Modeling and Stability Analysis of MTDC Grids for Offshore Wind Farms: A Case Study on the North Sea Benchmark System

Nilanjan Ray Chaudhuri, *Member, IEEE*, Rajat Majumder, *Member, IEEE*,

Balarko Chaudhuri, *Member, IEEE*, Jiuping Pan, *Senior Member, IEEE*, Reynaldo Nuqui, *Senior Member, IEEE*

Abstract—Modeling of VSC-based multi-terminal DC (MTDC) grids for modal analysis and stability studies is reported considering bipolar converters and DC cable network with metallic return path. The proposed model is flexible enough to accommodate different converter grounding options and unbalance on the DC side due to cable and/or converter outage. A simplified version of the benchmark test system for the envisioned offshore MTDC grid in the North Sea is studied. Modal participation factor analysis under nominal and converter outage conditions is used to ascertain the nature and root cause of the dynamic responses. Comparison of the time-domain simulation results using the proposed averaged model in Matlab/SIMULINK against a detailed switched model in EMTDC/PSCAD confirm the accuracy of the modeling approach.

Index Terms—Multi-terminal DC (MTDC), Wind, Voltage source converter (VSC), Stability, Modal analysis, Participation factor

I. INTRODUCTION

AS large offshore wind farms move deeper into the sea the case for a multi-terminal DC (MTDC) grid rather than individual point-to-point connections to shore is getting stronger to effectively share the wind resources among the European countries. An MTDC grid has been envisioned around the North sea to tap the rich wind resource of the region and also interconnect the UK and Nordic pool with continental Europe in future [1], [2].

VSC-based MTDC has received relatively less attention until recent past when modeling [3], control [4], [5], [6] and protection [7] issues were studied. Major challenges in operation of an MTDC grid includes appropriate primary control of converter stations to ensure autonomous sharing of power imbalance following a converter and/or cable outage. Interaction of primary controls with the neighboring AC networks including possible provision for AC system support like frequency control [8] etc. are critical from the point of view of operators. To study these issues a proper modeling and analysis framework for MTDC grids is essential which is presented in this paper through a case study on the simplified version of the benchmark test system for the envisioned offshore MTDC grid in the North Sea.

Support from EPSRC, UK under grant EESC P11121 is acknowledged. N.R. Chaudhuri and B. Chaudhuri are with Imperial College London, London, UK (e-mail: n.chaudhuri@imperial.ac.uk, b.chaudhuri@imperial.ac.uk). R. Majumder, J. Pan, R. Nuqui are with the ABB Corporate Research, Raleigh, NC, USA (e-mail: rajat.majumder@us.abb.com, jiuping.pan@us.abb.com, reynaldo.nuqui@us.abb.com).

A general asymmetric bipole¹ MTDC grid with the provision of metallic return network is considered here. The modeling framework is generic enough to accommodate a detailed pi section approximation of DC side cables and simulate different types of DC cable/converter faults followed by their outages. The proposed averaged model in Matlab/SIMULINK is validated against a detailed switched model in EMTDC/PSCAD software enabling easy integration of the MTDC grid with the multi-machine AC system models for stability studies.

II. MODELING

A. Converter Modeling

The converters were represented by their averaged model [9] in a synchronously rotating reference frame $d'-q'$ where the d' -axis is locked with the voltage E_{ac} at the point of common coupling (PCC) on the AC side of the converters (Fig. 1(a)) to ensure decoupled control of the active and reactive power. All notations in the modified reference frame are henceforth denoted with a prime. The AC system connected to the MTDC

Fig. 1. (a) modified reference frame for decoupled control (b) converter $P-Q$ control (c) converter $V_{DC}-Q$ control

grid is modeled in $d-q$ reference frame shown in Fig. 1(a) which needs to be transformed to $d'-q'$ frame and back while interfacing the MTDC grid variables with those of the AC system. From Fig. 2 the dynamics of the converter transformer

¹unlike symmetric bipole, an asymmetric bipole has the metallic return circuit to carry the current due to unbalance

current injection vector, shown in Fig. 2 and CCI_P is the incidence matrix of dimension $[N_{nodeP} \times N_{brP}]$ depending on the connectivity of different branches. The entries of CCI_P are $CCI_P(i, j) = 1$ (-1), if the i^{th} cable enters (leaves) the j^{th} node and $CCI_P(i, j) = 0$, otherwise.

Similarly, the connectivity between the nodes is determined by the matrix CCU_P which relates the node voltages with the voltage drop in the series $R-L$ branches by the equation

$$\bar{U}_{qr-p} = CCU_P \times \bar{U}_{q-p} \quad (5)$$

where $\bar{U}_{q-p}$ is the node voltage vector and $\bar{U}_{qr-p}$ is the vector of series branch voltage drops. Note that $CCI_P = -CCU_P^T$. Similar set of equations are applicable for the negative pole network too.

For the metallic return network (shown in Fig. 2), (3) can be modified as:

$$-I_{D-P} - I_{D-N} + I_{line-m} + I_{s-p} + I_{s-n} - I_{c-m} - \frac{U_{d-m}}{R_g} - I_f = 0 \quad (6)$$

All other above-mentioned equations are applicable for the metallic return network.

2) *Differential Equations:* For the positive pole network the converter station capacitor dynamics is given by:

$$U_{d-p} = \frac{1}{C_{s-p}} \int I_{s-p} dt + I_{s-p} R_{s-p} \quad (7)$$

where, R_{s-p} is the effective series resistance ESR (not shown explicitly in Fig. 2). For any cable node $q-p$ we can write:

$$\frac{1}{n_c C_{c-p}} \int I_{c-q-p} dt = U_{q-p} \quad (8)$$

Here, n_c depends on the node location, e.g. at any cable node other than the converter node, $n_c = 2$. The series $R-L$ branch dynamics can be described by:

$$L_p \frac{dI_{p-qr}}{dt} + R_p I_{p-qr} = U_{q-p} - U_{r-p} = U_{qr-p} \quad (9)$$

The negative pole and metallic return network dynamics can also be represented by similar equations.

C. Combined AC-MTDC Grid Modeling

The analytical model of the combined AC-MTDC grid is shown in Fig. 3. In an AC system the generators (both synchronous and asynchronous), excitation systems, loads and other dynamic components can be modeled by their respective differential and algebraic equations (DAEs) whereas the rest can be represented by the bus admittance matrix Y_{bus} . A set of algebraic equations satisfying the current balance of the injected current I and the current injection from the MTDC grid I_{MTDC} can be solved to find the AC network bus voltages. In case of an ideal voltage source with internal impedance on the AC side, the current balance equations are based on the KVL equations involving the PCC voltage and the internal source voltage.

The PCC bus voltages of the MTDC grid E_{ac} were locked with the d -axis of the synchronously rotating $d'-q'$ reference frame (Fig. 1 (a)). The AC currents injected by the positive and negative pole converters $I'_{acp/n}$ were calculated using the

Fig. 3. Block diagram of combined AC-MTDC grid analytical model.

converter and controller model shown in Fig. 1 (b) and (c). The DC side power $P_{dcp/n}$ of the converters was derived from the knowledge of PCC real power and losses - which was further used as input to the DC cable model to obtain the DC link voltages. The converter AC side current was transformed into the AC system $d-q$ reference frame before being used to solve the current balance equations.

D. Cable/Converter Outage Simulation

Fault and cable outage were simulated by lowering the resistance at the fault point followed by increase in terminal breaker resistances. Converter blocking and uncontrolled rectification through anti-parallel diodes during DC cable faults were included [11]. The faulted cable was isolated by inserting a high breaker resistance of $10^6 \Omega$ [12] at either end.

Converter outage was simulated by opening both the AC and the DC side breaker of the respective converter pole. The AC side breaker opening was simulated by increasing R_c in (1) and (2) to $10^6 \Omega$ [12] while the DC side breaker opening is done by reducing $P_{dcp/n}$ to zero.

The proposed model is generic and modular enough to handle any cable or converter outage leading to unbalanced (on DC side) operation under different grounding options.

III. NORTH SEA GRID BENCHMARK SYSTEM

A simplified version of the envisioned offshore MTDC grid in the North Sea, shown in Fig. 4 [2], is considered here

Fig. 4. Simplified benchmark test system for the envisioned offshore MTDC grid in the North Sea - corresponding to the year 2011 [2]

Fig. 5. Clustering of system poles based on participation factors in the nominal condition.

for the case study. The study network comprises of four bipole converter stations which are connected to three trans-national sub-sea DC cables as shown (in red) in Fig. 4. The system data is taken from [2] and is not repeated here except for the converter parameters shown in Table I. As shown in Fig. 4, the converter stations Conv-DK and Conv-NL are associated with Denmark and Netherlands, respectively, while Conv-DE28 and Conv-DE-A are part of Germany. Conv-DK is considered to be operating as an inverter under the $U_{dc} - Q$ control mode transporting about 800 MW under the nominal condition. Conv-DE28 and Conv-DE-A carry 400 MW each, operating as an inverter and rectifier, respectively. Conv-NL is in rectifying mode carrying 800 MW power.

The metallic return network of Conv-DK is grounded with 0.5Ω resistance while the rest are left ungrounded. Conv-DK keeps the DC link voltage constant at ± 150 kV while

the remaining stations operate in $P - Q$ control mode to maintain unity power factor at their respective PCCs. Real power injected into the AC system is considered to be positive and vice versa for the rest of the paper.

Note that the AC network representation in this model is not complete since the actual European grid is interconnected. However, in absence of such data we consider this model as a starting point for analysis. The averaged model of the above study network was built in Matlab/SIMULINK and validated against a detailed switched model in EMTDC/PSCAD.

IV. MODAL ANALYSIS

A. Participation Factor Analysis

The MTDC grid modeled by the differential-algebraic equations (DAEs) in (1)-(9) were linearized around a nominal condition and expressed in state-space form as:

$$\Delta \dot{x} = A \Delta x \quad (10)$$

where, Δx and A are the state-vector and state-matrix, respectively. The order of the state matrix for the study network considered was 111. The right (ϕ_i) and left (ψ_i) eigen vectors corresponding to the eigenvalues λ_i , $i = 1, 2, \dots, n$ satisfy:

$$A \phi_i = \lambda_i \phi_i, \quad \psi_i A = \lambda_i \psi_i \quad (11)$$

The k^{th} element of the right eigenvector ϕ_i measures the activity of the state variable x_k in the i^{th} mode while that of the left eigenvector ψ_i weighs the contribution of this activity to the i^{th} mode [13].

To get around the scaling problem often the above measures are combined to form the participation matrix, P whose elements $p_{ki} = \phi_{ki} \psi_{ik}$ are termed as the participation factor. It is a measure of the relative participation of the k^{th} state variable in the i^{th} mode and vice versa [13].

Fig. 6. Clustering of system poles based on participation factors with the outage of the positive pole converter in Conv-DE-A.

Fig. 7. Validation of the averaged model in Matlab/SIMULINK against detailed switched model in EMTDC/PSCAD. A pulse change to the real power reference of both the converters in Conv-DE28 is applied.

B. Nominal Condition

Modal participation factor analysis was carried out to ascertain the nature and the root cause of the dynamic response of the MTDC grid. It can be seen from the pole map of Fig. 5 (upper subplot) that the fastest modes are associated with the AC network (modeled by transformer and phase reactor dynamics) states and the participation of the DC network and the converter controller states increase as we move towards right on the s-plane. The lower subplot shows a zoomed view of the critical poles. Most of these are associated with the DC network except one pole pair which has high participation from the current controllers of the converters.

C. Converter Outage

Linear analysis was also done considering the outage of the positive pole converter in Conv-DE-A. It can be seen from Fig. 6 that the eigen-value plot shows minimal change from the nominal condition except that a new pole pair $-12.76 \pm j71.45$ having high participation from the converter current controller states.

V. NON-LINEAR SIMULATION RESULTS

As described in section II the MTDC grid and the converter modeling was done in the phasor form in a synchronously rotating d-q reference frame. Moreover the converters were represented by their averaged model. To assess the accuracy of the modeling approach a detailed switched model of the converters and the rest of the system was built in the EMTDC/PSCAD platform. Validation of the non-linear simulation results obtained from the phasor model in MATLAB/SIMULINK with the EMTDC/PSCAD model is considered to be essential and is shown below.

Fig. 8. Subplots (a), (b), (c): Power spectral density (P_{XX}) calculated from Welch Periodogram for determining the dominant modes in the DC link voltages of Conv-DK, Conv-DE28 and Conv-DE-A obtained from EMTDC/PSCAD as shown in Fig. 7. Subplot (d): modes obtained from eigen-value analysis of the averaged model.

A. Small Disturbances

Fig. 7 shows the dynamic response of the system following a pulse applied to P_{ref} of both the converters in Conv-DE28 at $t = 0.1$ s. Only the positive pole variables are shown here due to symmetry. Note that the DC link voltage of Conv-DK is tightly held at ± 150 kV (check the tight scale of Fig. 7(a)) - hence visually appearing to have larger ripple content than the others. Of course the power-flow patterns under the studied operating condition determined the steady state dc link voltages of the

Fig. 9. Comparison of the DC link voltage dynamics of the averaged model in Matlab/SIMULINK against detailed switched model in EMTDC/PSCAD following the outage of the positive pole converter in Conv-DE-A.

Fig. 10. Comparison of the dynamic behaviour of the averaged model in Matlab/SIMULINK against detailed switched model in EMTDC/PSCAD following the outage of the positive pole converter in Conv-DE-A.

converters. For example, around 800 MW flows from Conv-NL to Conv-DK via Conv-DE-A resulting in the highest dc link voltage (± 154 kV) at Conv-NL followed by Conv-DE-A (± 152 kV).

Overall, the responses exhibit very good match between the detailed switched model in EMTDC/PSCAD and the averaged model in Matlab/SIMULINK. The former was run with a fixed step size of $5.0 \mu\text{s}$ while the latter was run with a variable step solver. The dynamic variations observed in the EMTDC/PSCAD are clearly captured in the Matlab/SIMULINK model except for the high frequency switching ripples.

Power spectral density (P_{XX}) of the waveforms obtained from the EMTDC/PSCAD model was calculated using the Welch Periodogram technique to ascertain the principal modal content of the waveforms. The results for the DC link volt-

ages of Conv-DK, Conv-DE28 and Conv-DE-A are shown in Figs 8(a), (b) and (c) respectively. As expected, the Conv-DK DC link voltage has prominent side-bands around the switching frequency of 1980 Hz. Presence of lower frequency modes of 19.53 Hz, 29.3 Hz and 58.6 Hz are also apparent from Figs 8(a), (b) and (c).

As shown in Fig. 8(d), the eigenvalue plot obtained from the linear model highlights the presence of three modes with frequencies 18.7 Hz, 31.96 Hz and 59.36 Hz respectively which is in close agreement with the spectrum in Figs 8(a), (b) and (c). Side-bands due to the switching ripples were not captured in the eigenvalues due to the state-space averaging. Participation factor analysis shows that the aforementioned modes have high participation from the states in the DC side of the converter (see Section IV).

B. Converter Outage

As described in Section V-B the converter outage was simulated by opening both the AC and the DC side breakers. Figs 9 and 10 show the dynamic behavior of the MTDC grid following the outage of the positive pole converter of Conv-DE-A. As before the correlation between the frequency spectrum of the time-domain response and the modal analysis holds in this case but is not repeated here.

Although the positive pole converter in Conv-DE-A is absent, the switchyard in the DC side remains connected to the DC cable network. As shown in Fig. 9(b), the switching ripple in this switchyard voltage is noticeably reduced after the outage at $t=0.1$ s - compare with Fig. 9(c) showing the DC link voltage of the negative pole at the same converter station.

Figs 10(a), (d) and (b), (e) show the real power at the PCC (P_{pcc}) of the converter stations from the averaged model and the detailed switched model. Note that P_{pcc} at the positive pole of Conv-DE-A reduces to zero at a slower rate in the EMTDC/PSCAD simulation due to the operation of the AC side breaker at respective current zero of each phase - which is not the case for the phasor model. Since the slack converter station (Conv-DK) adjusts for the change in power, the dip in the corresponding P_{pcc} is higher in the MATLAB/SIMULINK model. In absence of one converter pole, an unbalance in the DC side occurs and the metallic return network connecting Conv-DK and Conv-DE-A carries current as high as 1.5 kA, see Figs 10(c) and (f).

As only Conv-DK is in $U_{dc} - Q$ control mode, it shares the full burden of unbalance in current. Autonomous power sharing can ensure division of this burden amongst all the converters based on their ratings. This is a topic of our ongoing research.

Overall, it can be seen that there has been a close match between the dynamic responses observed in the EMTDC/PSCAD model and the MATLAB/SIMULINK model. This confirms the validity of the proposed averaged model for modal analysis and stability studies.

VI. CONCLUSION

Modeling of VSC-based multi-terminal DC (MTDC) grids for modal analysis and stability studies has been done considering bipolar converters and cable network with metallic return path. The proposed model is flexible enough to accommodate different converter grounding philosophies and unbalance on the DC side due to cable and/or converter outage. Case study on a simplified version of the envisioned offshore MTDC grid in the North Sea was used for demonstrating the modeling and analysis. Modal participation factor analysis under nominal and converter outage conditions confirmed the nature and root cause of the dynamic responses. Comparison of the time-domain simulation results using the proposed averaged model in Matlab/SIMULINK against a detailed switched model in EMTDC/PSCAD confirm the accuracy of the modeling approach. This platform is currently being used for our ongoing research into interaction between the AC and MTDC systems with particular focus on primary controls for autonomous power sharing.

REFERENCES

- [1] J. Blau, "Europe plans a north sea grid," *IEEE Spectrum*, vol. 47, no. 3, pp. 12–13, 2010.
- [2] K. Rudion, A. Orths, P. B. Eriksen, and Z. A. Styczynski, "Toward a benchmark test system for the offshore grid in the north sea," in *IEEE Power and Energy Society General Meeting, 2010*, 2010, pp. 1–8.
- [3] A. A. van der Meer, R. Hendriks, and W. Kling, "Combined stability and electro-magnetic transients simulation of offshore wind power connected through multi-terminal VSC-HVDC," in *IEEE Power and Energy Society General Meeting, 2010*.
- [4] C. Hairong, W. Chao, Z. Fan, and P. Wulue, "Control strategy research of VSC based multiterminal HVDC system," in *IEEE PES Power Systems Conference and Exposition, 2006. PSCE '06*. 2006.
- [5] X. Lie, Y. Liangzhong, and M. Bazargan, "Dc grid management of a multi-terminal hvdc transmission system for large offshore wind farms," in *International Conference on Sustainable Power Generation and Supply, 2009. SUPERGEN '09*, 2009, pp. 1–7.
- [6] L. Fei, M. Zhiwen, Y. Xiaojie, and Z. Trillion, "The grid connected converter control of multi-terminal DC system for wind farms," in *Proceedings of the Eighth International Conference on Electrical Machines and Systems, 2005. ICEMS 2005*, vol. 2, 2005, pp. 1021–1023 Vol. 2.
- [7] L. Tang and B.-T. Ooi, "Protection of VSC-multi-terminal HVDC against DC faults," in *IEEE Annual Power Electronics Specialists Conference*, vol. 2, pp. 719–724, 2002.
- [8] T. M. Haileselassie and K. Uhlen, "Primary frequency control of remote grids connected by multi-terminal HVDC," in *IEEE Power and Energy Society General Meeting, 2010*.
- [9] S. R. Sanders, J. M. Noworolski, X. Z. Liu, and G. C. Verghese, "Generalized averaging method for power conversion circuits," *IEEE Transactions on Power Electronics*, vol. 6, no. 2, pp. 251–259, 1991.
- [10] C. Schauder and H. Mehta, "Vector analysis and control of advanced static var compensators," *IEE Proceedings on Generation, Transmission and Distribution*, vol. 140, no. 4, pp. 299–306, 1993.
- [11] Y. Jin, J. E. Fletcher, and J. O'Reilly, "Multiterminal dc wind farm collection grid internal fault analysis and protection design," *IEEE Transactions on Power Delivery*, vol. 25, no. 4, pp. 2308–2318, 2010.
- [12] PSCAD/EMTDC v. 4.2.1, Manitoba HVDC Research Centre, Winnipeg, MB, Canada.
- [13] P. Kundur, *Power system stability and control*, ser. The EPRI power system engineering series. New York; London: McGraw-Hill, 1994.