. Four pulse frequencies have been considered, viz., 20, 40, 60 and 80 Hz which correspond to the typical operating range of a medium-size four-stroke diesel engine. For the model prediction validation, three operating speeds will be considered, i.e., 27.0, 43.0 and 53.7 rps/ $\sqrt{K}$ and all four flow frequencies at these selected speeds will be analysed.

Since the experiment was conducted at cold-flow conditions (i.e., without the presence of a combustor but with only a small amount of upstream heating to avoid condensation at the turbine exit), the effects of heat transfer and bearing loss can be suppressed. This effectively eliminated the additional complexity that may have presented, e.g., the non-uniform heat flux in heat transfer as a result of volute cross-section [40–42], and the variation of lubri-

117 cant viscosity and bearing loss at elevated temperature [43, 44]. Therefore,
118 the experimental data can be considered as purely the turbine aerodynamic
119 performance.

120 **2. Mathematical Model**

121 *2.1. Computational domain*

122 The unsteady turbine model domain has undergone a series of advance-
123 ment throughout the previous works on this subject, viz. from a single
124 constant cross-sectional area pipe of equivalent volume representing the tur-
125 bine volute, to sections of tapered pipes with equivalent area variation and
126 volume changes, before multiplying the number of rotor entries to resemble
127 more realistic flow entry into the rotor wheel [30, 34–37]. As a consequence
128 of the new model domain which required more detailed volute geometrical in-
129 formation, the quality of the prediction improved significantly. For instance,
130 the varying cross-sectional volute model area made the prediction of dimen-
131 sional parameters possible whereas only non-dimensional parameters could
132 be predicted in the initial stage model. The addition of multiple rotor entries
133 around the volute azimuth angle also proved capable of capturing the actual
134 phase of the pulsating flow energy transfer into the rotor. These features
135 are essential to achieve a turbine model that could be easily integrated into
136 engine software for real time engine simulation.

137 The computational domain used in this paper follows that used in Ref.
138 [37], as illustrated in Fig. 1. The turbine volute (inlet to 360° azimuth angle)
139 is modelled as a series of pipes with varying areas, summarized in Table 1 [45].
140 The flow from the turbine inlet is assumed to merge into a single pipe from

four rotor entries located at 90° , 180° , 270° and 360° azimuth angle before entering the rotor boundary. Previous work assumed a closed end at 360° azimuth which led to noticeable tertiary oscillation on the pulsating actual power curve peak at higher pulse flow [36] owing to the artificial pressure wave reflection at the closed end. The flow re-entrance at the volute tongue from 360° azimuth was neglected (gas flow is assumed to be completely consumed at the end of the volute) and so gas flow tends to circulate inside volute domain instead of exiting from the rotor boundary as a result of the absence of free-swirl modelling.² Although such a simplification may appear unrealistic, it was found not to deteriorate the mass flow prediction [36, 37].

[Figure 1 about here.]

[Table 1 about here.]

2.2. Numerical method

The gas dynamics code used in this study is based on the conservative one-dimensional Euler formulation [34], expressed in vector form as

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{W})}{\partial x} + \mathbf{C} = 0 \quad (1)$$

156

$$\mathbf{W} = \begin{bmatrix} \rho F \\ \rho u F \\ \rho e_0 F \end{bmatrix}, \quad \mathbf{F}(\mathbf{W}) = \begin{bmatrix} \rho u F \\ (\rho u^2 + p) F \\ \rho u h_0 F \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 \\ -p \frac{dF}{dx} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \rho G F \\ -\rho q F \end{bmatrix} \quad (2)$$

²The free-swirl modelling would require at least two-dimensional modelling, i.e., mean flow path and additional radial dimensions.

157 The numerical solution discretizes³ the governing equations across the
158 model domain. From a pre-defined initial state, the domain instantaneous
159 flow state is solved using the classical two-step Lax-Wendroff scheme com-
160 bined with a TVD (total variation diminishing) flux limiter to ensure a shock-
161 capturing, second-order accurate flow prediction. The Swamee and Jain fric-
162 tion model [46] and the Reynolds analogy heat transfer model [47] are used
163 to evaluate the friction and heat transfer term (G and q) respectively in the
164 source term vector $\mathbf{C}$.

165 The model boundaries (located at turbine inlet, exit, between each pipe
166 segment in Fig. 1, and the rotor boundary to be discussed in the next section)
167 serve as the closure for the numerical calculation at each pipe domain end.
168 At each time step, and for a given combination of incident waves (λ_{in_1} and
169 λ_{in_2} in Fig. 2), these boundaries compute and provide information on their
170 influence on the domain flow state, i.e., the transmitted and reflected waves
171 (λ_{out_1} and λ_{out_2} respectively in Fig. 2).

172 [Figure 2 about here.]

173 Open-end boundaries have been used at the model inlet and exit. The
174 experimental instantaneous stagnation pressure and temperature are defined
175 at the inlet boundary condition which has been set to be anechoic to emulate
176 the long pipe upstream to the measurement plane in the actual test facility.
177 Ambient conditions have been defined at the model open-ended exit bound-
178 ary condition, which has been extended by a dimension equal to the exducer

³The most promising ratio of discretization length over domain diameter has been recommended to be 0.55 [37] and is used here.

179 diameter to allow for turbine downstream pressure fluctuations [37, 47]. Do-
 180 main pipes are connected using an area variation boundary whereas the rotor
 181 entries (at 90°, 180°, 270° and 360° azimuth) are modelled using junction
 182 boundaries [38, 47]. The selection of these boundaries has been validated
 183 in [37] with satisfactory instantaneous flow prediction. The simulation is
 184 executed in transient mode, and is considered to achieve convergence once
 185 sufficiently small changes⁴ in turbine and rotor inlet mass flow rate (less than
 186 0.01%) are observed.

187 *2.3. Non adiabatic pressure loss boundary*

188 The formulation of the NAPL boundary is in reference to the adiabatic
 189 pressure loss (APL) boundary presented by Benson [38] except for the gov-
 190 erning equations. The position diagram of an NAPL device is shown in Fig. 2,
 191 and the gas flow velocity notation is taken as positive for gas flow towards
 192 the boundary. Parameters upstream of the device are denoted with subscript
 193 1 and downstream as 2.

194 For an APL device, the conservation of energy and continuity equations
 195 across the boundary are expressed as [38]:

Continuity equation:

$$\rho_1 u_1 F_1 = \rho_2 u_2 F_2 \quad (3)$$

⁴For steady-flow simulation, these changes are monitored over time, whereas in pulse-flow simulation, these are monitored at the same pulse cycle angle over successive pulses.

Energy equation:

$$h_0 = h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad \text{or} \quad (4)$$

$$a_0^2 = a_1^2 + \frac{\kappa - 1}{2} u_1^2 = a_2^2 + \frac{\kappa - 1}{2} u_2^2 \quad (5)$$

196 In order to realistically represent a rotor, the conservation of energy across the
 197 boundary must be associated with an additional stagnation enthalpy transfer
 198 term (given as Δh_{turb} in Eqn. 6), equal to the specific work transfer from the
 199 rotor wheel. Since it eventually leads to gas flow stagnation temperature loss,
 200 the boundary will be referred as a – *non-adiabatic pressure loss* boundary.
 201 The continuity equation remains unchanged from the APL device in Eqn. 3
 202 since the mass flow rate is always conserved across the boundary.

$$h_0 = h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} + \Delta h_{\text{turb}} \quad \text{or} \quad (6)$$

$$a_0^2 = a_1^2 + \frac{\kappa - 1}{2} u_1^2 = a_2^2 + \frac{\kappa - 1}{2} u_2^2 + (\kappa - 1) \Delta h_{\text{turb}} \quad (7)$$

203 The pressure loss coefficient through a pressure loss boundary can be defined
 204 as [38]:

$$K = \frac{\kappa \cdot \Delta p}{\rho_1 u_1^2} \quad (8)$$

205 Further defining the *non-adiabatic parameter*, d which consists of the stag-
 206 nation enthalpy transfer term, Δh_{turb} as:

$$d = \frac{2 \Delta h_{\text{turb}}}{a_2^2} M_2^2 \quad (9)$$

207 It is shown in Appendix A that while simultaneously fulfilling the pressure
 208 and stagnation enthalpy loss, an unique boundary upstream Mach number,
 209 M_1 at particular flow condition may be obtained by solving:

$$M_1^2 = \{2K(b+d) + a\alpha\} - \frac{\sqrt{\{2K(b+d) + a\alpha\}^2 - 4(b+d)\{(b+d)K^2 - \alpha\}}}{2\{(b+d)K^2 - \alpha\}} \quad (10)$$

$$\text{where} \quad a = \frac{2}{\kappa - 1} \quad b = aM_2^2 + M_2^4 \quad \alpha = \left(\frac{F_1}{F_2}\right)^2$$

210 Note that when there is no stagnation enthalpy change across the device,
 211 i.e., an adiabatic flow process across the boundary ($d = 0$), Eqn. 10 be-
 212 comes identical to that in the APL boundary condition formulation [38, 47].
 213 From the derivation in Appendix A, the resultant boundary reflected and
 214 transmitted wave characteristics are respectively defined as:

$$\lambda_{\text{out}_1} = A_{A_1} \left(A_1^* - \frac{\kappa - 1}{2} U_1^* \right) \quad (11)$$

$$\lambda_{\text{out}_2} = A_{A_{2c}} \left(A_2^* - \frac{\kappa - 1}{2} U_2^* \right) \quad (12)$$

215 The parameter with asterisk superscript are those further normalized by
 216 their corresponding entropy level, A_A . The flow direction through the bound-
 217 ary can be identified via comparing the upstream and downstream incoming
 218 flow characteristics,⁵ i.e.,

⁵Arbitrarily chosen initially, and corrected if necessary once flow direction has been established.

$$\text{Positive flow} \quad \lambda_{\text{in}_1}^* > \lambda_{\text{in}_2}^* \quad (13)$$

$$\text{Reverse flow} \quad \lambda_{\text{in}_1}^* < \lambda_{\text{in}_2}^* \quad (14)$$

$$\text{No flow} \quad \lambda_{\text{in}_1}^* = \lambda_{\text{in}_2}^* \quad (15)$$

219 Note that the mathematical calculation of the NAPL boundary is compatible
 220 for reverse flow by switching the boundary upstream and downstream sta-
 221 tions. However, the experimental turbine reverse flow characteristic is rarely
 222 present, hence it is not possible to validate it here. Nevertheless, since the
 223 typical turbine upstream pressure is constantly higher than atmospheric, this
 224 will not raise additional concern. Even if intermittent reverse flow occurred
 225 (e.g., during pulse flow conditions), the associated pressure loss coefficient
 226 would achieve a very large magnitude (corresponding to low M_2 , which will
 227 be presented in the subsequent section), i.e., consistent with reverse flow
 228 through the rotor wheel. Thus the flow direction is expected to resume its
 229 conventional direction once the inlet PR has increased above atmospheric.
 230 During no flow conditions, the pipe domain flow characteristics will not be
 231 updated, i.e., the flow does not experience any change. The instantaneous
 232 actual power extracted by the turbine at each time step is then computed
 233 via:

$$\dot{W}_{\text{act}} = \dot{m}_1 \cdot \Delta h_{\text{turb}} \quad (16)$$

234 The solution procedure for the NAPL boundary is identical to the APL
 235 formulation described in [38], thus can be summarized by the generalized

236 form given in Fig. 3 (in reference to equations in Appendix A). At a partic-
 237 ular time step, the solution for Eqn. 10 can be found through an incremental
 238 search across a range of M_2 where the convergence is expected to occur,
 239 gradually refining its range after each pass of sub-iteration.

240 [Figure 3 about here.]

241 2.4. Solution for pressure loss & non-adiabatic parameter

242 The pressure loss coefficient, K for an APL boundary is typically char-
 243 acterized against the boundary upstream Reynolds number or downstream
 244 Mach number at different solidities,⁶ which are obtained empirically from
 245 steady flow testing [38]. For a turbocharger turbine, a similar solution has
 246 been shown possible [48] and the pressure loss coefficient curves are grouped
 247 according to turbine reduced operation speeds, N_s , as shown in Fig. 4a. Like-
 248 wise, the enthalpy change, Δh_{turb} in the non-adiabatic parameter, d of the
 249 NAPL boundary can be characterized against the empirical steady-flow per-
 250 formance data from which it will be instantaneously interpolated according
 251 to local flow conditions prior to pulsatile operation.

252 [Figure 4 about here.]

253 Since the pressure loss coefficient, K and non-adiabatic parameter, d are
 254 both functions of the flow state and turbine operating speed, their influence
 255 on the new flow state can be summarized as in Fig. 5.

⁶The ratio of blocked area to the total cross-sectional area exposed to gas flow at the boundary.

[Figure 5 about here.]

The steady-state calibrated boundary data for the subject single-entry nozzleless turbine is shown in Fig. 4. The calibration procedure involved incrementally searching for the unique combination of pressure loss coefficient, K and stagnation enthalpy loss, Δh_{turb} that would match the steady-flow performance within tolerance. The pressure loss coefficient, without the consideration of enthalpy change across the rotor boundary [37], has been plotted in Fig. 4a to give a sense of how the inclusion of the non-adiabatic parameter affects the turbine model flow state. It is apparent in the non-adiabatic process, for a given turbine PR, the downstream Mach number is always lower than that calculated assuming an adiabatic process, and the difference increases with turbine PR (higher M_2). This is because the loss in gas flow stagnation enthalpy further reduces the gas flow energy. Nonetheless, the shifted Mach number points still fall on a unique speed line curve, which corresponds to the turbine MFP-PR characteristic at that particular speed.

In order to account for different operating temperatures to which the turbine rotor may be subjected, the calibrated specific enthalpy change for the non-adiabatic parameter is further reduced by the boundary upstream stagnation temperature, T_{01} . The resultant reduced specific enthalpy change at different turbine operating speeds is shown in Fig. 4b. Note that this reduced specific enthalpy change is analagous to the power parameter used in typical quasi-steady turbine models [38]. A separate hot flow steady-state simulation is also performed in Section 3 to validate this normalization method, by comparing the turbine non-dimensional performance parameters.

280 *2.5. Meanline integration method*

281 Throughout this paper, the new model predictions will be constantly
 282 compared against the established meanline integration method prediction
 283 [37] apart from the experimental data. For this reason, this methodology
 284 will be briefly discussed in this section. The meanline integration method
 285 uses the same model domain as in Fig. 1. Similar to the previous section, the
 286 subscripts 1 and 2 here denote the rotor boundary upstream and downstream
 287 stations respectively. From the converged flow state at the rotor boundary,
 288 the meanline integration method first obtains the “averaged” flow condition
 289 among the four rotor entries in Fig. 1, i.e.,

$$\dot{m}_1 = \sum_{\text{entry}=0}^4 \dot{m}_{\text{entry}} \quad (17)$$

$$p_1 = \sum_{\text{entry}=0}^4 \frac{F_{\text{entry}}}{F_1} p_{\text{entry}} \quad (18)$$

$$T_1 = \sum_{\text{entry}=0}^4 \frac{\dot{m}_{\text{entry}}}{\dot{m}_1} T_{\text{entry}} \quad (19)$$

290 The rotor inlet velocity is then constructed, from which the rotor inci-
 291 dence and passage losses will be considered according to the formulation in
 292 [49, 50], respectively defined as

$$L_{\text{incidence}} = \begin{cases} \frac{1}{2} K_{\text{incidence}} (W_1 \sin \beta')^2 & \text{if } \beta' < \frac{\pi}{4} \\ \frac{1}{2} K_{\text{incidence}} W_1^2 \left(\frac{1}{2} + |\beta'| - \frac{\pi}{4} \right) & \text{if } \beta' \geq \frac{\pi}{4} \end{cases} \quad (20)$$

$$L_{\text{passage}} = \frac{1}{2} K_{\text{passage}} [W_1^2 \cos^2 (|\beta_1 - i_{\text{opt}}|) + W_2^2] \quad (21)$$

where $\beta' = \beta_1 - \beta_{\text{blade}_1} - i_{\text{opt}}$

$$\text{and } i_{\text{opt}} = \tan^{-1} \left[\frac{-1.98 \cdot \tan \theta_1}{Z(1 - 1.98Z)} \right]$$

293 The turbine instantaneous actual power in this case is evaluated from
 294 the velocity triangle changes across the rotor boundary minus the clearance
 295 loss (due to rotor tip clearance) [51] and disk friction loss (the viscous loss
 296 between rotating rotor and stationary volute) [52], i.e.,

$$\dot{W}_{\text{act}} = \dot{m}_1 (U_1 C_{\theta_1} - U_2 C_{\theta_2}) - (L_{\text{clearance}} + L_{\text{disk friction}}) \quad (22)$$

$$L_{\text{clearance}} = \frac{(U_1 C_{\theta_1} - U_2 C_{\theta_2}) \frac{\text{clearance gap}}{r_{2\text{tip}}}}{1 - \frac{r_{2\text{hub}}}{r_{2\text{tip}}}} \quad (23)$$

$$L_{\text{disk friction}} = \frac{0.02125 \cdot U_1^2 \rho_1^2}{\dot{m}_1 \left(\frac{\rho_1 U_1 r_1}{\mu} \right)^{0.2}} \quad (24)$$

297 3. Steady-state Performance Validation

298 Comparison of the steady-state cold-flow turbine swallowing capacity,
 299 MFP and actual output power, $\dot{W}_{\text{act}}$ predictions against experimental data
 300 is given in Fig. 6. The meanline integration method predictions are the same
 301 as those presented in previous works [36, 37]. The comparison between the

302 turbine MFP predictions (evaluated using Eqn. 25) generated by the NAPL
 303 boundary and the meanline integration method is given in Fig. 6a.

$$\text{MFP} = \frac{\dot{m}_1 \sqrt{T_{01}}}{p_{01}} \quad (25)$$

304 Both methods show good agreement with the experimental data with
 305 an inaccuracy of less than 5% throughout a wide range of operating PR and
 306 different turbine operating speeds. For the actual power prediction in Fig. 6b,
 307 the meanline method showed larger deviation from the experimental data at
 308 the lower turbine operating speed (27.0 rps/ $\sqrt{K}$) since the rotor passage loss
 309 coefficient was only calibrated at 43.0 rps/ $\sqrt{K}$ turbine speed. This suggests
 310 that the turbine losses experience smaller changes towards higher operating
 311 speeds as compared to lower speeds. This observation is also consistent
 312 with the calibrated reduced specific enthalpy curves in Fig. 4b for the NAPL
 313 boundary method where the 43.0 and 53.7 rps/ $\sqrt{K}$ curves lie closer to one
 314 another than the 27.0 rps/ $\sqrt{K}$ speed. The NAPL boundary method on the
 315 other hand, showed no deviation larger than $\pm 5\%$ throughout all steady-state
 316 simulation cases since it was calibrated at all three operating speeds.

317 [Figure 6 about here.]

318 The hot-flow steady-state simulation was also performed to identify the
 319 validity of the characterized reduced specific enthalpy (Fig. 4b) at a different
 320 operating temperature. A turbine upstream stagnation temperature of 700 K
 321 is used in the hot-flow steady-state simulations (instead of ~ 340 K in cold-
 322 flow conditions). Since only the turbine aerodynamic performance will be

323 examined here, it is assumed the turbine operates under adiabatic conditions
324 (i.e., without external heat transfer as if it is perfectly insulated) despite
325 working under hot gas flow conditions. With such an assumption, the turbine
326 non-dimensional parameters (viz., MFP and total-static efficiency) will equal
327 those for cold flow conditions.

328 The turbine MFP, mass flow rate, total-static efficiency and the actual
329 power are presented in Fig. 7 in comparison to the cold-flow predictions and
330 experimental data. The turbine non-dimensional mass flow parameter, MFP
331 is identical (Fig. 7a) regardless of the operating temperature, yet the hot-flow
332 mass flow rate is expectedly lower than under cold-flow operation (Fig. 7b).
333 The lower mass flow rate is due to the lower gas flow density at the higher
334 temperature, and the mass flow variation against operating temperature is
335 governed by the non-dimensional MFP. Likewise, the actual output power at
336 hot-flow conditions is also comparably higher than during cold-flow operation
337 (Fig. 7d), yet its ratio to the gas flow isentropic power – the total-static
338 efficiency between different operating temperatures is exactly alike (Fig. 7c).
339 The hot-flow simulation indicated that the calibrated turbine performance
340 parameters at the rotor boundary (Fig. 4) are capable of accounting for
341 different turbine operating temperatures, which is crucial during on-engine
342 simulation.

343 [Figure 7 about here.]

344 4. Pulse Flow Performance Prediction

345 The unsteady turbine model pulse flow performance predictions will now
346 be presented at four pulse flow frequencies (20–80 Hz) at 27.0, 43.0 and 53.7

347 rps/ $\sqrt{K}$ turbine speeds. The unsteady turbine model predictions will be
348 compared to the pulsating flow experimental data [39] and the predictions
349 by the meanline integration method [37].

350 Fig. 8 shows the comparison of the turbine instantaneous mass flow rate
351 predictions by both the NAPL boundary and meanline integration methods
352 against the experimental data at 20–80 Hz pulse flow and at all three turbine
353 operating speeds. The magnitude of the mass flow pulses is higher at higher
354 turbine operating speeds. At a glance, both the NAPL boundary and the
355 meanline integration methods deliver equally good mass flow rate predictions
356 with the primary fluctuation magnitude and pulse phase matched reasonably
357 well with the experimental data. A similar trend can be observed at all pulse
358 frequencies.

359 [Figure 8 about here.]

360 The comparison of the instantaneous actual power prediction between
361 the NAPL boundary and the meanline integration method against the ex-
362 perimental data is shown in Fig. 9. Again, the predictions by both methods
363 are fairly similar, in terms of actual power magnitude and pulse features.
364 Crucially, the NAPL boundary is also capable of capturing the actual en-
365 ergy transfer phase into the rotor wheel. Note that the instantaneous actual
366 power curves in Fig. 9 are illustrated in their as-recorded phase, i.e., without
367 any phase-shifting. This will appear to be out-of-phase to the instantaneous
368 mass flow curves in Fig. 8. Similar to the mass flow curve, the magnitude of
369 the actual power pulses increase with turbine operating speed. Nonetheless,
370 there are a few noticeable discrepancies between these two predictions, e.g.,

the slight difference in the secondary fluctuation and its features (markedly at 130–190° pulse cycle at the higher turbine operating speed in Fig. 9). The temporal fluctuations seen in the meanline prediction (around 30–60° pulse cycle) at higher pulse flow frequency and turbine speed are not visible in the curve from the NAPL boundary method, nor in the experimental data.

[Figure 9 about here.]

Both prediction methods also showed similar a departure trend from the experimental data, i.e., the actual power primary peak at $53.7 \text{ rps}/\sqrt{K}$ speed. Such a mismatch is probably due to the fact that the flow component approaching the rotor is no longer operating in a quasi-steady manner at this operating range. From meanline method analysis [37], it was discovered that the rotor upstream absolute flow angle remained largely constant throughout the pulse cycle which was somewhat inconsistent with the relevant CFD findings [53] which showed the rotor upstream absolute flow angle changing up to 20°. In order to capture this flow phenomenon, the tangential velocity (swirl) component must be modelled separately, which is only possible via the two-dimensional modelling mentioned earlier in Section 2.1, and is out of scope of this work. After all, the model developed here is to be incorporated into a conventional one-dimensional engine gas dynamics code, hence the consistent gas dynamics governing equations set (Eqns. 1 and 2) has been maintained, as well as its dimension.

The instantaneous predictions in Figs. 8 and 9 can be further quantified as prediction deviation density plots in spectral form, to assess the prediction quality with regards to the empirical data and between prediction methods.

395 The instantaneous mass flow rate and actual power prediction deviations
 396 ($\dot{m}_{\text{dev}}$ and $\dot{W}_{\text{dev}}$) are computed using Eqns. 26 and 27 respectively. The pre-
 397 diction deviations are normalized by the maximum mass flow/actual power
 398 of the corresponding operating condition since the pulse magnitude varies
 399 according to operating speed and pulse flow frequency. The density plots
 400 of model prediction deviation are shown in Figs. 10 and 11. The abscissa
 401 shows the percentile of the deviation whereas the ordinate represents the
 402 number (density) of instantaneous prediction points falling within a particu-
 403 lar deviation range. To give a sense of the prediction performance at different
 404 operating speeds and pulse frequencies, the histograms in Figs. 10 and 11 are
 405 further broken down into operating speed and pulse frequency constituents.

$$\dot{m}_{\text{dev}} (\%) = \frac{\dot{m}_{\text{pred}} - \dot{m}_{\text{exp}}}{\dot{m}_{\text{expmax}}} \times 100\% \quad (26)$$

$$\dot{W}_{\text{dev}} (\%) = \frac{\dot{W}_{\text{actpred}} - \dot{W}_{\text{actexp}}}{\dot{W}_{\text{actexpmax}}} \times 100\% \quad (27)$$

406 [Figure 10 about here.]

407 [Figure 11 about here.]

408 The distribution plots of instantaneous mass flow rate prediction devia-
 409 tion given in Fig. 10 for the meanline and the NAPL boundary methods are
 410 highly comparable. In fact, the mean $\pm$ standard deviation ($\overline{\dot{m}_{\text{dev}}} \pm \sigma_{\dot{m}_{\text{dev}}}$) for
 411 the meanline and the NAPL boundary prediction methods are $1.4\% \pm 8.2\%$
 412 and $0.8\% \pm 8.3\%$ respectively. Since this is a percentage error, the ideal value
 413 would be nil, meaning no deviation from experimental results. In this case,

414 the NAPL boundary method yielded a lower deviation of mean at 0.8% as
 415 compared to the 1.4% resulting from the meanline integration method. An-
 416 other aspect of this data is the standard deviation, which provides a sense of
 417 the spread of this error percentage. Fig. 10 shows that the instantaneous mass
 418 flow deviations from the experimental data given by both solution methods
 419 have a comparable spread. It can also be seen that the mass flow rate predic-
 420 tion quality across different operating speeds and pulse frequencies are also
 421 fairly consistent between both prediction methods, illustrated by the more
 422 or less similar histogram sub-section height and their distribution pattern
 423 between Fig. 10a and 10b.

424 Similar considerations as the mass flow can be made for the instanta-
 425 neous actual power prediction (Fig. 11). The mean $\pm$ standard deviation
 426 ($\overline{\dot{W}_{\text{dev}}} \pm \sigma_{\dot{W}_{\text{dev}}}$) for the meanline and the NAPL boundary predictions meth-
 427 ods are $1.9\% \pm 13.5\%$ and $0.8\% \pm 12.8\%$ respectively. Again, the mean pre-
 428 diction deviation from the NAPL boundary method showed slightly higher
 429 adherence to the experimental data than the meanline integration method,
 430 i.e., 0.8% against 1.9%. The spread of instantaneous actual power predic-
 431 tion points are generally wider than for the mass flow prediction in Fig. 10,
 432 owing to the mismatch of the actual power peak at higher turbine speeds
 433 illustrated in Fig. 9. Nevertheless, due to lesser tertiary fluctuation at these
 434 pulse peaks in the NAPL boundary method, the deviation spread in Fig. 11b
 435 is marginally narrower (12.8%) than the meanline method (13.5%) from their
 436 corresponding means. Fig. 11 also shows that the actual power prediction
 437 performance of both the NAPL boundary and meanline integration methods
 438 are comparable at different pulse frequencies and turbine speeds, indicated

439 by the largely similar histogram sub-section heights and their distribution
440 pattern between Figs. 11a and 11b.

441 The histogram distribution in Figs. 10 and 11 have been found not to
442 follow a normal distribution via a normality test particularly towards the
443 tail ends of the distribution curve, thus limiting the validity of any further
444 statistical analyses that may be performed. Nevertheless, such a highly popu-
445 lated deviation plot around zero deviation is still a good indication of turbine
446 model performance. Furthermore the NAPL boundary method formulated
447 in this paper, i.e., a relatively simple method of achieving an unsteady tur-
448 bine model in a conventional one-dimensional engine gas dynamic code, has
449 been shown to deliver highly similar predictions to the meanline integra-
450 tion method – a more theoretical method of evaluating turbocharger turbine
451 actual power yet more tedious and sophisticated to accomplish.

452 Another important comparison aspect to be taken into consideration is
453 the amount of model inputs required by the meanline integration and NAPL
454 boundary methods. Table 2 summarizes the performance parameter and geo-
455 metrical information needed in setting up the unsteady turbine models using
456 these two boundary methods. In both methods, the steady-flow performance
457 maps (for model performance tuning/rotor boundary characterization) and
458 volute geometry along azimuth angle (for constructing the domain in Fig. 1)
459 are mandatory. While the NAPL boundary method requires only the rotor
460 inducer and exducer flow area (in order to define the boundary area ratio, α
461 in Eqn. 10), the meanline integration method demands more detailed rotor
462 dimensions (i.e., blade number, angles, diameters and clearance gap as indi-
463 cated in the lower-half of Table 2) in order to evaluate the four losses under

464 consideration (Eqns. 20–24). Hence, the comparisons in Table 2 and Figs. 8–
465 11 indicate that the NAPL boundary method can be a more favourable choice
466 in achieving the same quality of pulse flow prediction without the need for
467 extensive rotor geometry measurements.

468 [Table 2 about here.]

469 4.1. *Unsteady pressure propagation*

470 Fig. 12 shows the unsteady pressure wave propagation along the model
471 domain (using the NAPL boundary method) at 43.0 rps/ $\sqrt{K}$ turbine speed
472 for the 20–80 Hz pulse flow cases. Instantaneous static pressure is recorded at
473 eight locations along the model domain, viz., turbine inlet, 0° , 45° , 90° , 135° ,
474 180° , 225° and 270° azimuth angle. The mean/cycle-averaged static pressure
475 for each pulse frequency case at these particular locations is also given in the
476 legend of each subplot. In general, it can be seen that the pulse flow shape
477 and magnitude experiences the largest change between the turbine inlet and
478 45° azimuth angle (from Figs. 12a to 12b), with the mean static pressure
479 remaining nearly constant throughout the remainder of the turbine volute
480 with a small gradual drop in its magnitude.

481 At higher pulse frequency, the magnitude of the secondary peak becomes
482 closer to the primary pulse as the gas flow travels further downstream into the
483 turbine volute due to the associated wave action activity. Such observation
484 shows that the nozzle-shaped volute amplifies the pulse magnitude as the gas
485 flow cross-sectional area gradually gets smaller. This also helps to explain
486 the presence of the strong secondary peak beyond 120° pulse cycle in the
487 experimental measurement plane mass flow profile at 80 Hz flow while the

488 test facility rotary pulse generator [8, 9, 39] only generates one triangular
489 pulse within each 0–360° pulse cycle. It is worth mentioning that the finding
490 in Fig. 12 is also in line with the experimental instantaneous volute pulsating
491 pressure traces reported in [11].

492 [Figure 12 about here.]

493 The realistic wave action prediction indicates that the combination of the
494 turbine model domain (in Fig. 1) and the NAPL rotor boundary is capable
495 of capturing the major flow structures within the turbocharger turbine un-
496 der pulsating flow. Such a model capability is indispensable during engine
497 simulation since any pressure (wave) reflections from the rotor boundary and
498 volute domain will affect the back pressure (restriction) on the engine model.

499 **5. Computation time comparison**

500 Although it has been shown, from a prediction quality perspective, that
501 the NAPL boundary method can be an effective substitute for the meanline
502 integration method in evaluating the unsteady turbine actual power, it is still
503 worthwhile to analyse the feasibility of solving the difficulty of the meanline
504 integration method mentioned in Section 1.1 in the future work. For this
505 assessment, the computational time performance of these two methods will be
506 compared, i.e., another vital criterion of an effective unsteady turbine model
507 in an engine gas dynamic code, since engine model analyses are typically
508 performed transiently.

509 The computational time comparison will emphasize the time required in
510 obtaining the boundary pressure loss coefficient and non-adiabatic parameter

511 since the remaining computational steps (updating the flow field character-
 512 istics and flow propagation in the turbine domain) are identical regardless
 513 of which method is used. The computing time comparison is carried out
 514 for three different turbine steady-state constant pressure ratios ($PR = 1.3$,
 515 1.6 and 1.9) at $43.0 \text{ rps}/\sqrt{K}$ turbine operating speed. In this way, the ro-
 516 tor upstream tangential velocity in the meanline integration method can be
 517 evaluated easily without any conflict in the phase-difference. Different PR
 518 operating points will result in different flow states across the rotor boundary;
 519 hence the actual computation time is expected to be slightly different.

520 Fig. 13 presents the actual averaged computing time for the meanline and
 521 the NAPL boundary methods executed on a conventional desktop worksta-
 522 tion. The averaged values are taken over five computation cycles (time steps).
 523 The focus here is on the relative computing time difference between these
 524 methods, labelled as percentile in Fig. 13. In general, the NAPL boundary
 525 requires approximately 38.6% less computational time compared to the mean-
 526 line integration method, and the difference becomes larger towards lower op-
 527 erating PR (to as much as 44.6% at $PR = 1.3$). The longer computational
 528 time of the meanline method is due to its procedural step in solving the
 529 meanline loss equations (Eqns. 20–24) which consist of several implicit itera-
 530 tions whereas for the NAPL boundary method, the procedure involves only
 531 the linear interpolation in obtaining the reduced specific enthalpy change and
 532 pressure loss coefficient data from Fig. 4. Such a difference in computing time
 533 is not negligible considering that one-dimensional gas dynamic simulations
 534 are often transient and the time step can be rather small⁷ depending on the

⁷In the case of the pulse flow analyses presented in Section 4, the time step can be as

535 model domain complexity. Moreover throughout the pulse cycle under stud-
536 ied (which replicates the exhaust pulse flow a turbine may be encountered
537 during on-engine operation), the gas flow PR is at a low magnitude for most
538 instance.

539 [Figure 13 about here.]

540 This assessment although brief, has shown that the NAPL boundary
541 method is capable of matching the same level of accuracy of turbine perfor-
542 mance prediction as that using the meanline integration method in a shorter
543 computational time. Therefore, the use of the meanline model in engine
544 modelling could be limited to extrapolating the conventionally narrow man-
545 ufacturer’s steady-state turbine maps (such as those practised by [54]). In
546 the occasion where detailed rotor geometrical information is not accessible,
547 a physically-based method of extrapolating turbine performance maps [55]
548 may be used. Instead of typical polynomial-based methods employed in con-
549 ventional engine simulations, these turbine map extrapolation methodologies
550 provide a more physically realistic trend of turbine performance variation par-
551 ticularly towards very low and high PR range. The broadened steady-flow
552 performance maps can then be used in characterizing the NAPL boundary
553 (i.e., obtaining the pressure and enthalpy loss curves in Fig. 4) for engine
554 transient simulation use.

555 6. Conclusion

556 In this study, the following conclusions may be drawn:

small as 1 ms.

- 557 1. A non-adiabatic pressure loss (NAPL) boundary has been formulated
558 for evaluating the pulse flow turbocharger turbine actual power as an
559 alternative to the meanline integration method reported in previous
560 study [37].
- 561 2. The formulation of the NAPL boundary is based on the adiabatic coun-
562 terpart, but with the inclusion of a turbine work transfer – gas flow
563 stagnation enthalpy loss term.
- 564 3. The unsteady turbine model is calibrated against steady-flow perfor-
565 mance data, and the non-dimensionality of boundary data has been
566 verified via hot steady-flow simulation.
- 567 4. The NAPL boundary method showed highly comparable predictions
568 with the meanline integration method and experimental data at differ-
569 ent pulse flow frequencies and operating speeds. This is despite the sim-
570 plification of not evaluating the turbomachinery losses in real-time, but
571 relying on instantaneous interpolation of the pre-characterized bound-
572 ary data.
- 573 5. As such, the NAPL boundary method required less computational time
574 compared to the meanline integration method – by 38.6% on average.
- 575 6. The NAPL boundary method required significantly less rotor geomet-
576 rical inputs in comparison to the meanline integrated method. This
577 enhanced the overall model applicability.
- 578 7. The pulse pressure trace in the model domain closely resembles the
579 literature empirical data trend [11], thus confirming the validity of the
580 finite unsteady turbine model domain in resolving the physical wave
581 action taking place in pulsating flow conditions.

582 8. It is suggested that the systematic use of numerical turbine models
583 in engine simulation codes will be possible by using the conventional
584 meanline model/physical-based extrapolation method to broaden the
585 conventionally narrow turbocharger turbine steady-flow maps (instead
586 of the typical numerical extrapolation method). The widened perfor-
587 mance map can then be used to obtain the unsteady model boundary
588 data/parameter (as described in this paper) for on-engine simulation.

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Nomenclature

a	speed of sound (m/s)
A	dimensionless speed of sound
A_A	dimensionless entropy level
$\mathbf{C}$	source vector
C_θ	rotor tangential velocity
d	non-adiabatic term
e	specific internal energy (J/kg)
F	cross-sectional area (m ²)
$\mathbf{F}$	flux vector
G	friction source term
h	specific enthalpy (J/kg)
K	loss coefficient/pressure loss coefficient
L	loss
$\dot{m}$	mass flow rate
M	Mach number
MFP	pseudo-non-dimensional mass flow parameter
N_s	dimensionless speed parameter
NAPL	non-adiabatic pressure loss
p	pressure (Pa)
PR	pressure ratio
q	specific heat transfer rate (J/kg · s)
t	temporal dimension
T	temperature (K)
u	velocity (m/s)

Greek letters

α	boundary cross-sectional area ratio
β	relative flow angle (rad)
Δ	increment/changes
κ	ratio of specific heats
λ	Riemann variable
ρ	density (kg/m ³)
σ	standard deviation
θ	absolute flow angle (rad)
μ	gas flow dynamic viscosity

Subscripts

0	total or stagnation value
1	upstream of subject station
2	downstream of subject station
blade	parameter associated to rotor blade
B	parameter associated to boundary
c	corrected form of parameter
clearance	parameter associated to clearance loss
d	derived parameter
dev	prediction deviation
disk friction	parameter associated to disk friction
entry	parameter associated to rotor entry
exp	experimental value
hub	dimension at rotor hub

U	dimensionless velocity/rotor tangential velocity	in	incoming or incidence parameter
$\mathbf{W}$	state vector	incidence	parameter associated to incidence loss
W	relative velocity	max	maximum value
$\dot{W}_{\text{act}}$	work transfer (kW)	n	uncorrected form of parameter
x	spatial dimension	out	outgoing or transmitted parameter
Z	blade number	passage	parameter associated to passage loss
		pred	prediction value
		ref	reference value
Superscript			
*	values normalized with respect to entropy level	tip	dimension at rotor tip
		turb	parameter associated to turbine

Figure 1: Schematic diagram of the one-dimensional turbine model domain.

Figure 2: Position diagram of a pressure loss device.

Figure 3: Flow diagram showing the solution procedure for the generalized pressure loss (APL and NAPL) boundary.

(a)

(b)

Figure 4: Calibrated (a) pressure loss coefficient and (b) reduced specific enthalpy change profiles at three turbine operating speeds.

Figure 5: Influence chart showing the relationship between boundary losses, domain existing and new flow states.

Figure 6: Steady-state turbine (a) MFP and (b) corresponding deviations, (c) actual power predictions and (d) the corresponding deviations compared to experimental data plotted against turbine PR.

Figure 7: Steady state turbine (a) MFP, (b) mass flow rate, (c) total-static efficiency and (d) actual power predictions at hot-flow condition compared to cold-flow experimental data and predictions.

Figure 8: Instantaneous mass flow rate prediction comparison at (a) 20 Hz, (b) 40 Hz, (c) 60 Hz and (d) 80 Hz pulse flow.

Figure 9: Instantaneous actual power prediction comparison at (a) 20 Hz, (b) 40 Hz, (c) 60 Hz and (d) 80 Hz pulse flow.

Figure 10: Distribution plot of instantaneous mass flow rate prediction deviation, $\dot{m}_{dev}$ according to pulse flow frequency and turbine operating speed.

Figure 11: Distribution plot of instantaneous actual power prediction deviation, $\dot{W}_{dev}$ according to pulse flow frequency and turbine operating speed.

Figure 12: Pulse pressure distribution around model domain at (a) turbine inlet, (b) 0°, (c) 45°, (d) 90°, (e) 135°, (f) 180°, (g) 225° and (h) 270° azimuth angle.

Figure 13: Averaged computation time (in milliseconds) between meanline integration and NAPL boundary methods.

Table 1: One-dimensional turbine model domain geometry.

Location	Hydraulic diameter (mm)	Cumulative length (mm)
Turbine inlet	67.7	0
0° azimuth	57.2	181.35
90° azimuth	49.1	292.95
180° azimuth	41.1	397.65
270° azimuth	30.3	494.15
360° azimuth	10.5	579.15
Turbine exit	75.0	659.15

Table 2: Comparison of model inputs between meanline integration and NAPL boundary method.

Rotor boundary method	Meanline integration	NAPL boundary
<i>Steady-flow performance</i>		
PR vs. MFP map	✓	✓
PR vs. Total-static efficiency/ actual power map	✓	✓
<i>Volute geometry</i>		
Volute inlet area	✓	✓
Volute exit area	✓	✓
<i>Rotor geometry</i>		
Inducer area	✓	✓
Exducer area	✓	✓
Inducer blade angle	✓	
Exducer blade angle	✓	
Inducer tip diameter	✓	
Exducer tip diameter	✓	
Exducer hub diameter	✓	
Blade number	✓	
Inducer tip-to-shroud clearance	✓	
Exducer tip-to-shroud clearance	✓	

✓ = required

592 Appendix A. Derivation of NAPL boundary

593 In this section, the detailed derivation for non-adiabatic pressure loss
 594 (NAPL) boundary will be presented. The derivation procedure is based on
 595 the methodology presented by [38]. From the:

Continuity equation (Eqn. 3):

$$\rho_1 u_1 F_1 = \rho_2 u_2 F_2 \quad (\text{A.1})$$

Energy equation (Eqn. 5):

$$a_0^2 = a_1^2 + \frac{\kappa - 1}{2} u_1^2 = a_2^2 + \frac{\kappa - 1}{2} u_2^2 + (\kappa - 1) \Delta h_{\text{turb}} \quad (\text{A.2})$$

Pressure loss coefficient (Eqn. 8):

$$K = \frac{\kappa \cdot \Delta p}{\rho_1 u_1^2} \quad (\text{A.3})$$

596 Rearranging energy equation (Eqn. A.2) with Mach number definition $M = \frac{u}{a}$
 597 gives

$$\left(\frac{a_1}{a_2} \right)^2 = \frac{\frac{2}{\kappa-1} + M_2^2}{\frac{2}{\kappa-1} + M_1^2} + \left[\frac{2 \Delta h_{\text{turb}}}{a_2^2} \cdot \frac{1}{\frac{2}{\kappa-1} M_1^2} \right] \quad (\text{A.4})$$

598 and combining the continuity equation (Eqn. A.1) with ideal gas equation
 599 definition $a^2 = \frac{\kappa p}{\rho}$ yields

$$\frac{p_2}{p_1} = \frac{F_1}{F_2} \cdot \frac{a_2}{a_1} \cdot \frac{M_1}{M_2} \quad (\text{A.5})$$

600 Rearranging the pressure loss coefficient definition (Eqn. A.3) with the ideal
 601 gas definition gives

$$K = \left(\frac{p_1 - p_2}{p_1} \right) \frac{1}{M_1^2} \quad \text{or} \quad (\text{A.6})$$

$$\frac{p_2}{p_1} = 1 - K M_1^2 \quad (\text{A.7})$$

602 Combining Eqns. A.4–A.7 give an expression that relates the boundary up-
 603 stream and downstream steady flow Mach number to its pressure loss coef-
 604 ficient, K , i.e.,

$$\frac{\frac{2}{\kappa-1} + M_2^2}{\frac{2}{\kappa-1} + M_1^2} + \left[\frac{2\Delta h_{\text{turb}}}{a_2^2} \cdot \frac{1}{\frac{2}{\kappa-1} + M_1^2} \right] = \left(\frac{F_1}{F_2} \right)^2 \frac{M_1^2}{M_2^2 (1 - K M_1^2)^2} \quad (\text{A.8})$$

605 Equation A.8 appears in second order polynomial equation form. By
 606 simplifying the constant terms (or at least assuming they remain unchanged
 607 over a particular time step), the solution for Eqn. A.8 can be expressed as

$$M_1^2 = \{2K(b+d) + a\alpha\} - \frac{\sqrt{\{2K(b+d) + a\alpha\}^2 - 4(b+d)\{(b+d)K^2 - \alpha\}}}{2\{(b+d)K^2 - \alpha\}} \quad (\text{A.9})$$

$$\text{where} \quad a = \frac{2}{\kappa-1} \quad b = aM_2^2 + M_2^4 \quad d = \frac{2\Delta h_{\text{turb}}}{a_2^2} M_2^2 \quad \alpha = \left(\frac{F_1}{F_2} \right)^2$$

608 The boundary starred Riemann variables, λ_B^* (flow characteristics nor-
 609 malized by entropy level, A_{A_B}) are defined as

$$\text{Incoming} \quad \lambda_{\text{inB}}^* = A_B^* + \frac{\kappa-1}{2} U_B^* \quad (\text{A.10})$$

$$\text{Outgoing} \quad \lambda_{\text{inB}}^* = A_B^* - \frac{\kappa-1}{2} U_B^* \quad (\text{A.11})$$

Equation A.10 can be further rearranged as

$$\frac{\lambda_{\text{inB}}^*}{A_{\text{B}}^*} = 1 + \frac{\kappa - 1}{2} \frac{U_{\text{B}}^*}{A_{\text{B}}^*} = 1 + \frac{\kappa - 1}{2} M_{\text{B}} \quad (\text{A.12})$$

$$\text{hence} \quad A_{\text{B}}^* = \frac{\lambda_{\text{inB}}^*}{1 + \frac{\kappa - 1}{2} M_{\text{B}}} \quad (\text{A.13})$$

Upstream of the boundary, $\text{B} = 1$; $M_{\text{B}} = M_1$; downstream, $\text{B} = 2$; $M_{\text{B}} = -M_2$. Therefore for $\text{B} = 2$, the non-dimensional speed of sound (again, normalized against its entropy level) is

$$A_2^* = \frac{\lambda_{\text{in2}}^*}{1 + \frac{\kappa - 1}{2} M_2} \quad (\text{A.14})$$

Since $A_1^* = \left(\frac{p_1}{p_{\text{ref}}} \right)^{\frac{\kappa - 1}{2\kappa}}$ and $A_2^* = \left(\frac{p_2}{p_{\text{ref}}} \right)^{\frac{\kappa - 1}{2\kappa}}$, hence

$$A_1^* = \left(\frac{p_1}{p_2} \right)^{\frac{\kappa - 1}{2\kappa}} A_2^* \quad (\text{A.15})$$

Combining Eqns. A.6, A.14 and A.15 yields

$$A_1^* = \left(\frac{1}{1 - K M_1^2} \right)^{\frac{\kappa - 1}{2\kappa}} \left[\frac{\lambda_{\text{in2}}^*}{1 - \frac{\kappa - 1}{2} M_2} \right] \quad (\text{A.16})$$

From Eqn. A.4:

$$\left(\frac{A_1}{A_2} \right)^2 = \frac{\frac{2}{\kappa - 1} + M_2^2}{\frac{2}{\kappa - 1} + M_1^2} + \left[\frac{d}{M_2^2} \cdot \frac{1}{\frac{2}{\kappa - 1} M_1^2} \right] \quad (\text{A.17})$$

where

$$\frac{d}{M_2^2} = \frac{2\Delta h_{\text{turb}}}{a_2^2} \quad \text{and} \quad \frac{A_1^*}{A_2^*} = \frac{\frac{A_1}{A_{A1}}}{\frac{A_2}{A_{A2}}} = \frac{A_1}{A_2} \cdot \frac{A_{A2}}{A_{A1}}$$

617 From Eqns. A.6 and A.15, the boundary upstream-to-downstream non-
 618 dimensional speed of sound, A_B^* can be related to the boundary pressure loss
 619 coefficient and the solution of upstream Mach number from Eqn. A.9 (that
 620 implicitly accounted for the non-adiabatic process) as

$$\frac{A_1^*}{A_2^*} = \left(\frac{1}{1 - KM_1^2} \right)^{\frac{\kappa-1}{2\kappa}} \quad (\text{A.18})$$

621 By combining Eqns. A.17–A.18, the entropy change across the boundary is
 622 defined as

$$\frac{A_{A2}}{A_{A1}} = \left(\frac{1}{1 - KM_1^2} \right)^{\frac{\kappa-1}{2\kappa}} \left[\frac{\frac{2}{\kappa-1} + M_1^2}{\frac{2}{\kappa-1} + M_2^2 + \frac{d}{M_2^2}} \right]^{\frac{1}{2}} \quad (\text{A.19})$$

623 and the corrected downstream entropy level due to entropy difference across
 624 boundary is

$$A_{A2c} = \left(\frac{A_{A2}}{A_{A1}} \right) A_{A1} \quad (\text{A.20})$$

625 The un-corrected boundary downstream inlet characteristic, normalized
 626 by the corrected downstream entropy level is given as

$$\lambda_{\text{in}2n}^* = \frac{\lambda_{\text{in}2n}}{A_{A2c}} \quad (\text{A.21})$$

627 The remaining normalized flow variables, i.e., U_1^* and U_2^* are obtained
 628 from

$$U_1^* = M_1 \cdot A_1^* \quad (\text{A.22})$$

$$U_2^* = M_2 \cdot A_2^* \quad (\text{A.23})$$

629 where A_1^* is evaluated from Eqn. A.16 and A_2^* from Eqn. A.14 using $\lambda_{\text{in}_2}^* =$
 630 $\lambda_{\text{in}_2\text{n}}^*$ computed from Eqn. A.21. The derived value of the upstream inlet
 631 flow characteristic against which flow convergence across boundary will be
 632 compared against is therefore

$$\lambda_{\text{in}_1\text{d}} = A_{A_1} \left(A_1^* + \frac{\kappa - 1}{2} U_1^* \right) \quad (\text{A.24})$$

633 Lastly, the corrected downstream incoming characteristic due to the entropy
 634 change is expressed as

$$\lambda_{\text{in}_2\text{c}} = \lambda_{\text{in}_2\text{n}} + A_2^* \left(A_{A_2\text{c}} - A_{A_2\text{n}} \right) \quad (\text{A.25})$$

635 and the corresponding boundary upstream and downstream outgoing char-
 636 acteristics are

$$\lambda_{\text{out}_1} = A_{A_1} \left(A_1^* - \frac{\kappa - 1}{2} U_1^* \right) \quad (\text{A.26})$$

$$\lambda_{\text{out}_2} = A_{A_2\text{c}} \left(A_2^* - \frac{\kappa - 1}{2} U_2^* \right) \quad (\text{A.27})$$

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