A multiscale frame work for EHL and micro-EHL

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Abstract

In this paper a heterogeneous multiscale method is introduced to analyze the micro-elastohydrodynamic lubrication (micro-EHL) of bearings with topological features. Two scales are adopted in the analysis; the large scale describes the entire bearing domain, and the small scale simulations describe the Fluid Structure Interaction (FSI) at the small scale features. Conservation of mass and momentum of the lubricant, and bearing's elastic deformation are solved for. The relationship between the pressure gradient and mass flow is obtained from homogenised small scale FSI simulations, and applied on a global scale via a scattered data interpolation method. The elastic deformation of the textured bearing surface is addressed at both the large and small scales, by decomposing the displacement influence matrix into the diagonal terms and non-diagonal terms (sorted at the small scale and large scale respectively).

The multiscale method is demonstrated as being capable of modelling the global pressure and film thickness for a bearing with surface texture, while maintaining the accuracy of the small scale modelling features. The illustrative geometry considering is that of a linear converging pad bearing in two dimensions. The solutions are compared with those obtained using the lubrication theory for the smooth surface case, and good agreement is obtained. The method is then demonstrated for geometries incorporating topographical features.

Keywords

Multiscale Method; Micro-EHL; Surface Roughness Analysis and Models; Tilting-Pad Bearings

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1 Introduction

Elastohydrodynamic lubrication (EHL) has been to focus of a number of studies since the mid-1960s. The most common description of this lubrication regime is Reynolds equation (1), which has elucidated the important physical phenomena in this fluid regime. In the last 20 years surface texture has been of increasing interest with experimental investigations showing that such changes to the bearing surface can lead to an improved load bearing capacity and/or lower friction (2). The potential of such topographical features to improve bearing performance along with the use of lower viscosity lubricants in a number of applications, resulting in increased importance of roughness, has led to new demands being placed on lubrication analysis. Such analysis raises questions as to the viability of describing the fluid film using the lubrication approximation and its associated assumptions (3). This has led to renewed interest in applying Stokes or Navier-Stokes (N-S) flow equations to the problem of lubrication analysis.

Comparison between the Reynolds, Stokes and full N-S solutions has been made by a number of researchers in recent years (4-9). These studies investigated the textured surfaces lubrication problems and compared the solutions based on the different equations under different operating and topography parameters.

Odyck van and Venner (4) compared the pressure results in a lubricated geometry using Reynolds and Stokes equations for a single local feature. It was concluded that a significant difference in excess of 10% between the Reynolds and the Stokes solution measured by the smooth surface load occurs when the aspect ratio (the local film thickness to the feature lengthscale) is larger than 0.2. Similarly, Tichy and Bou-Said (5) investigated periodic rough surfaces, and found large difference from Reynolds solutions compared with those from Stokes, when the aspect ratio is larger than 0.1. Argir et al. (6) analysed a single macro-roughness effect using the N-S equations, where the macro-roughness wavelength and its amplitude are of the same order of magnitude as the lubricated film thickness. It was determined that the net pressure gain is a purely inertia effect due to not only the macro-roughness, but also the increasing Reynolds number. Although the classic Reynolds or Stokes equations are not adequate when inertial effects present, the generalized Reynolds equation as described by Wilson and Duffy (10) is capable of introducing inertial effects into the classical lubrication approximation. In 2005 Sahlin et al. (7) confirmed the above findings that a 'net pressure build up' is obtained with a micro-grooved surface, a result that cannot be arrived at if the problem is described by Stokes flow, due to the linearity of the equations. Dobrica and Fillon (8) concluded that the texture aspect ratio has an equally important influence with Reynolds number in determining inertial effects. It was also concluded that inertial effects cannot be analysed by single texture cell models, and it may play a negative role in pressure build-up on a partial inlet textured surface (textures partially generated at the inlet lubrication area of bearing surfaces). Similar conclusions were also made by Cupillard et al. (9) that inertial effect can have an adverse effect on bearing load if the texture geometrical parameters were not optimized. San Andres and Childs (11) studied bearing for rotordynamics problems where the speed is very high and large scale inertia effects cannot be neglected.

Both deterministic (where the topographical features are fully described and the small scale features are resolved over the entire lubrication domain) and homogenisation (where the small scale features are modelled and the results of these simulations are applied to the large scale simulation) based models have been used to analyse bearings with surface textures. Although the N-S equations have the potential to more comprehensively describe the lubrication physics, most studies have been based on the Reynolds equation, either in deterministic models (2, 12-14) or homogenisation models (5, 15-18). There are few deterministic models developed that apply the N-S equations (5, 19). The large difference in scales between the local feature and the global contact domain makes the computational simulation of the entire problem extremely challenging. However, the distinct separation of the scales makes the homogenisation method an appropriate means of describing the problem, and some of the literatures are reviewed in the following.

The 'average flow model' was firstly developed by Patir and Cheng (20) in 1978. The Reynolds equation was modified with flow factors, which treats the two scales separately. The flow factors are obtained from the local surface texture with periodic boundaries, and then coupled in a global scale simulation. Sahlin et al. (16) developed a homogenisation model for hydrodynamic full film lubrication analysis of bearings with periodic surface roughness, in which the flow factors were generated by the homogenised results of the compressible Reynolds equation. More recently, Sahlin et al. (17) further extended their model from hydrodynamic to mixed lubrication regime, from a representative sinusoidal roughness to a measured real surface topography. Significant progress was made that the local asperity deformation (both elastic and plastic) was computed by introducing a model of contact mechanics. In this case the deformation of the bearing surface was derived from the solid-solid contact, and Fluid-Structure Interaction (FSI) was not accounted for at either large or small scales, and the micro-fluid flow was described by the Reynolds equation.

Some investigators have used the N-S equations to describe the small scale problem, of particular note is the work of de Kraker et al. (21, 22). The incompressible N-S equations at the micro-scale were solved for and an averaged Reynolds equation was used at the large scale (with flow factors accounting for the small scale simulations). In de Kraker et al. (21) the plastic asperity deformations at large scale were analyzed by contact mechanics, however, the micro-EHL effects were not considered. A similar model was developed by Hewson et al. (23), in which the discrete-cell gravure roll coating process was modelled, where Stokes flow was applied to describe the small scale flow features. From the small scale solutions a relationship between the flow rate and pressure gradient was homogenised, and then solved in the large scale domain (with a flexible plastic substrate rather than bearing surface modelled). The linearity of Stokes flow description at the small scale simplified the integration of the small scale model into the large scale simulation.

Based on a Heterogeneous Multiscale Method (HMM) developed by E and Engquist (24), a two-dimensional (2D) multiscale (MS) model for micro-EHL analysis is developed in this study. The model allows both the local and global EHL effects to be described. The N-S equations are used to describe the small scale flow. The relationship between pressure gradient and mass flow is homogenised and applied to global solution via interpolation of the small scale data, which is not linear due to the elastic deformation of the bearing surface and

micro flow. Numerical solutions from the MS approach are verified with those from the lubrication approximation for the smooth surface, and some micro-EHL solutions of bearings with topographical features are demonstrated.

2 Theory

2.1 Heterogeneous Multiscale Method (HMM)

HMM is a general technique that has been applied to a number of problems where there is a distinct separation of scales (24). It assumes that the large scale model is known, in which some terms are explicitly unknown, and chooses a conventional large scale solver as the starting point. In the process of implementing this large scale solver, HMM replaces function evaluations that involve unknown quantities, by the results from numerical simulations using the small scale model. The key to the application of the HMM approach proposed here is how the two scales are coupled. For this particular application, the small scale results provide the homogenised pressure gradient-mass flow rate relationship, while the large scale applies this solution via a global pressure distribution and conservation of mass. As a sequential method has been used in the current work the macro state variables influence the homogenised function via the interpolation of the small scale solutions. The limitation of this approach is that the large scale inertial effects are not included. It only accounts for small scale inertial effects, and the neglect of large scale inertial effects is consistent with the lubrication theory.

2.2 Large scale simulation

The large scale simulation describes the EHL in the global lubrication domain. Similar to classic EHL approaches, the hydrodynamic pressure are solved simultaneously with the bearing elastic deformation. The difference between the current study and classic EHL is that the governing equation for pressure is a homogenised pressure gradient-mass flow equation, rather than the Reynolds equation.

2.2.1 Pressure governing equations

In the present study, the fluid flow is described by a governing pressure gradient equation and mass conservation equation, expressed as:

$$\frac{dp}{dx} = f(g, p, q) \quad [1]$$

$$\frac{dq}{dx} = 0 \quad [2]$$

The pressure gradient ($\frac{dp}{dx}$) is a homogenised function of the cell pressure (p), mass flow rate (q) and cell gap (g). The three parameters (p, g, q) in the right-hand-side of Eq. [1] are the only large scale derived parameters that influence the small scale flow. These data are obtained from small scale simulations, with details presented in Section 2.3. The Dirichlet

boundary condition applies to Eqs. [1] and [2] that the pressure at the global inlet (p_{in}) and outlet (p_{out}) boundaries is equal to zero:

$$p_{in} = p_{out} = 0 \quad [3]$$

Eq. [1] is an ordinary differential equation; together with its boundary conditions it describes a boundary value problem.

2.2.2 Separation of the deformation matrix

The elastic deformation of the bearing surface due to fluid pressure is calculated in a similar way as in classic EHL analysis. See for example (25) for more details.

Total displacement influence matrix ($\mathbf{K}$) is the flexibility matrix calculated using elasticity theory (26), also known as ‘deformation coefficient matrix’. The total deformation matrix for the surface is expressed as:

$$\delta = \mathbf{K} \times \mathbf{p} \quad [4]$$

Diagonal terms ($\mathbf{K}_1$) is the diagonal matrix composed of all diagonal elements of the total displacement influence matrix ($\mathbf{K}$). Physically its element K_{ii} means the displacement at point i due to a unit local pressure p_i (i.e. load per unit length for line contacts).

Non-diagonal terms ($\mathbf{K}_2$) is the deformation coefficient matrix ($\mathbf{K}$) with the diagonal elements removed. Physically its element K_{ij} means the displacement at location i due to a unit neighbouring pressure p_j (for $j \neq i$).

The diagonal terms and non-diagonal terms are separated from the total deformation matrix that:

$$\mathbf{K} = \mathbf{K}_1 + \mathbf{K}_2 \quad [5]$$

So that the total deformation in Eq. [4] are accounted for separately at both scales:

$$\delta = \mathbf{K}_1 \times \mathbf{p} + \mathbf{K}_2 \times \mathbf{p} \quad [6]$$

The non-diagonal terms ($\mathbf{K}_2 \times \mathbf{p}$) account for the non-local large scale deformation (i.e. the deformation at non-local points in the large scale domain due to a local pressure). The diagonal terms ($\mathbf{K}_1 \times \mathbf{p}$) account for the local deformation (i.e. the deformation at a local point due to a local pressure – both the micro-EHL and the local EHL effects as described in the following Section 2.3).

2.3 Small scale simulations

The small scale problem is described by the flow equations (in this case isothermal, laminar flow as governed by the N-S equations) and those governing the elastic deformation of the small scale features. The coupling is achieved through the application of the Arbitrary Lagrangian Eularian (ALE) method, which defines the reference coordinates ($\bar{X}$, $\bar{Y}$) in the solid domain and spatial coordinates (x , y) in the fluid domain. The material deformation is solved in the reference coordinates and the N-S equations are solved in the spatial

coordinates where the mesh is governed by the solid deformation. The solid material deforms due to fluid pressure, and this deformation consequently changes the geometry of the fluid domain and the fluid pressure.

The computational domain used to model the small scale problem is based on the small scale features as well as the diagonal terms of the deformation matrix. Since a small scale feature is regarded as a point at the large scale, the local deformation is assumed as a spring column. Thus, an equivalent height of the solid domain (t') is introduced in order to ensure that the local deformation in the small scale simulations is the same as the column deformation obtained from the diagonal matrix $\mathbf{K}_1$.

$$t' = k_1 \times E' \quad [7]$$

where, a constant k_1 is chosen to represent the elements in matrix $\mathbf{K}_1$. E' is the equivalent elastic modulus, for the plain strain model (26):

$$E' = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \quad [8]$$

where E and ν is the Young's modulus and Poisson's ratio of the bearing material respectively. By applying an equivalent material height to the problem, the local deformation at the fluid-structure interface (micro-EHL) is accurately described while the overall spring stiffness of the material at the larger scale is also maintained. It should be stressed that this approach is only valid when the material height is an order of magnitude greater than that of the topographical feature, as it is in the case considered in this paper.

2.3.1 Fluid flow model

In the fluid domain of a unit cell, the compressible isothermal flow is governed by the N-S equations given in the form below:

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - 2\eta/3(\nabla \cdot \mathbf{u})\mathbf{I}] \quad [9]$$

$$\nabla \cdot (\rho\mathbf{u}) = 0 \quad [10]$$

with ρ denotes the fluid density, $\mathbf{u}$ the velocity vector, p the fluid pressure, η the fluid viscosity, $\mathbf{I}$ the unit tensor.

The boundary conditions are shown in Fig. 1(a). The lower boundary CD is a sliding wall with speed U . The upper fluid structure interface is a no slip boundary. The AD and BC boundary is periodic with velocity (scaled for the strain in the mesh) and semi-periodic for the pressure field. The deformation of the small scale domain presents some challenges to the periodicity of the domain as length of the inlet and outlet boundaries are usually different. This difference in length is accounted for by scaling the local strain of the fluid boundary meshes, and the reference coordinates (undeformed mesh) are calculated as:

$$\begin{cases} \bar{Y}_L(i) = \frac{y_L(i)}{1+\varepsilon_L} \\ \bar{Y}_R(i) = \frac{y_R(i)}{1+\varepsilon_R} \end{cases} \quad [11]$$

where, ε is the strain, i is the mesh series number, subscript L and R denotes the left and right boundary respectively. The mass flow rate (product of velocity and density) is identical

at the periodic boundaries but the pressure has a discrepancy Δp . This is consistent with the HMM, where the near-periodicity is maintained in the inlet and outlet velocity conditions over the length of a small scale feature. As the scale separation increases this becomes an increasingly valid assumption. Indeed as the small scale length tends to zero, and the small scale feature vanishes the problem can be solved for analytically, resulting in the lubrication equation.

Given an inlet pressure profile (the pressure at the point 'A' in Fig. 1(a), denoted as p_0) and an initial gap (g , which includes the undeformed gap and non-diagonal terms of deformation, excludes the diagonal terms of deformation), the solution field of fluid velocity and pressure can be obtained by solving this model. The homogenised pressure gradient ($\frac{dp}{dx}$) in a unit cell is calculated as:

$$\frac{dp}{dx} = \frac{\Delta p}{L} \quad [12]$$

The pressure difference (Δp) between the left and right hand side boundaries is the same. The mass flow rate (q) is:

$$q = \rho \int_0^g u \, dy \quad [13]$$

The cell inlet pressure (p_0), pressure gradient ($\frac{dp}{dx}$), gap (g) and mass flow rate (q) are required data for the interpolation in the large scale simulation.

As the pressure is not linearly distributed in a cell due to the small scale topography and solid deformation, an average pressure (p^*), i.e. load per unit length, is defined to represent the cell pressure in large scale solutions:

$$p^* = \int_0^L p \, dx / L \quad [14]$$

It is this pressure that is applied to the global load bearing description of the bearing. The shear stress τ (shear force per unit length) for Newtonian fluid is calculated as:

$$\tau = \int_0^L \eta \frac{du}{dy} \, dx / L \quad [15]$$

2.3.2 Plain strain model

In the solid domain in Fig. 1(a), a plain strain model is applied in the reference coordinates frame. The strain-stress relationship is described by the following equations (26):

$$\varepsilon_x = \frac{1-\nu^2}{E} \left(\sigma_x - \frac{\nu}{1-\nu} \sigma_y \right) \quad [16]$$

$$\varepsilon_y = \frac{1-\nu^2}{E} \left(\sigma_y - \frac{\nu}{1-\nu} \sigma_x \right) \quad [17]$$

where σ is normal stress. The top boundary IJ is fully constrained, with zero displacement in both the x and y directions. The vertical boundary AI and BJ is constrained in the x direction only. The FSI boundary (marked in Fig. 1a) is free and subject to the local fluid pressure p . The solid displacements (u_2, v_2), as an output of this model, are subsequently used to define the spatial coordinates frame for the fluid flow model via the ALE method. The local

deformation (δ_1) to represent the unit cell is defined as the average of the vertical displacement at the cell boundaries, point A and B in Fig. 1(a):

$$\delta_1 = \frac{\delta_A + \delta_B}{2} \quad [18]$$

which is consistent with the diagonal terms $\mathbf{K}_1 \times \mathbf{p}$ in Eq. [6] by using the equivalent material height l' defined in Eq. [7].

2.4 Coupling of the pressure and deformation

The large scale solution procedure starts from an initial guess of pressure, which here is chosen to be the corresponding Reynolds solution of smooth surface. The deformation due to non-diagonal terms (δ_2) is calculated based on the initial guess of pressure, and added by the undeformed gap (g_u). The pressure is then solved based on the updated geometry with the pressure gradient-mass flow rate relationship (Eq. [1]), along with the local elastic deformation of the bearing surface obtained from the small scale solutions. This process is repeated until the relative error in pressure between two iterations satisfies the required precision (0.001 in the current study). For each iteration the non-diagonal deformation is relaxed with a relaxation factor of 0.5. The flow chart of the entire computing procedure is shown in Fig. 2.

The Eqs. [1] and [2] is solved using a shooting method to obtain the flow rate for which the outlet pressure is zero. The flow chart of this solving process is shown in Fig. 3.

2.5 The lubrication theory

Lubrication theory was used to compare with the multiscale method, through replacing the small scale simulations described in Section 2.3 by Reynolds equation. For the compressible Reynolds equation, the analytical formula of the pressure gradient and mass flow rate is:

$$q = -\frac{\rho h^3}{12\eta} \frac{dp}{dx} + \frac{\rho h(U_1 + U_2)}{2} \quad [19]$$

where $U_{1,2}$ is the sliding velocity of bearing surface 1 and 2, respectively. The film thickness (h) including the rigid gap, elastic deformation (δ) and texture topography (s) is calculated as:

$$h = h_a + \frac{x}{L}(h_b - h_a) + \delta + s \quad [20]$$

The elastic deformation in matrix form is calculated as:

$$\delta = \mathbf{K} \times \mathbf{p} \quad [21]$$

Given the same boundary conditions in Eq. [3], the Reynolds Eq. [19] is solved in a similar way to that of the MS approach as described in Section 2.4.

3 Numerical Method

3.1 Geometry and materials

The global geometry of the current micro-EHL model is a 2D linear pad bearing with micro pocket textures, as shown in Fig. 1(b). The pad face is a layer of PTFE with a thickness (t) of

2 mm. The pad backing is assumed to be rigid. The textured pad surface is assumed to be stationary, while the counter face is rigid and ideally smooth, and moves at a speed U . The undeformed inlet and outlet gap is h_a and h_b respectively. Micro-pockets are assumed to distribute uniformly at the pad bearing surface. The geometry of a unit pocket cell with length L and depth d is shown in Fig. 1(a). The lubricant is assumed to be incompressible and isoviscous in the current framework, (i.e., the variations of density or viscosity with pressure are neglected). The geometric parameters and material properties are listed in Table 1.

3.2 Calculation of the influence matrix $\mathbf{K}$

In the current study, only the elastic deformation of the pad bearing surface is taken into account, while bending is not considered. The influence matrix $\mathbf{K}$ was calculated using the finite element method as described by Rodkiewicz and Yang (27). The top boundary of the pad is constrained with zero displacement in both the x and y directions. A mapped mesh was used and mesh independence confirmed with 2000 elements. This mesh independence is shown in Table 2,

3.3 Homogenisation of the pressure gradient equation

The homogenised relationship between the pressure gradient and mass flow rate links the small scale and large scale simulations. This relationship is obtained via interpolation. In order to obtain an accurate representation of the small scale model, a range of small scale simulations were undertaken for a range of gaps (g), pressure gradients ($\frac{dp}{dx}$) and cell inlet pressures (p_0). A linear interpolation function was adopted, based on a Delaunay triangulation of the data using Quickhull algorithm as implemented in Matlab (28). To obtain effective data samples for the interpolation, the range of input parameters are selected, based on the corresponding results of the smooth surface case of Reynolds equation. This approach ensured that the small scale parameters considered spanned those of the large scale simulation.

The entire small scale solutions are computed using COMSOL Multiphysics. The variables are transformed to the non-dimensional forms for convenience of numerical computing:

$$X, Y = \frac{x, y}{L}, G = \frac{g}{L}, P = \frac{pL}{\eta U}, Q = \frac{q}{\rho LU} \quad [22]$$

Subsequently, the pressure gradient equation is obtained via linear interpolation,

$$\frac{dP}{dX}(i) = f \left[\widetilde{P}_0, \widetilde{G}, \widetilde{Q}, \frac{dP}{dX}, P(i), G(i), Q \right], i = 1, \dots, n \quad [23]$$

and pressure is calculated:

$$P(i+1) = P(i) + \frac{dP}{dX}(i) \cdot \Delta X, i = 1, \dots, n-1 \quad [24]$$

where the parameters with tilde indicate the known data which have been obtained from small scale analysis. Q is an unknown constant. n denotes the mesh points at the large scale domain, $n = 100$ was chosen in the present study. A similar interpolation approach was

used to determine the load per unit length (p^*), local deformation (δ_1) and shear force per unit length (τ), the latter two were obtained once the pressure profile had been obtained as they are not explicitly required in the solution method. The bearing carrying load is calculated as:

$$w = \int_0^{L_p} p^* dx \quad [25]$$

The friction coefficient (μ) is obtained:

$$\mu = \frac{\int_0^{L_p} \tau dx}{w} \quad [26]$$

4 Results and discussion

The results are presented and discussed in the three following sections. The numerical accuracy of the interpolation procedure described in Eq. [23] is studied in subsection 4.1. The large scale solutions for the smooth and textured surface cases are presented in subsection 4.2. The small scale solutions are analyzed in subsection 4.3.

4.1 Interpolation accuracy

The data interpolation is of major importance to the accuracy of the HMM framework proposed here. The sensitivity of the density of sample data is based on the smooth surface (because the interpolation is independent with the surface features). The fluid viscosity of 1 Pa s and cell length of 20 μm were used for the small scale analysis. Seven cases with different numbers of data points (m) from 125 to 4000 were selected as specified in Table 3. Note that the mass flow rate is dependent on three-dimensional (3D) variables: gap, pressure and pressure gradient, thus the data structure is 3D. The data point distribution for each dimensional data is uniform.

The relative errors in pressure (err_p) referring to the solution of the densest data $m = 4000$ are shown in Fig. 4, calculated by the following formula:

$$err_p^m = \sum_{i=1}^n |p_i^m - p_i^{4000}| / \sum_{i=1}^n p_i^{4000}, m = 125, \dots, 4000 \quad [27]$$

When the data density increases the large scale solutions steady converges. The relative error in the global pressure distribution is less than 1% if the number of data points greater than 1000.

4.2 Large scale solutions

Two different fluid viscosities (1 and 0.01 Pa s) are considered. For the high viscosity case, the MS and Reynolds solutions are firstly compared in terms of pressure and deformation for the smooth surface case in Fig. 5. Two different cell lengths (L) of 20 μm (small cell) and 200 μm (large cell) are examined within the MS approach. As there is no topography in each cell it was expected that the MS solution would be the same as that of the lubrication approach. Indeed it can be seen that the MS solutions agree well with the Reynolds solution with the relative error in the pressure distribution of 0.95% for the small cell case and 2.02% for the

large cell case (the term cell is used here to describe the size of the small scale simulation, however, no small scale feature is present). It is interesting to note that the error between the two cases reduces as the length of the small scale feature reduces, and the underpinning assumption that there is a separation of scales becomes increasingly valid. Importantly, as this paper introduces a method for applying the MS approach to EHL, the bearing surface deformations compare well between the MS and lubrication approaches (see Fig. 5). The small scale structural deformation captures the diagonal terms in the deformation matrix well with the relative error of 1.73% as shown in Fig. 5(c). MS global pressure, film thickness, and deformations are then compared between the textured surfaces in Fig. 6. The homogenised pressure is sensitive to the cell length and the small cell textured bearing predicts larger integral quantity of pressure.

For the low viscosity case, the pressure field, film thickness and deformations are compared between the two textured surfaces in Fig. 7. Similar conclusions as the high viscosity case are found in the bearing textured with small cells that the load is larger compared with the bearing textured with large cells. The maximum pressure, load and friction coefficient for all cases in this study are presented in Table 4. Although both textured cases result in a reduction in the load bearing capacity compared with the smooth surface case, there is a significant reduction in bearing friction (for example in the high viscosity case, 22.6% and 14.3% for the small and large textured cells respectively). This reduction of friction coefficients is because the shear stress is reduced due to an increase of the gap in the local features.

4.3 Small scale solutions

The small scale pressures, deformations and velocity streamlines at two different locations in the large scale domain (for high viscosity and small cell case) are plotted in Fig. 8, which can be regarded as the enlarged and detailed images at the corresponding locations in Fig. 6. These small scale results have been obtained by applying the large scale solutions at these two locations as boundary conditions to the small scale simulations. As can be seen in Fig. 9, the change in the inlet and outlet height is not large and the resulting difference in velocity between the inlet and outlet is therefore very similar. This is consistent with the underlying assumption that the small scale solution varies slowly from one cell to the next.

The fluid velocity streamlines show a flow pattern with near symmetric recirculation, highlighting the small effect that inertia plays on the small scale flow in the cases studied here. However it is expected that as Reynolds number is increased, the local streamline will show a different pattern as found by Cupillard et al. (9) and de Kraker et al. (22).

5 Summary

A heterogeneous multiscale framework has been developed for the EHL and micro-EHL analysis. The full N-S equations together with local elastic deformations are solved in the small scale. The elastic deformation of the bearing surface is addressed at both the large and small scales, by decomposing the influence matrix into the diagonal terms and non-

diagonal terms (sorted at the small scale and large scale respectively). A pressure gradient and mass flow rate relationship links the two scales, which is homogenised via interpolation of the data from small scale solutions, and applies to the large scale domain. The solutions of pressure and elastic deformation of the bearing surface obtained by the multiscale approach have been found agree well with those from the lubrication approach for the smooth surface case.

Since the current study is focused on the framework of multiscale micro-EHL method, only the solutions of incompressible flow are demonstrated. The application of compressible flow, cavitation and non-Newtonian fluid could also conceivably be considered in the framework as could formal optimization methodologies. There are two levels that the current model might be extended to high dimension. The first is where the small scale geometry considered is 3D with the homogenisation of such a solution limited to 1D (as with the current model). While this would permit 3D topography to be considered it would require the long-bearing assumption to still be applied. The second extension to the model is where a 3D small scale and 2D large scale simulation are developed. This is considerably more challenging, in the main due to the non-linearities of the small scale flow description leading to a coupling of the small scale flow in the two cross film directions.

6 Nomenclature

L	Cell length, m
L_p	Pad length, m
d	Cell depth, m
E	Young's modulus, Pa
E'	Equivalent elastic modulus, Pa
g	Input gap for small scale simulation, m; $g = g_u + \delta_2$
g_u	Undeformed gap, m
h_a, h_b	Undeformed inlet and outlet gap in large scale lubrication domain, m
h	Film thickness, m
$\mathbf{K}$	Displacement influence matrix, m^3/N
$\mathbf{K}_1$	Diagonal terms of matrix $\mathbf{K}$, m^3/N
$\mathbf{K}_2$	Non-diagonal terms of matrix $\mathbf{K}$, m^3/N
m	Number of sample data in small scale solutions for interpolation
n	Number of mesh points in large scale domain
p	Fluid pressure, Pa
p_0	Cell inlet pressure, Pa
p^*	Cell average pressure, Pa
q	Mass flow rate, $\text{kg}/(\text{m}\cdot\text{s})$
s	Texture geometry in the y direction, m
t	PTFE layer thickness, m
t'	Equivalent solid height, m
$\bar{X}, \bar{Y}$	Reference coordinates, m
X, Y	Dimensionless spatial coordinates
x, y	Spatial coordinates, m
$\mathbf{u}$	Fluid velocity vector, m/s
u, v	Fluid velocity components, m/s
U	Sliding velocity of the smooth surface, m/s
ρ	Fluid density, kg/m^3
η	Fluid viscosity, $\text{Pa}\cdot\text{s}$
δ	Deformation, m
δ_1	Diagonal terms of deformation, m
δ_2	Non-diagonal terms of deformation, m
ν	Poisson's ratio
μ	Friction coefficient

τ	Shear stress, Pa
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Tables and Figures

Table 1 Material properties and operating parameters

Inlet gap, h_a	30 μm
Exit gap, h_b	10 μm
Pad length, L_p	20 mm
PTFE layer thickness, t	2 mm
Cell length, L	20 and 200 μm
Cell depth, d	20 μm
Elastic modulus of PTFE, E	0.5 GPa
Poisson's ratio of PTFE, ν	0.4
Fluid density, ρ	870 kg/m ³
Fluid viscosity, η	0.01 and 1 Pa s
Sliding speed of the smooth surface, U	1 m/s

Table 2 Mesh sensitivity to the deformation matrix **K**

Mesh	Relative error (%)
500	1.60
1000	0.64
1500	0.19
2000	0.09

Table 3 Data selection of small scale solutions for interpolation

Total number (m)	Mesh point number in:		
	Initial gap (g)	Pressure gradient (dp/dx)	Cell inlet pressure (p_0)
125	5	5	5
250	10	5	5
500	10	10	5
1000	10	10	10
2000	20	10	10
3000	30	10	10
4000	40	10	10

Table 4 Summary of the maximum pressure, load and friction coefficient

Viscosity (Pa s)	Surface	Method	$p_{\max}$ (MPa)	w (kN)	μ
1	Smooth	Reynolds	6.51	83.5	0.0084
		MS_small cell	6.53	83.4	0.0084
		MS_large cell	6.43	82.5	0.0082
	Textured	MS_small cell	6.55	82.9	0.0065
		MS_large cell	6.19	79.1	0.0072
0.01	Smooth	Reynolds	0.44	5.38	0.0016
		MS_small cell	0.44	5.35	0.0016
		MS_large cell	0.44	5.35	0.0016
	Textured	MS_small cell	0.41	4.91	0.0009
		MS_large cell	0.30	3.67	0.0013

Fig.1 Micro-cell (a) and global (b) geometry of the pad bearing in line contacts ($|AE|=|HB|=0.3L$, $|FG|=0.2L$, the x-distance of $|EF|$ and $|GH|$ is $0.1L$.)

Fig.2 Flow chart of large scale pressure and deformation iteration

Fig.3 Shooting method for pressure solving

Fig.4 Data sample sensitivity on interpolation accuracy (smooth surface, high viscosity)

(a)

(b)

(c)

Fig.5 Global pressures and deformations for the smooth surface and high viscosity case (1 Pa s): (a) pressure and film thickness comparison between MS and Reynolds approach; (b) total, small scale and large scale deformations; (c) local deformation predicted by the MS method (for the small cell case), the diagonal terms V.S. the spring model with the equivalent material height.

(a)

(b)

Fig.6 Global solutions for the textured surfaces and high viscosity case (1 Pa s): (a) pressures and film thickness; (b) deformations

(a)

(b)

Fig.7 Global solutions for the textured surfaces and low viscosity case (0.01 Pa s): (a) pressures and film thickness; (b) deformations

Fig.8 The small scale pressures (at the middle of the gap), vertical deformation (at the FSI boundary) and velocity streamlines at two different locations (high viscosity and small cell case): (a) $x = 0.005$ m and (b) $x = 0.015$ m.

(a)

(b)

Fig. 9 The fluid velocity distributions at the small scale periodic boundaries at two locations (high viscosity and small cell case): (a) $x = 0.005$ m and (b) $x = 0.015$ m.