The Effect of Particle Porosity on Liquid Holdup in Heap Leaching

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ABSTRACT

The inter- and intra-particle porosities of heaps have two distinct length scales (of order millimetres between the particles versus tens of microns within the particles) and therefore the dominant flow mechanisms within and around the particles are quite different. This paper investigates the effect of particle porosity on heap hydrodynamics by comparing the behaviour of a model system consisting of non-porous glass beads with a system of actual ore particles. The overall liquid holdup behaviour of these two systems initially appears quite different. However, when the effect of the liquid holdup around the particles is separated from that within the particles, the same theoretical flow model can be applied to both the model and ore systems. This demonstrates that correlating the liquid flow to the overall liquid holdup is problematic and that the effect of the inter- and intra-particle liquid holdup should be considered separately. This is important as the amount of liquid held within the ore particles in these experiments was nearly as large as that held around the particles. The model for the external liquid flow proposes a power law relationship between the relative flow rate (flow rate divided by residual holdup) and the excess relative holdup (the steady state liquid holdup divided by the residual holdup minus one) with an exponent of two. It was found that the pre-factor in this relationship was quite a strong function of particle size for the spherical glass beads, but relatively constant for the more angular ore particles.

Keywords: Heap leaching, Hydrodynamics, Hydrometallurgy, Liquid holdup

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1. Introduction

The fluid flow in heap leaching is unsaturated, consisting of both leaching solution and air (Bartlett, 1992). The liquid flow characteristics are a very important aspect of the leaching process, influencing both the overall recovery and apparent kinetics of the system (Murr et al., 1981; Yusuf, 1984). There have been a number of studies of the fluid flow in heap leaching systems, but these studies have either tended to be purely empirical or produced qualitative descriptions of the fluid flow behaviour (eg. Roman, 1977; Murr et al., 1981; Yusuf, 1984; de Andrade Lima, 2006). More theoretical modelling of the fluid flow in heaps has typically involved using Richard's equation, which is a description for fluid in particle beds often used in ground water and oil reservoir modelling (Decker, 1996; Decker and Tyler, 1999; Cross et al., 2006).

The problem with all these approaches is that they either treat the liquid holdup as an empirical input parameter, which limits the predictive ability of the method, or they propose a direct relationship between the liquid holdup and the flow permeability of the system. In this paper, it will be demonstrated that the use of such a direct relationship is not entirely appropriate for two different reasons. Firstly, these systems exhibit hysteresis, with the steady state liquid holdup depending not only the current flow rate, but also on the flow rate history (Ilankoon and Neethling, 2012). It will also be demonstrated that the presence of porous particles has a marked influence on the liquid holdup and flow behaviour.

The packed bed that constitutes a typical heap has porous particles that are mainly in the size range of millimetres to a few centimetres. Thus, the porosity of the packed particles has two distinct length scales, namely that of the channels between the particles (i.e. interstitial space), which will typically have a length scale of order millimetres, and that within the particles (i.e. intra-particle space), which will typically have a length scale of order tens of microns and smaller. The Bond number, which is the ratio of gravity to capillary forces, will be around 1 for the fluid flow between the particles, indicating that this flow is in the transition region between capillary and gravity dominated flow. However, the existing micropores within the particles will have Bond numbers that are many orders of magnitude less than 1, indicating capillary dominated flow (note that a Bond number well below 1 does not mean that gravity does not affect the flow rate, it rather means that the shape of the flow paths is not influenced by gravity). This distinct separation of length scales means that liquid holdup within the particles will not have the same effect on liquid flow as the holdup between the particles and must thus be considered separately.

The main objective of this study is to experimentally investigate and model the effect of the particle porosity on the overall flow through a heap by comparing the behaviour of a model system consisting of non-porous glass beads with an ore system consisting of similar sized copper ore particles. A theoretical flow model to describe the flow between the particles in a way that it accounts for hysteresis in both systems will be developed.

2. Experimental Design and Methods

The liquid holdup measurements in a circular Perspex bed of 243 mm diameter and a height of 300 and 500 mm were performed gravimetrically by suspending the column from a high precision load cell. In order to verify that the load cell was able to measure the relatively small changes in liquid content needed in this work, the gravimetric measurements were compared to an independent measurement based on the volume of drained liquid (see Ilankoon and Neethling, 2012, for the method and results), with both methods giving virtually identical results and thus providing confidence in the load cell based measurements.

The glass bead system (i.e. model system) consisted of randomly packed mono-dispersed spheres of 2, 10, 14 and 18 mm. This size range covers the particle size typically found in column and industrial heap leaching.

Similar experiments to those performed with the non-porous glass beads were repeated with slightly porous copper ore particles. A sample of copper ore of around 285 kg was collected from Kennecott Utah Bingham Canyon mine.

Kennecott Utah Bingham Canyon mine is a low grade ore deposit that contains finely disseminated sulphide minerals, primarily copper and iron sulphides within a predominantly quartz monzonite host rock (Lufkin, 2010; Rio Tinto, 2009). This porphyry copper ore body has zones of both primary (containing mainly Chalcopyrite (CuFeS_2) and Bornite (Cu_5FeS_4)) and secondary (containing Chalcocite (Cu_2S) and other species) sulphide mineralisation. The current grade of this deposit is less than about 0.75% copper (Lufkin, 2010). After drilling and blasting, the rock is crushed to less than 250 mm (10 inches) in diameter in a gyratory crusher (Rio Tinto, 2012). The ore sample used in this project was obtained from the product of this primary crusher. The original ore sample thus has a size distribution that is wider and coarser than that typically encountered in heap leaching and is more typical of a dump leaching size distribution. The original size distribution is not especially relevant to the results obtained as narrow size intervals were obtained from the sample for use in the experiments presented in this work. Sieve analysis was performed using the sieve sizes of 2, 4, 8, 11.2, 13.2, 16, 20, 26.5, 31.5, 37.5 and 45 mm. The resultant particle size distribution curve is shown in Figure 1.

All the ore experiments were conducted in the 300 mm column, as there were insufficient particles in each size range to fill a 500 mm column. Typically, about 18 kg of particles were required to fill the 300 mm column. The bottom plate of the column has apertures of 2 mm, thus setting the smallest size of particles that could be tested. The 2-4 mm particle size range did not contain sufficient material to fill the column and thus the smallest size range tested was the 4-8 mm fraction.

Nine different narrow size fractions of ore particles were used in these tests, namely 4-8 mm, 8-11.2 mm, 11.2-13.2 mm, 13.2-16 mm, 16-20 mm, 20-26.5 mm, 26.5-31.5 mm, 31.5-37.5 mm and 37.5-45 mm (Figure 2).

Figure 1: Particle size distribution of the original copper ore sample.

Figure 2: Ore particles of selected narrow size ranges used in this study.

The water accessible porosity of the ore was obtained by subtracting the weight of ore particles which were soaked for 3 days from the dry weight of the particles. The soaked particles were screened and patted down with a cloth to remove any external water. This can cause some inaccuracies and therefore porosity values are only estimates. The water accessible porosity is likely to be lower than the actual porosity as some of the pore volume

might not be connected to the particle surface. The porosity values are within the range of 2-7%. Larger water accessible porosities were found in the smaller ore particles (eg. 7.1% in the 4-8 mm fraction versus 2.2% in the 37.5-45 mm fraction). The most likely reason for this is that the average distance to the surface is smaller in the smaller particles and therefore there are less likely to be unconnected pore spaces in the smaller particles. Another possible contribution is that more porous regions of the ore might fracture more easily and thus be over-represented in the smaller size fractions.

Figure 3 shows both the model system and the ore system together with the other main components of the 1-D experimental rig.

Figure 3: The column leaching experimental rig. The left figure shows the model system packed with glass beads and the right figure shows the ore system.

A novel liquid distributor was designed so that liquid could be evenly distributed over the surface of the packed bed (Figure 3 and see Ilankoon and Neethling, 2012). The distributor consists of 32 individual drip points with equal flow from each point. The liquid distributor was mounted over the packed bed as a separate unit, thus not affecting the weight measured by the load cell. The superficial flow rates within the range of 0.0075-0.12 mm/s were used, the lower values of which are within the relevant range of solution application rates in industrial leaching and studies of column leaching (Roman et al., 1974; Cariaga et al., 2003; de Andrade Lima, 2006). The full details of the experimental setup can be found in Ilankoon and Neethling (2012).

2.1. Steady state and residual liquid holdup measurements with the model system

Liquid was introduced at the lowest rate (i.e. 0.0075 mm/s) to the non-porous system to measure the steady state liquid holdup, which was achieved after approximately an hour. After recording the steady state liquid content, the bed was allowed to drain until no more liquid comes out of the bottom of the column and a constant residual holdup was achieved. This drainage time is usually between 15-20 minutes. The liquid flow is then turned back on at the next liquid addition rate (i.e. 0.015 mm/s) and the procedure is repeated over the other three flow rates (0.03, 0.06 and 0.12 mm/s). After the highest flow rate, the same procedure was performed while reducing the flow rate following the same sequence, once again measuring the respective steady state and residual liquid holdup values (see Figure 4). The output of this procedure is a set of steady state liquid holdup values and corresponding residual liquid contents (Ilankoon and Neethling, 2012).

Figure 4: Steady and residual liquid holdup during the increasing and decreasing flow rates in 500 mm column packed with 2 mm particles. The superficial velocities in mm/s are shown (Ilankoon and Neethling, 2012).

3. Inter-particle flow model

A theoretically based model has been developed by the authors (Ilankoon and Neethling, 2012), which describes low saturation flow through a packed bed of non-porous particles including the effect of liquid holdup hysteresis. In subsequent sections, it will be shown how

this model can be modified to account for the particles possessing porosity, as is the case in heap leaching systems.

At low superficial velocities liquid flows through the particles as rivulets and these flow features were observed during the liquid flow experiments. In addition, in the drained bed some of the liquid connections remain to form the residual holdup. The residual holdup (θ_r) within the particle bed consists of liquid held between neighbouring particles by capillarity. The amount of liquid that will be held in an individual connection will depend on the gravitational force as well as the capillarity, the ratio of which is the Bond number. As it influences the shape of the connections, contact angle will also influence the amount of liquid held in them. This means that the residual holdup is proportional to the number of liquid filled connections per unit area ($N_{rivulets}$) in the bed as equation 3.1, with a proportionality constant A_r (the average cross section area of the residual liquid in a rivulet) that depends on the Bond number and contact angle only.

$$\theta_r = N_{rivulets} A_r \quad (3.1)$$

A relative velocity (v_s^*) can thus be defined, which is proportional to the average flow per rivulet ($v_s/N_{rivulets}$), where v_s is superficial liquid velocity.

$$v_s^* = \frac{v_s}{\theta_r} \quad (3.2)$$

A relative holdup (θ^*) that is proportional to the holdup per rivulet ($\theta/N_{rivulets}$) can also be defined, where θ is steady state liquid holdup.

$$\theta^* = \frac{\theta}{\theta_r} \quad (3.3)$$

Liquid is held in reservoirs between the particles by capillarity and these separate reservoirs are connected by flowing drops and rivulets. In Ilankoon and Neethling, 2012, a theoretical model for the relationship between the relative velocity and relative liquid holdup was developed:

$$v_s^* = K(\theta^* - 1)^2 \quad (3.4)$$

$$\text{where } K = \rho g A_r / \mu C_{drag}$$

The pre-factor, K , in equation 3.4 depends on the Bond number, the shape of the rivulet (which will depend on contact angle), its tortuosity (which will depend on the particle packing) and as well as rheological properties of the liquid. A_r is likely to be a function of the particle size and the Bond number and not a function of the flow rate (Ilankoon and Neethling, 2012). This theoretical model proposes a squared relationship between the v_s^* and $\theta^* - 1$ for the model system and the proportionality, K is expected to change as the particle size changes.

4. Model system and the ore system

As was demonstrated in Ilankoon and Neethling, 2012, this model fits the experimental data for the non-porous model system very well (see Figure 10). Therefore, to check the validity

of the same model for the porous ore system (Figure 3 right), preliminary experiments were performed with dry ore particles.

The first thing that was observed when carrying out the experiments with the dry ore particles was that the breakthrough time for the first drop to come out the bottom was dramatically longer than with the model system (about one hour compared to five to six minutes in the model system when using a 300 mm column) (see Table 1). The reason for this difference is that, unlike the glass spheres which have no porosity, the ore particles are slightly porous (2-7% of water accessible porosity) and, having very fine channels and a relatively low contact angle, there is strong capillary suction into the ore particles. This means that as soon as a water droplet touches an unsaturated ore particle, the liquid gets sucked into the ore particle by capillarity. This water is also distributed to neighbouring dry particles by capillary wicking.

Further, with a dry bed start-up condition it is very hard to tell which portion of the liquid within the bed is held in the particles and which is held between the particles, factors that will be important for the subsequent modelling in this work. Determining the residual liquid holdup is also very hard as the ore column continues to drip very slowly for more than 24 hours rather than stopping dripping within half an hour, as is the case with the model system.

4.1. Steady state external liquid holdup of the ore system

To distinguish the two fluid classes in the ore system the experiments were performed with saturated particles (note that it is the particles that are initially saturated and not the bed). This was achieved by placing the particles in water for three to four days and then screening them and patting them down with a cloth to remove any external water. The particles were then placed in the column and the liquid flow to the column started straight away. As both the dry mass of the particles and their wetted weight is known, it is possible to calculate the amount of liquid initially held within the particles (i.e. water accessible porosity). The liquid content within the saturated ore particles represents around half of the total liquid in the system during the steady state portion of this particular test (Ilankoon, 2012).

Assuming that as long as liquid is being added to the system, capillarity will ensure that the particles remain saturated or close to saturated, then any extra liquid within the system is being held between the particles and the external steady state holdup values were calculated to cover the experimental conditions described above. Note that this liquid content is a nominal external liquid content calculated by subtracting the weight of the column from the initial weight of the column containing the saturated ore particles. Unlike when starting with a bed of dry particles, starting with saturated particles, the breakthrough time is comparable to that of the non-porous glass bead system (see Table 1).

Table 1: Comparison of the breakthrough times for all the particle sizes in the model and ore systems.

Model System (300 mm)		Ore System (300 mm)	
Particle Size (mm)	Breakthrough Time (s) at 0.0075 mm/s	Particle Size Range (mm)	Breakthrough Time (s) at 0.0075 mm/s
2	396	8-11.2 (Initially dry)	2855
10	318	4-8 (pore saturated)	1709
14	250	8-11.2 (pore saturated)	878
18	239	11.2-13.2 (pore saturated)	852
		13.2-16 (pore saturated)	827
		16-20 (pore saturated)	562
		20-26.5 (pore saturated)	530
		26.5-31.5 (pore saturated)	518
		31.5-37.5 (pore saturated)	503
		37.5-45 (pore saturated)	340

4.2. Residual liquid holdup of the ore system

When attempting to ascertain the residual holdup in the column by turning off the flow, a large amount of liquid drains out in the first ten minutes but it then continues to drain very slowly for many hours, compared to the non-porous system, where the column stops dripping after about 20 minutes, with most of the liquid lost in the first few minutes (again in a 300 mm column). This results in quite different liquid content profiles as a function of time during the drainage portions of these experiments (Figure 5), with the glass bead's liquid content asymptoting to a constant value very rapidly, while the ore system has a liquid content that initially decreases rapidly, but then instead of asymptoting to a constant value continues to slowly decrease and does so for many hours (see Figure 5 right).

Figure 5: Variation of the liquid holdup during the drainage of the 300 mm column at superficial velocity of 0.12 mm/s containing 10 mm glass beads (left) and 8-11.2 mm ore particles (right).

While the liquid content profiles look quite different, the profiles for the flow out of the columns are very similar, especially at short drainage times (see Figure 6). At longer times there is an important difference, with the flow out of the column rapidly dropping to zero in the glass bead system, while for the ore system the flow out drops off very rapidly not to zero, but to a very small and near constant value (see Figure 6). The model system's curve is a bit less smooth, but this is mainly due to the low liquid contents involved and thus the finite precision of the measurements.

Figure 6: Comparison of the drainage velocity for the glass beads and ore particles in a 300 mm column. The initial liquid addition rate was 0.12 mm/s.

In Figure 7, it can be seen that this small flow is two orders of magnitude slower than the initial flow (which is the same as the flow through the bed at steady state) and that it remains very constant over quite a long period of time.

Figure 7: Drainage velocity of the column for 8-11.2 mm ore particles in 300 mm column at superficial velocity of 0.12 mm/s (log-normal axes).

It was found that the drainage rate became relatively constant after about 25-30 minutes of drainage (see Figure 7) and that for every flow rate for a particular particle size this value was uniform (see Figure 8). This final drainage rate (v_{end}) remains virtually constant for many hours before tapering off.

Figure 8: End drainage velocity of different ore fractions at different superficial liquid velocities in the 300 mm packed bed.

Visually, this flow is accompanied by the slow drying out of the ore particles from the top of the column downwards, with the flow tapering off as the drier ore region approaches the bottom of the column. It is thus a good assumption that this small flow is caused by liquid motion within the ore particles. At steady state, this flow is not a major contribution to overall fluid motion, being over an order of magnitude smaller than the total liquid flux, but it does have a significant effect on the final drainage of the column due to the length of time over which it acts. Therefore, it is possible to use this to estimate the residual liquid held between the ore particles.

If the flow within the particles, rather than around them, is assumed to be constant at short times and have the same flow rate as it asymptotes to longer times, then the residual liquid holdup between the particles can be estimated by adding the liquid flowing out of the particles back onto the liquid content as given by equation 4.1.

$$\theta_{ex} = \theta_e + \frac{v_{end}t}{Z} \quad (4.1)$$

where θ_{ex} is the estimated external liquid between the particles, v_{end} is the roughly constant flow out of the column at longer times and Z is the length of the column. θ_e is the liquid

content obtained by subtracting the weight of the bed from the initial weight of the bed containing saturated ore particles.

Unlike the actual liquid content of initially saturated particles, the estimated external liquid content between the ore particles, θ_{ex} , rapidly asymptotes towards a constant value (see Figure 9) and this liquid content after 40 minutes drainage was taken as the residual holdup for the externally held liquid. This asymptotic value will be used in the model represented by equation 3.4 since the liquid contents required in this model are those held outside the particles.

Figure 9: Actual and estimated liquid holdup for 8-11.2 mm ore particles during the drainage in 300 mm column at a superficial velocity of 0.12 mm/s.

As the initial and final weights of liquid in the column, as well as the weight of drained liquid were measured, the amount of evaporated liquid could be calculated. This was negligible over the standard 40 minute drainage experiments.

5. Results and Discussion

This estimated value for the externally held liquid was used to calculate the relative flow rate (v_s^*) and the relative liquid holdup (θ^*) for each superficial velocity in the ore system (similar to equations 3.2 and 3.3):

$$v_s^* = \frac{v_s}{\theta_{ex}(t=40 \text{ minutes})} \quad (5.1)$$

$$\theta^* = \frac{\theta_{ex(t=0)}}{\theta_{ex(t=40 \text{ minutes})}} \quad (5.2)$$

The liquid holdup model formulated for the model system predicts a squared relationship between the two parameters, v_s^* and $\theta^* - 1$. The same model can be tested for the ore system. To investigate this, each v_s^* values for all the ore size fractions are plotted against the corresponding $\theta^* - 1$ values. Figure 10 illustrates this correlation for all the size ranges of ore particles used in this experimental study and it also shows that the data for the model glass bead system for comparison. It shows that the data follows a similar power law relationship for all the particle sizes of ore particles studied.

The theoretical modelling (Ilankoon and Neethling, 2012), assumes that the cross-sectional shape, especially the aspect ratio, of the rivulet remains constant. If some of the flow takes the form of a wetting film rather than a rivulet, then the validity of the assumption of a constant shaped rivulet will be violated. Film flow would result in a cubic relationship between v_s^* and $\theta^* - 1$ (Ilankoon, 2012). While the data in Figure 10 is only slightly steeper than squared, this slight deviation could be due to some of the liquid flow occurring in films rather than rivulets. The extra film flow compared to the glass bead system could be due to differences in contact angle and more angular particles.

Figure 10: The relationship between the relative flow rate (v_s^*) and the additional liquid content of that rivulet ($\theta^* - 1$) for both model and ore systems.

While Figure 10 shows that the ore and model systems have a similar form for the relationship between relative liquid velocity and holdup, it also shows that the pre-factor in the relationship for the model system is approximately a factor of five greater than that for the ore system. There are two reasons for this. Firstly, the tortuosity of the rivulets will be higher in the ore system. This has the effect of increasing the apparent drag coefficient and thus decreasing the pre-factor (K). The other reason is that the contacts in the ore system are more angular than those in the glass beads. This means that they will hold less liquid, which will result in a lower average residual area of the rivulet (A_r) and thus a lower value of K .

A difference in contact angle could also account for some of the difference as it will influence both the shape of the flowing rivulet (and thus the drag coefficient) and the size of the residual connections.

5.1. Pre-factor (K)

In Figure 11, the pre-factor in the relationship between the relative flow rate and the relative holdup is plotted against the particle size for both the glass beads and the ore particles. The geometric mean size (square root of the product of the smaller and larger particle sizes) of each size fraction of ore particles was used to represent the particle size.

Figure 11: Comparison of the pre-factor (K) in the model and the ore systems.

It can be seen that K is quite a strong function of the particle size in the case of the spherical glass beads, more than doubling over the particle size range measured. In the ore system,

though, there is far less variation in the pre-factor, with an average value of 0.0063 ± 0.0008 m/s, with very little systematic change in this value with particle size.

The reason for this is probably due to the angularity of the ore particles. In angular particles the local shape of the particles as seen by a rivulet flowing over it and, particularly, the local shape of the particle in the vicinity of particle to particle contacts is not a strong function of the particle size. For instance, the rivulet and connection shape will be influenced by the angles between facets and the number of facets coming together at points, neither of which necessarily depends on the macroscopic size and shape of the ore particles. On the other hand, the local shape of the glass particles experienced by the rivulets in the model system is directly related to the size of the particles.

That the pre-factor is virtually independent of particle size for the ore particles does not mean that the total liquid holdup is independent of particle size. This only says that the relationship between the relative flow rate and the relative holdup is virtually independent of the particle size in the angular particles in the ore system.

5.2. Residual liquid holdup

Figure 12 shows the residual liquid holdups in the ore system with a superficial velocity of 0.12 mm/s as a function of geometric mean size of ore particles.

Figure 12: Variations of the residual liquid holdup as a function of geometric mean size for the different size fractions of ore particles in the 300 mm packed bed.

The residual liquid content is quite a strong function of the particle size, as can be seen from Figure 12, which means that the actual (rather than relative) liquid content will depend quite strongly on the particle size. The main reason for this is that the number of potential rivulet

paths through the bed increases as the particle size decreases and therefore the number of residual connections within the drained bed also increases.

6. Simulation of liquid drainage of the ore system

Since we have a relationship for the liquid flux as a function of the liquid content, it can be used to predict the evolution of the liquid content in the column as it drains. This will take the form of a partial differential equation (PDE) as the liquid content in the column will vary both spatially and temporally as it drains.

The liquid content of the ore bed can be expressed as:

$$\frac{\partial \theta_e}{\partial t} = -\nabla F \quad (6.1)$$

where θ_e is external liquid holdup, F is liquid flux within the bed.

If residual liquid holdup (θ_r) is constant with respect to time (t) and distance (Z), equation 6.1 becomes:

$$\frac{\partial(\theta_e/\theta_r)}{\partial t} = -\frac{\partial}{\partial Z} \left(\frac{F}{\theta_r} \right) \quad (6.2)$$

Assuming that the effect of capillarity on the external liquid drainage is small, the quantity on the right hand side in equation 6.2 can be approximated using equation 3.4.

$$\frac{F}{\theta_r} = K \left(\frac{\theta_e}{\theta_r} - 1 \right)^2 \quad (6.3)$$

The assumption that capillarity has little impact in the vertical direction can be made since the inter-particle spaces are quite large. The capillary pressures will be of order surface tension divided by particle spacing, which is of order millimetres and the square root of the liquid content. As the liquid contents are a few percentage, the capillary pressure will thus be of order tens to hundred of Pascals. Since the column is tens of centimetres tall, the pressure gradient associated with capillarity will thus be of order 100-1000 Pa/m. This compares to the effect of gravity, which is of order 10,000 Pa/m (i.e ρg). This implies that gravity will be the dominant factor in the flow, with capillarity as a secondary, though not necessarily totally insignificant factor. This is a separate consideration to that expressed by the Bond number, which dictates whether the shape of the rivulet (not the flow down it) is dominated by gravity or capillarity.

The ratio of the steady state external and the residual liquid holdup (θ/θ_r) is defined as θ^* in this work. Substituting equation 6.3, into equation 6.2 (and noting that gravity is downwards and thus if K is positive the sign changes):

$$\frac{\partial \theta^*}{\partial t} = \frac{\partial}{\partial z} [K(\theta^* - 1)^2] \quad (6.4)$$

Equation 6.4 was solved numerically using a first order upwind method. As the PDE is first order in both space and time, an upwind method is required for numerical stability. Since the liquid flow is always downwards, an upwind scheme involves a backward difference in space. To allow the equations to be solved explicitly, a forward difference in time is used. The discretised equation that describes the evolution of the liquid content in the packed bed can thus be written as follows.

$$\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} = K \left[\frac{(\theta_i^j - 1)^2 - (\theta_{i-1}^j - 1)^2}{\Delta z} \right] \quad (6.5)$$

where i is index of the space, j is the index of the time.

As an initial condition, it was assumed that the liquid content was constant at the average liquid content of the system:

$$\theta^*(0, Z) = \theta_e / \theta_r \quad (6.6)$$

As this simulation covers the portion of the experiment when the liquid flow is turned off, the top boundary condition is:

$$Q_L = 0 \quad (6.7)$$

where Q_L is liquid flow into the column. There was no boundary condition at the bottom of the bed as the order of the PDE is one.

In all the simulation presented below the flux due to flow through the particles, v_{end} , was added onto that obtained from the above equations, which are only for the flow around the particles.

The following figures (Figure 13 and 14) compare the agreement between the experimental and simulated data for two superficial velocities. Since the steady state and residual liquid holdups are known experimentally, the value of K can be obtained using these values together with the known liquid flux through the system. This means that these predictions of the drainage rates out of the bed are made with NO adjustable fitting parameters.

Figure 13: Simulated and experimental drainage curves for a superficial velocity of 0.12 mm/s with 8-11.2 mm ore bed (solid line is simulated, diamonds are the experimental results).

Figure 14: Simulated and experimental drainage curves for a superficial velocity of 0.03 mm/s with 8-11.2 mm ore bed (solid line is simulated, diamonds are the experimental results).

Figure 13 shows the comparison of two sets of data for a 300 mm column packed with 8-11.2 mm ore particles for a superficial velocity of 0.12 mm/s. Figure 14 shows the same for a superficial velocity of 0.03 mm/s. A good fit to the experimental data can be observed, with very high R^2 values obtained despite the lack of adjustable fitting parameters. It implies that the estimated residual liquid holdup values using equation 4.1 are reliable and the methodology which was developed in this work to estimate the residual liquid holdup between the ore particles is reasonably accurate.

7. Conclusions

The same model form described the relationship between the liquid flux and liquid holdup in the ore system as in the model glass bead system, but only when the effect of the liquid content held within the ore particles was separated from that held around the particles. Unlike the model system, the main model parameter, K , is not a strong function of the particle size. However, the steady and residual liquid holdups are strong functions of the particle size of the ore system.

This paper has demonstrated that a consistent, widely applicable model form can be obtained for the relationship between liquid flux and liquid holdup, but only if the effect of the externally held liquid is separated from that held internally. This is especially important when modelling transients in the liquid holdup, such as during the initial wetting of the heap and the final drain down of the heap, both of which are aspects that are often problematic for existing heap flow simulations. Distinguishing between these two contributions to the fluid flow could also ultimately result in improved mass transport models.

The porphyry copper ore used in this system has comparatively low porosity. In a system with higher porosity, such as oxide gold ores or where the ore has been agglomerated, the effects discussed in this paper are likely to be even more pronounced and thus the need to separate the contribution of the internally and externally held liquid will be even greater.

Narrow size intervals of the ore particles were used in order to understand the dependencies and mechanisms involved in a comparatively simple system. Industrial heap leaching systems have much wider particle size distributions than those studied in this work, with the shape of this distribution introducing an additional dependency for the parameters in the fluid flow model. While understanding the effect of the particle size distribution is important for predicting the performance of real heaps and is the subject of ongoing work, the underlying mechanisms are likely to be the same, with the need to separate the effect of internally and externally held liquid remaining important.

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