Optimising reinforcement design in D regions using non-linear finite-element analysis

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This paper presents an automated procedure for designing reinforcement in D regions where plane sections do not remain plane after loading. The reinforcement is designed to resist stresses calculated in a non-linear finite-element analysis. The design procedure is implemented into NonOpt, which is a Fortran program that works in conjunction with the finite-element program DIANA. NonOpt gives three alternative methods for optimising the reinforcement design, of which strategy C is the subject of this paper. It finds the minimum weighted sum of the reinforcement ratios in two-dimensional structures such as deep beams subject to design constraints and practical requirements.

Notation

$A_{c,ef}$	effective concrete area
a	aggregate size
c	as defined in Figure 2
F_S	safety factor
f_l	mean stress in longitudinal reinforcement
f_t	mean stress in transverse reinforcement
f_{yd}	design yield strength of the reinforcement
M	sum of the number of HB and VB bands
n_d	number of mesh divisions
n_r	number of mesh refinements
$n_{r,cr}$	current level of mesh refinement
s_l, s_t	crack spacings normal to the longitudinal axis of the (s_l) and transverse reinforcement (s_t)
s_{ma}	average crack spacing
v_{ci}	crack shear stress
w	crack width
α	angle of inclination of principal compressive stress to longitudinal axis
β	softening coefficient
γ_{lt}	average shear strain in cracked concrete
$\delta \rho_{int}^{n_{r,cr}}$	incremental step in the reinforcement mesh
ϵ_d	principal compressive strain in concrete
ϵ_l	normal strain in longitudinal direction
ϵ_r	principal tensile strain in concrete
ϵ_t	normal strain in transverse direction
$ \epsilon_{d,per} $	maximum permissible compressive strain
ρ	reinforcement ratio
ρ'	reinforcement ratio within the effective concrete area $A_{c,ef}$
ρ_l, ρ_t	longitudinal and transverse reinforcement ratios
ρ_{min}, ρ_{max}	minimum and maximum permissible reinforcement ratios
σ_d	principal compressive stress in concrete

σ_r	principal tensile stress in concrete
τ_{lt}	shear stress in reinforced concrete element

Introduction

All structures can be subdivided into B or Bernoulli regions in which plane sections remain plane after loading and so-called D or disturbed regions in which the strain distribution is significantly non-linear (Schlaich *et al.*, 1987). D regions occur adjacent to supports, concentrated loads, corners, bends and openings with examples including deep beams, squat shear walls, halving joints, corbels and beam–column connections. The design of B regions is much better covered in codes of practice than that of D regions, which is clearly just as important for structural safety. D regions are typically designed using strut-and-tie models (STMs), linear elastic finite-element analysis or judgement based on past experience.

Strut-and-tie modelling is a generalisation of the truss analogy for shear in which a structure is modelled as a truss with compression typically being resisted by concrete and tension by reinforcement. Design standards such as ACI 318 (ACI, 2011) and EC2 (BSI, 2004) provide guidance on the development of STM and the calculation of member strength. Despite this, the development of STM is complex because there are many possible solutions, of which only some are acceptable for reasons of ductility and serviceability (Kuchma *et al.*, 2010). Difficulties also arise in the determination of the dimensions, axial resistance and stiffness of concrete struts (Tjhin and Kuchma, 2002; Vollum and Parker, 2008). Consideration of these issues has led many researchers (e.g. Sagaseta and Vollum, 2010) to develop increasingly complex STMs that are typically only suitable for the problem in hand. The present research considers non-linear finite-element analysis (NLFEA) as an alternative to STMs for the design of D regions.

Linear elastic finite-element analysis (FEA) is used routinely for the design of structures like deep beams and shear walls at the ultimate limit state. The results of linear FEA can be used to develop improved STM (Schlaich *et al.*, 1987). Alternatively, the reinforcement can be designed to enable individual elements or groups of elements to resist the stresses calculated in the FEA. The main limitation of the latter approach is that it neglects the internal redistribution of stress that occurs owing to material non-linearity. Relatively little attention has been given to the use of NLFEA for the automatic design of reinforcement in D regions. Tabatabai and Mosalam (2001) developed a computational platform for optimising the reinforcement design in D regions in which the design begins with a linear FEA. A linear programming tableau is established to minimise a linear objective function subject to a set of prescribed linear constraints. The minimisation is performed by superimposing a series of self-equilibrating stress states onto the linear elastic solution. The program gives an estimate of the minimum reinforcement weight required for a user-defined reinforcement field. The resulting reinforcement designs are assessed at the serviceability (SLS) and ultimate (ULS) limit states using NLFEA. The assessment involves manually checking reinforcement stresses, crack widths and deflections at the SLS, as well as practical requirements. The reinforcement field is amended manually and the minimisation process is repeated until the design constraints are satisfied. User-specified design constraints can only be satisfied in an ad hoc fashion by trial and error owing to the lack of compatibility between the material relationships assumed in the reinforcement design and the subsequent NLFEA. The procedure is only semi-automated and requires considerable user intervention.

More recently, Fernández Ruiz and Muttoni (2007) presented a semi-automated approach for developing stress fields based on NLFEA. They modelled concrete as elastic-perfectly plastic in compression with zero tensile resistance. Reinforcement is modelled as linearly elastic in the initial NLFEA and as bilinear in the assessment of the design with NLFEA. The first step is to develop a finite-element (FE) model in which the reinforcement arrangement is either based on user experience or consists of an orthogonal grid with the minimum area of reinforcement required for crack control. The final reinforcement areas are calculated as $A_s = A_{s \min} \sigma_s / f_{yd} \geq A_{s \min}$, where σ_s is the reinforcement stress from the initial FEA and f_{yd} is the design yield strength of the reinforcement. The reinforcement is then updated in the FE model and a new FEA is carried out with bilinear reinforcement to assess the structural performance. The approach is clearly a significant improvement on current design practice. A limitation is that the reinforcement design is based on a stress field calculated with linearly elastic reinforcement neglecting tension stiffening. The current work addresses both these issues.

Proposed design procedure

This paper proposes an alternative procedure for the design of D regions in which the reinforcement is designed for stresses calculated in a non-linear finite-element analysis. The procedure

is iterative owing to the dependence of the stresses on the reinforcement arrangement. The procedure uses the same constitutive relationships in the reinforcement design, as well as the NLFEA. This is advantageous as it allows explicit performance-based design constraints, such as crack widths, to be specified at the design stage, unlike the methods of Tabatabai and Mosalam (2001), and Fernández Ruiz and Muttoni (2007). The current formulation of the design procedure incorporates the equations of the modified compression field theory (MCFT) (Collins *et al.*, 2008), but any similar constitutive model, such as that of Hsu (1993), could be used. The equations of the MCFT are summarised in Table 1. Equations 1 to 3 and 6 to 8 in Table 1 are presented in the forms used by Hsu (1993), which are equivalent to the corresponding equations of the MCFT. Equations 4 to 5 and 15 in Table 1 are unique to the MCFT. This paper describes the general framework of the optimisation strategy. The objective function is defined as the weighted sum of all the individual reinforcement ratios inside the structure. It is minimised subject to the design constraints, which are expressed as safety factors.

The design starts with a linear FEA of the structure, which is reinforced initially with minimal reinforcement. The reinforcement is designed initially to resist the stresses from the linear FEA. Practical reinforcement arrangements are obtained by grouping the finite elements into horizontal and vertical bands. Subsequently, a NLFEA is performed with the updated reinforcement and the design constraints are rechecked. The design terminates if the convergence criterion is satisfied. Otherwise the reinforcement is redesigned using the stresses from the last NLFEA until the convergence criterion is satisfied.

The design procedure is implemented in NonOpt, which is a Fortran program, developed by the authors, that works in conjunction with the finite-element program DIANA (TNO DIANA, 2007). This paper describes the procedures used to optimise the reinforcement design in NonOpt, as well as the overall architecture of the program (see Figure 1). Further details can be found in Amini Najafian (2011). NonOpt performs the entire design procedure, including the execution of DIANA, without user intervention. Presently, NonOpt gives the user three alternative strategies for optimising the reinforcement design, of which strategies A and C are described in this paper. All three strategies optimise the reinforcement design subject to predefined design constraints, as well as practical considerations. As way of introduction, strategy A reinforces groups of linked elements with the greatest of the individual areas of reinforcement required in the linked elements. Strategy C has two subsets, which are denoted strategies C_A and C_B . Strategy C_A minimises the sum of the individual reinforcement ratios that define the reinforcement within the structure, whereas strategy C_B minimises the overall reinforcement weight. Strategy B (Amini Najafian, 2011), which is not discussed further, is designed for computational efficiency. It is similar in concept to strategy C, but less efficient at optimising the overall reinforcement design. Strategy A is most efficient for practical design, whereas strategy C is designed to

Equilibrium

Strain compatibility

Average stresses

1. $\sigma_l = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + \rho_l f_l$
2. $\sigma_t = \sigma_d \sin^2 \alpha + \sigma_r \cos^2 \alpha + \rho_t f_t$
3. $\tau_{lt} = (-\sigma_d + \sigma_r) \sin \alpha \cos \alpha$

Average strains

6. $\epsilon_l = \epsilon_d \cos^2 \alpha + \epsilon_r \sin^2 \alpha$
7. $\epsilon_t = \epsilon_d \sin^2 \alpha + \epsilon_r \cos^2 \alpha$
8. $\gamma_{lt}/2 = (-\epsilon_d + \epsilon_r) \sin \alpha \cos \alpha$

Stresses at crack (see Figure 3(c))

4. $f_{slcr} = (\sigma_l + \tau_{lt} \cot \alpha - v_{ci} \cot \alpha) / \rho_l$
5. $f_{stcr} = (\sigma_t + \tau_{lt} \tan \alpha + v_{ci} \tan \alpha) / \rho_t$

Crack widths

9. $w = \epsilon_r s_{m\alpha}$
10. $s_{m\alpha} = \frac{1}{\frac{|\sin \alpha|}{s_l} + \frac{|\cos \alpha|}{s_t}}$

Material

Reinforcement

11. $f_l = f_l(\epsilon_l)$
12. $f_t = f_t(\epsilon_t)$

Concrete

13. $\sigma_d = \sigma_d(\epsilon_d, \beta)$
14. $\sigma_r = \sigma_r(\epsilon_r)$

Shear stress on crack

15. $v_{ci,max} = \frac{0.18 \sqrt{f'_c}}{0.31 + 24w/(a + 16)}$

Table 1. Equations of the modified compression field theory
(Collins *et al.*, 2008)

Figure 1. Architecture and flowchart of NonOpt

investigate the potential benefits of minimising the reinforcement that are examined later in this paper.

Design constraints

The design constraints are specified in terms of maximum permissible reinforcement stresses and maximum permissible mean strains in the reinforcement, as well as concrete. For example, concrete crushing is prevented by limiting its maximum compressive strain to $|\varepsilon_{d,per}|$. Safety factors are defined for each design constraint as the ratio of its permissible to actual value. The reinforcement ratios are also constrained to lie between ρ_{min} and ρ_{max} .

Crack widths can be restricted to w_{max} by limiting the principal

tensile strain ε_r to $\varepsilon_{r,per} = (w_{max}/s_{m\alpha})$ where $s_{m\alpha}$ is the average crack spacing. The crack spacing $s_{m\alpha}$ is defined in Equation 10 in Table 1 in terms of the crack spacings normal to the longitudinal (s_l) and transverse reinforcement (s_t). The normal crack spacings are defined in accordance with the recommendations of Bentz (2000) as $s = 2c + 0.1(d_b/\rho')$ in which s is s_l or s_t as appropriate, ρ' is the reinforcement ratio within the effective concrete area $A_{c,ef}$ and d_b is the reinforcement bar diameter. The dimension c and the effective concrete area $A_{c,ef}$ are defined in Figure 2.

The key equations of the MCFT (see Table 1) are expressed in terms of the mean stresses and strains in the reinforcement and concrete, f_l , f_t , ε_l , ε_t , σ_d , σ_r , ε_d and ε_r . It follows that it is also necessary to ensure that the reinforcement is not overstressed at

Figure 2. Effective concrete area used in calculation of crack spacing

cracks, as shown in Figures 3a to 3c. Figures 3b and 3c illustrate the requirements for equilibrium in terms of average stresses and the local stresses at a crack. The crack normal stress f_{ci} in Figure 3c is assumed to be zero in the current version of the MCFT (Collins *et al.*, 2008). It is only possible to check whether or not the stresses at cracks, f_{slcr} , f_{stcr} and v_{ci} (see Figure 3(c)), are within their permissible range $f_{slcr,per}$, $f_{stcr,per}$ and $v_{ci,per}$ as Equations 4 and 5 in Table 1 are indeterminate to one degree. For this purpose, the maximum reinforcement stresses are initially calculated with $v_{ci} = 0$. The crack shear stress v_{ci} is only invoked if the reinforcement is overstressed in one direction. In this case, v_{ci} is found from equilibrium assuming that the stress in the critical reinforcement direction equals its maximum permissible value. The element is safe if $|v_{ci}| \leq v_{ci,per}$ and the reinforcement stress is within its permitted range in the other direction. The safety factor for the maximum stresses is defined as $F_S = \min(|f_{slcr,per}/f_{slcr}|, |f_{stcr,per}/f_{stcr}|)$ when $v_{ci} = 0$ or the reinforcement is overstressed in both directions, and as $F_S = v_{ci,per}/|v_{ci}|$ when $v_{ci} \neq 0$ and the least stressed reinforcement is not overstressed.

Figure 3. Average and local stresses: (a) applied stresses, (b) average stresses and (c) local stresses at a crack

Bunching of elements

A practical reinforcement arrangement is obtained by grouping the finite elements into horizontal (HB) and vertical (VB) bands, as illustrated in Figure 4, where the borders of the VB and HB bunches are indicated with dotted and dashed lines, respectively. The reinforcement ratios in the VB and HB completely define the reinforcement throughout the structure. All the elements in a HB have the same horizontal reinforcement ratio ρ_1 . Similarly, all the elements in a VB have the same vertical reinforcement ratio ρ_2 . The elements at the intersection of a HB and VB constitute a VHB bunch as shown in Figure 4 with dashed-dotted lines. The total number of independent reinforcement ratios equals the sum of the number of HB and VB bands, which is denoted M . The total reinforcement weight is proportional to $V_{total} = \sum_{i=1}^M A_i \rho_i$ in which A_i is the surface area of band i and ρ_i is its reinforcement ratio. The reinforcement is designed to resist the stresses calculated in the last FEA subject to the design constraints. Strategy A sets the area of reinforcement in each VB and HB equal to the maximum of the ratios calculated in its linked

Figure 4. Definition of VB, HB and VHB bunches in a FE model

bunches (Amini Najafian, 2011). Strategy C finds the absolute minimum of the sum of the resulting weighted reinforcement ratios which is given by

$$16. \quad X_{\text{total}} = \sum_{i=1}^M \alpha_i \rho_i$$

As discussed previously, strategy C has two subsets, which are denoted C_A and C_B . The weighting function α_i is taken as 1 in strategy C_A , which minimises the sum of the reinforcement ratios and A_i in strategy C_B , which minimises the overall reinforcement weight in the structure.

Optimisation of reinforcement design

The optimisation procedure considers all possible combinations of the M independent reinforcement ratios in the structure. The initial step is to generate a multi-dimensional (MD) mesh in which the axes represent the reinforcement ratios ρ_i . The sum of the M individual reinforcement ratios $\rho_{\text{total}} = \sum_{i=1}^M \rho_i$ varies from $M\rho_{\text{min}}$ to $M\rho_{\text{max}}$ where ρ_{min} and ρ_{max} are the minimum and maximum permissible reinforcement ratios. The minimisation is described initially for a single VHB with reinforcement ratios ρ_l and ρ_t .

Optimisation of reinforcement in a single VHB

In this case, a two-dimensional (2D) mesh is defined with n_d divisions. The objective is to minimise $\rho_{\text{total}} = \rho_l + \rho_t$ subject to the design constraints. The first step is to generate an equally spaced mesh in the ρ_l - ρ_t plane in which ρ_l and ρ_t vary between ρ_{min} and ρ_{max} as shown in Figure 5. The equation $\Lambda_i = \rho_{li} + \rho_{ti} = 2\rho_i$ defines a line in the mesh parallel to the lines $\Lambda_{\text{min}} = \rho_{\text{total}} = 2\rho_{\text{min}}$ and $\Lambda_{\text{max}} = \rho_{\text{total}} = 2\rho_{\text{max}}$. The solution procedure involves moving sequentially through the mesh on parallel lines Λ_i from Λ_{min} to Λ_{max} until a solution is found. The direction of movement along the parallel lines is illustrated with

Figure 5. Mesh generation and refinement (2D)

arrows in Figure 5. The design constraints are checked at every Gauss point in the VHB at successive coordinates on parallel lines Λ_i until a solution is found. This involves first solving the rotating crack equations of the MCFT with the current reinforcement ratios and then checking the design constraints, which are expressed in terms of safety factors, as described previously. If all the design constraints are met, the analysis moves to the next Gauss point in the VHB bunch and continues until all the Gauss points are checked. However, if any design constraint is violated, the current reinforcement combination is declared invalid and the coordinate on the ρ_l - ρ_t plane is updated.

The procedure is repeated until a reinforcement combination, (ρ_l, ρ_t) , is found which satisfies the design constraints at every Gauss point in the VHB bunch. At this point, either the reinforcement mesh is refined as shown in Figure 5 or the remaining points on the current parallel line are checked to determine whether there are any additional solutions. The mesh is refined between the parallel line passing through the solution and the adjacent parallel line with δ_ρ less total reinforcement, where $\delta_\rho = (\rho_{\text{max}} - \rho_{\text{min}})/n_d$ is the incremental step in the mesh. The safety factor of a VHB is taken as the least of the safety factors at any Gauss point inside the bunch. The reinforcement combination with the greatest safety factor is adopted in cases where multiple solutions exist with the same value of ρ_{total} . Strategy A designs the reinforcement in each VHB using the procedure described above. The reinforcement in each VB and HB is subsequently set equal to the maximum of the ratios calculated in its linked bunches (Amini Najafian, 2011).

Optimisation of multiple reinforcement ratios

The solution procedure becomes M -dimensional when M reinforcement ratios are involved in the minimisation, as illustrated in Figure 6 for three reinforcement ratios. In this case, a constant value of $\rho_{\text{total}} = \sum_{i=1}^3 \rho_i$ depicts a plane in the three-dimensional (3D) mesh. Increasing ρ_{total} from $3\rho_{\text{min}}$ to $3\rho_{\text{max}}$ in incremental

Figure 6. Coordinates in a plane with constant value of ρ_{total} in a 3D mesh

steps of δ_ρ covers the permissible domain in a series of parallel planes. The movement starts at the first coordinate in the mesh where $\rho_{\text{total}} = 3\rho_{\text{min}}$ and continues in and between the parallel planes until a solution is found. At this point the mesh is either refined between the parallel plane passing through the current solution and the adjacent parallel plane with δ_ρ less total reinforcement or a search is made for additional solutions in the current plane, as well as beyond if the reinforcement weight is being minimised. The solution with the greatest safety factor is adopted if there are multiple solutions with the same X_{total} .

In the case of a M -dimensional mesh, the axes are the independent reinforcement ratios, i.e. $\rho_1, \rho_2, \dots, \rho_M$. The increments in the reinforcement mesh are defined as $\delta_\rho = (\rho_{\text{max}} - \rho_{\text{min}})/n_d$, where n_d is the number of divisions along each axis. A constant value of $\rho_{\text{total}} = \sum_{i=1}^M \rho_i$ represents a hyperplane with $M - 1$ dimensions. The sum of the reinforcement ratios in each hyperplane is given by $\rho_{\text{total}} = M\rho_{\text{min}} + n\delta_\rho$ where $n = 0, 1, \dots, Mn_d$. The computations start at coordinate 1 where all the reinforcement ratios equal ρ_{min} . Subsequently, all possible combinations of the reinforcement ratios are checked in successive parallel hyperplanes in which $\rho_{\text{total}} = M\rho_{\text{min}} + \delta_\rho, M\rho_{\text{min}} + 2\delta_\rho, \dots, M\rho_{\text{min}} + Mn_d\delta_\rho = M\rho_{\text{max}}$ until a solution is found.

Generation of MD meshes

The initial step in the generation of a MD mesh is to determine the valid domain of its hyperplanes. The domain bounds of a parallel hyperplane depend on its position, as illustrated in Figure 7 for a 2D mesh. Consideration of Figure 7 shows that $\rho_{i,\text{min}} \geq \rho_{\text{min}}$ and $\rho_{i,\text{max}} \leq \rho_{\text{max}}$. Each hyperplane is defined by the equation $\rho_{\text{total}} = M\rho_{\text{min}} + n\delta_\rho$, where $n = 0, 1, \dots, Mn_d$. The first step in determining the valid domain for a hyperplane is to find the value of n beyond which no reinforcement ratio equals ρ_{min} . This is done by setting one of the variables equal to ρ_{min}

Figure 7. Valid domain for parallel line in 2D mesh

and the remaining $M - 1$ variables equal to ρ_{max} . The last parallel hyperplane in which ρ_{min} is possible (i.e. case 3 in Figure 7) is defined by

$$17. \quad \rho_{\text{min}} + (M - 1)\rho_{\text{max}} = M\rho_{\text{min}} + n\delta_\rho$$

Replacing ρ_{max} with $\rho_{\text{min}} + n_d\delta_\rho$ in Equation 17 and simplifying gives $n = (M - 1)n_d$. It follows that the minimum reinforcement ratio can only equal ρ_{min} in parallel hyperplanes with $n \leq (M - 1)n_d$. The minimum possible reinforcement ratio $\rho_{i,\text{min}}$ can be found in hyperplanes with $n > (M - 1)n_d$ by replacing ρ_{min} in the left-hand side of Equation 17 with $\rho_{i,\text{min}}$. It follows that $\rho_{i,\text{min}}$ is given by

$$\rho_{i,\text{min}} = \rho_{\text{min}} + (n - (M - 1)n_d)\delta_\rho$$

$$18. \quad \text{where } n > (M - 1)n_d$$

The maximum reinforcement ratio $\rho_{i,\text{max}}$ first equals ρ_{max} in the parallel hyperplane in which

$$19. \quad \rho_{\text{max}} + (M - 1)\rho_{\text{min}} = M\rho_{\text{min}} + n\delta_\rho$$

Substituting $\rho_{\text{max}} = \rho_{\text{min}} + n_d\delta_\rho$ in Equation 19 and simplifying gives $n = n_d$, which implies that the maximum reinforcement ratio can only equal ρ_{max} in parallel hyperplanes with $n \geq n_d$. The maximum reinforcement ratio is found in hyperplanes with $n < n_d$ by replacing ρ_{max} with $\rho_{i,\text{max}}$ in the left-hand side of Equation 19. In this case

$$20. \quad \rho_{i,\text{max}} = \rho_{\text{min}} + n\delta_\rho \text{ where } n < n_d$$

As the reinforcement ratios lie between ρ_{min} and ρ_{max} , the possible domain for the variation of each reinforcement ratio ρ_i can be stated in the following form for any n :

$$\max \{ \rho_{\text{min}}, \rho_{\text{min}} + (n - (M - 1)n_d)\delta_\rho \}$$

$$21. \quad \leq \rho_i \leq \min \{ \rho_{\text{min}} + n\delta_\rho, \rho_{\text{max}} \}$$

For programming purposes, it is more efficient to express the reinforcement ratios in the MD space in integer format. In this case, the minimum reinforcement ratio ρ_{min} is defined as zero. The incremental step in the mesh is defined as $\delta_{\rho_{\text{int}}}^{n_{r,\text{cr}}} = n_d^{n_r - n_{r,\text{cr}}}$, in which n_d is the number of mesh divisions, n_r is the number of mesh refinements and $n_{r,\text{cr}}$ is the current level of mesh refinement, which can be 0, 1, $\dots$, n_r . It follows that the incremental step in the reinforcement mesh is $n_d^{n_r}$ in the unrefined mesh and 1 in the final refined mesh. Consequently, $\rho_{\text{max,int}} = n_d^{n_r+1}$. Consider a mesh with four divisions and one level of mesh refinement for

which $n_d = 4$ and $n_r = 1$. The integer values of the mesh divisions are 4 in the unrefined mesh, for which $n_{r,cr} = 0$, and 1 in the refined mesh. Consequently, $\rho_{\max,int} = 16$ in the refined mesh, as well as the unrefined mesh.

The first step in the generation of a hyperplane is to determine the valid domain of each of its reinforcement ratios using Equation 20, which is more efficiently expressed in integer format as Equation 22.

$$22. \quad \max \left\{ 0, (n - (M - 1)n_d^{n_{r,cr}+1})\delta_{\rho_{int}}^{n_{r,cr}} \right\} \leq \rho_{i,int} \leq \min \left\{ n\delta_{\rho_{int}}^{n_{r,cr}}, \rho_{\max,int} \right\}$$

in which n varies from 0 to $Mn_d^{1+n_{r,cr}}$.

Consider a 3D mesh as an example. The first step in the generation of the mesh is to determine the valid domain of its reinforcement ratios (ρ_1 , ρ_2 and ρ_3) using Equation 22. After that, ρ_3 is varied in its valid domain and for each value of ρ_3 all the possible combinations of ρ_2 and ρ_1 for which $\rho_1 + \rho_2 =$

$\rho_{\text{total}} - \rho_3$ are included in the movement in the current 2D plane. Table 2 shows the results of this procedure for a 3D mesh with two divisions and no mesh refinement. The valid reinforcement ratios are 0, 1 and 2 in the integer coordinate system.

This procedure is readily extended to higher dimensions. The starting point is to find the valid domain for each reinforcement ratio as before. After that ρ_M is varied in its valid domain and for each value of ρ_M all the possible combinations of ρ_1 to ρ_{M-1} for which $\sum_{i=1}^{M-1} \rho_i = \rho_{\text{total}} - \rho_M$ are included in the movement in the current $(M - 1)$ D hyperplane. Mesh refinement is accomplished by updating $n_{r,cr}$, and setting the maximum and minimum values of ρ_{total} to its value in the current and previous hyperplane.

Design procedure

The design procedure is illustrated in the flowchart in Figure 8, which is discussed below. The bracketed numbers in the text refer to the steps in the flowchart.

(a) After reading the input data and generating the various files

$\Sigma \rho_i$	ρ_1	ρ_2	ρ_3	$\Sigma \rho_i$	ρ_1	ρ_2	ρ_3
$3\rho_{\min}$	$\rho_{\min}$	$\rho_{\min}$	$\rho_{\min}$	0	0	0	0
$3\rho_{\min} + \delta_\rho$	$\rho_{\min}$	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	1	0	0	1
	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	1	0	1	0
	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	$\rho_{\min}$	1	1	0	0
$3\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	2	0	0	2
	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	2	0	1	1
	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	2	1	0	1
	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	2	0	2	0
	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	2	1	1	0
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	$\rho_{\min}$	2	2	0	0
$3\rho_{\min} + 3\delta_\rho$	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	3	0	1	2
	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	3	1	0	2
	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	3	0	2	1
	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	3	1	1	1
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	$\rho_{\min} + \delta_\rho$	3	2	0	1
	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	3	1	2	0
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min}$	3	2	1	0
$3\rho_{\min} + 4\delta_\rho$	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	4	0	2	2
	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	4	1	1	2
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	$\rho_{\min} + 2\delta_\rho$	4	2	0	2
	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	4	1	2	1
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + \delta_\rho$	4	2	1	1
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min}$	4	2	2	0
$3\rho_{\min} + 5\delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	5	1	2	2
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	$\rho_{\min} + 2\delta_\rho$	5	2	1	2
	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + \delta_\rho$	5	2	2	1
$3\rho_{\min} + 6\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	$\rho_{\min} + 2\delta_\rho$	6	2	2	2

Table 2. Reinforcement ratios in a 3D mesh ($n_d = 2$ and $n_r = 0$)

Figure 8. Flowchart of design strategy C

- needed for the FEA, NonOpt defines the VHB and identifies the reinforcement ratios in each VHB bunch (1–3).
- (b) The reinforcement ratios are initialised to $\rho_{\min}$ and a linear FEA is carried out (4–5).
- (c) NonOpt initialises the data required for generating a MD mesh that includes all the possible reinforcement combinations in the structure (6). Before searching through the points in the MD mesh, NonOpt solves the equations of the MCFT at every Gauss point in each VHB for every possible reinforcement combination in the bunch and stores the safety factors of each VHB (7–8). This is done to prevent repetitive analyses, which would otherwise arise as the same values of (ρ_l, ρ_t) occur many times in the MD space for each VHB bunch.
- (d) NonOpt finds solutions by sequentially checking the safety factors of each VHB as it moves through the coordinates of the MD mesh from $\rho_{\text{total}} = M\rho_{\min}$ to $M\rho_{\max}$ (9). The solution procedure moves to the next coordinate in the MD mesh as soon as a design constraint is violated unless the VHB is reinforced with $\rho_{\max}$ in both directions. In this case, the critical element cannot be designed for the current stress distribution. Continuing to the next coordinate would simply

result in the entire structure being reinforced with $\rho_{\max}$. Critical points may become allowable in the next NLFEA owing to the dependency of the FE results on the reinforcement ratios.

- (e) The solution procedure initially finds the first coordinate in the current hyperplane that satisfies the design constraints. Subsequently, NonOpt minimises either ρ_{total} (strategy C_A) or the reinforcement weight (strategy C_B). In strategy C_A, NonOpt adopts the solution with the greatest safety factor where multiple solutions exist with the same ρ_{total} . In strategy C_B, NonOpt finds the solution with the least $V_{\text{total}} = \sum_{i=1}^M A_i \rho_i$ among those with the same ρ_{total} in the current hyperplane. Subsequently, NonOpt refines the mesh between the penultimate and current hyperplanes if $n_r \geq 1$ (11) and searches for the next solution with less V_{total} in the refined mesh. Having found a solution, the mesh refinement is continued until $n_{r,\text{cr}} = n_r$, at which point the design constraints are checked at all the remaining points in the current hyperplane. The final solution is taken as that with the lowest value of V_{total} , which is depicted $V_{\text{total min},1}$. It is possible, however, that the minimum weight solution for the current stress distribution lies in an adjacent hyperplane with a greater

ρ_{total} . This solution is found by continuing the movement in the refined mesh up to the hyperplane in which there is no solution with $V_{total} \leq V_{totalmin,1}$ (12–13). The final solution is that with the absolute minimum V_{total} . In the case of multiple solutions with the same V_{total} , the solution with the greatest safety factor is adopted.

- (f) The reinforcement is updated in DIANA and a NLFEA is carried out (14–15). The convergence criterion is checked at every Gauss point using the results from DIANA. If the convergence criterion is not satisfied, the procedure returns to box 6 and the reinforcement is redesigned for the current stresses from the last NLFEA. This process is repeated until the convergence criterion is fulfilled at every Gauss point (17).

Convergence criterion

The design procedure can be terminated when the design constraints are first satisfied at every Gauss point in the structure. It is, however, possible to minimise the reinforcement further owing to the dependency of the NLFEA results on the reinforcement ratios. Therefore, NonOpt gives the user two options, namely C1 and C2, for continuing the optimisation procedure after the design constraints are first satisfied. In option C1, the user specifies a minimum number of FEA in addition to the design constraints. The number of reinforcement design iterations is 1 less than the total number of FEAs as a NLFEA is carried

out after every reinforcement design. The design terminates when the design constraints are satisfied throughout the structure and at least the specified minimum number of FEAs have been completed. Option C2 requires the reinforcement ratios to converge within a prescribed tolerance. In this case, the design terminates if the design constraints are met and the reinforcement ratios have converged in successive reinforcement minimisations.

Case study

This study considers the deep beam shown in Figure 9, which had previously been designed by Schlaich *et al.* (1987) and Fernández Ruiz and Muttoni (2007). The beam is designed using strategies A, C_A and C_B with the VB and HB arrangement shown in Figure 9. Strategy A minimises the sum of reinforcement ratios in each VHB and sets the reinforcement ratio in each VB or HB equal to the maximum of the ratios required in any of its linked VHB bunches. Strategy C_A minimises the sum of the reinforcement ratios in the VB and HB, whereas strategy C_B minimises the total weight of reinforcement in the structure as described previously. Tension stiffening is included in NonOpt, as well as DIANA. It should be noted that a perfect bond is assumed in NonOpt, as well as DIANA, which is not the case in reality. Therefore, it is essential to extend the reinforcement by at least an anchorage length beyond the edge of the VHB where it is no longer required according to NonOpt. The reinforcement weights quoted in this paper do not include the additional reinforcement

Figure 9. Deep beam with opening. Dimensions in mm

required for anchorage. Convergence criterion C1 is adopted with a minimum of three FEAs to ensure at least one reinforcement design is carried out after a NLFEA.

Material properties and design constraints

The concrete compressive strength is taken as 30 MPa with $\varepsilon'_c = -0.002$ as stated in EC2 (BSI, 2004). The concrete compressive stress–strain relationship is modelled in accordance with the recommendations of Collins and Porasz (1989) as

$$23. \quad \sigma_d = -\beta f'_c \frac{m(\varepsilon_d/\varepsilon'_c)}{(m-1) + (\varepsilon_d/\varepsilon'_c)^{mk}}$$

in which

$$24. \quad m = 0.8 + \frac{f'_c}{17} \quad (\text{MPa})$$

$$25. \quad k = 1; \quad \varepsilon'_c < \varepsilon_d < 0$$

$$26. \quad k = 0.67 + \frac{\beta f'_c}{62} \quad (\text{MPa}); \quad \varepsilon_d \leq \varepsilon'_c$$

$$27. \quad \beta = \frac{1}{1 + k_c} \leq 1$$

Results	Strategy								STM	Fernández	Schlaich
	A				C _A (minimum $\sum_{i=1}^M \rho_i$)				orthogonal	Ruiz and	<i>et al.</i>
	Initial	Final	Initial	Final	Initial	Final	Initial	Final	Final	Muttoni	STM ^a
n_d	10	10	10	10	10	10	20	20	—	—	—
HB1	0.0134	0.0096	0.0134	0.0096	0.0096	0.0058	0.0115	0.0077	0.0104	0.0098	0.0063
HB2	0.0058	0.0020	0.0058	0.0058	0.0058	0.0020	0.0058	0.0039	0.0020	0.0020	0.0020
HB3	0.0058	0.0058	0.0058	0.0058	0.0058	0.0058	0.0039	0.0039	0.0035	0.0020	0.0023
HB4	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
HB5	0.0134	0.0058	0.0134	0.0058	0.0096	0.0020	0.0096	0.0020	0.0035	0.0020	0.0023
HB6	0.0210	0.0096	0.0210	0.0096	0.0172	0.0058	0.0153	0.0058	0.0173	0.0082	0.0117
HB7	0.0020	0.0058	0.0020	0.0058	0.0020	0.0058	0.0020	0.0039	0.0020	0.0020	0.0020
HB8	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB1	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB2	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB3	0.0020	0.0020	0.0020	0.0020	0.0096	0.0020	0.0039	0.0020	0.0020	0.0020	0.0020
VB4	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0035	0.0020	0.0023
VB5	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB6	0.0096	0.0020	0.0096	0.0020	0.0134	0.0096	0.0134	0.0039	0.0035	0.0020	0.0023
VB7	0.0134	0.0096	0.0134	0.0058	0.0172	0.0096	0.0172	0.0077	0.0173	0.0052	0.0117
VB8	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0039	0.0020	0.0020	0.0020
VB9	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB10	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB11	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB12	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
VB13	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
Sum	0.1104	0.0762	0.1104	0.0762	0.1142	0.0724	0.1066	0.0667	0.0870	0.0592	0.0669
D^b	—	—	—	—	—	—	—	—	—	—	94
Weight: kg	1168	858	1168	828	1087	758	1051	690	923	622	794

^a The reinforcement weight is calculated assuming that the ties in HB3 and HB5 are 5.5 m long

^b D denotes weight of diagonal bars in kg

Table 3. Reinforcement ratios and weights in deep beam with opening

$$28. \quad k_c = 0.27 \left(-\frac{\varepsilon_r}{\varepsilon'_c} - 0.37 \right)$$

Following cracking, the principal concrete tensile stress is calculated as follows, in accordance with the recommendations of Vecchio and Collins (1986).

$$29. \quad \sigma_r = \frac{f_{cr}}{1 + \sqrt{(200\varepsilon_r)}}$$

in which the concrete tensile strength f_{cr} is taken as $f_{cr} = 0.33\sqrt{f'_c}$.

The reinforcement is modelled bilinearly with elastic modulus $E_s = 200$ GPa, yield strength $f_y = 500$ MPa and post-yield tangent modulus $E'_s = 0.842$ GPa. The strain at the maximum force is taken as 0.05, which satisfies the requirements of Ductility Class B in EC2. Steel bearing plates are modelled as linearly elastic with an elastic modulus of 200 GPa and a Poisson ratio of 0.3.

The principal compressive strain in the concrete is limited to $|\varepsilon_{d,per}| = 0.0017$, which is 85% of the strain at the peak stress. The maximum permissible reinforcement stresses at crack are taken as $f_{sler,per} = f_{str,per} = 536$ MPa, which is the stress at 90% of the assumed characteristic strain at the maximum force. No limits are applied to the maximum principal tensile strain in the concrete or the mean strain in the reinforcement as the case studies only consider the ULS. The permissible reinforcement

ratios are taken as $\rho_{min} = 0.002$ and $\rho_{max} = 0.04$ in accordance with the requirements of EC 2 (BSI, 2004) for walls and columns. The number of reinforcement mesh divisions is taken as either $n_d = 10$ ($\delta_\rho = 0.0038$) or $n_d = 20$ ($\delta_\rho = 0.0019$), as noted in Table 3 without any mesh refinement. Crack spacings are calculated assuming the maximum aggregate size, concrete cover and bar spacing equal 20, 25 and 200 mm respectively.

Results

Table 3 lists the initial and final reinforcement ratios and weights given by each strategy. The final reinforcement ratios for strategy CB, where greater than ρ_{min} , are also shown in Figure 10 along with the directions of the minimum principal strains. The initial reinforcement ratios are calculated for the linear elastic stress field, whereas the final ratios are calculated for the stress field from the penultimate NLFEA, which is influenced by the reinforcement arrangement. The final reinforcement weight is significantly less than the initial weight for all three strategies. This is significant since it shows that the common practice of designing reinforcement in D regions on the basis of stresses calculated in an elastic FEA can be overly conservative. Table 3 also shows that the total reinforcement weight given by strategy CB reduces from 758 kg to 690 kg when the number of divisions in the reinforcement mesh is increased from $n_d = 10$ to $n_d = 20$. By way of comparison, the reinforcement weight given by strategy A reduces from 858 kg to 745 kg when n_d is increased from 10 to 20. Strategy A gives the greatest final reinforcement weight and strategy CB the least, as expected, but the use of strategy CB only reduces the weight by around 10%. This is of

Figure 10. Reinforcement ratios and directions of minimum principal strains for strategy CB ($n_d = 20$)

practical significance since strategy A is significantly less computationally demanding than strategy C.

Comparison with other design methods

Schlaich *et al.* (1987) proposed the STM shown in Figure 11(a) for the design of the reinforcement in the deep beam shown in Figure 9. They assumed that the diagonal tie resisted 50% of the vertical reaction at the left-hand support. This assumption allows all the member forces in the STM to be calculated from statics. The resulting reinforcement arrangement, as rationalised by Fernández Ruiz and Muttoni (2007), is shown in Figure 11(b). The authors also designed the reinforcement using the STM shown in Figure 11(a), but without the diagonal tie and its

equilibrating struts. The ties were proportioned using a yield strength of 500 MPa. Table 3 shows the reinforcement ratios corresponding to both these designs, as well as that of Fernández Ruiz and Muttoni (2007), which is based on NLFEA without tension stiffening. Consideration of Table 3 shows that all the design methods give reinforcement weights of the same order of magnitude, but there are some significant differences in the reinforcement arrangements. The reinforcement tends to be more smeared in the NLFEA based designs than the STM designs, particularly that with orthogonal reinforcement as the design makes better use of the minimal reinforcement required by EC2. Table 3 shows that the reinforcement weight given by strategy C_B reduces toward that calculated by Fernández Ruiz and Muttoni (2007) as the number of reinforcement mesh divisions is increased from 10 to 20.

Concluding remarks

An automated non-linear optimised design procedure is proposed for the design of D regions. The reinforcement is designed to resist stresses calculated with NLFEA. Practical reinforcement arrangements are achieved by grouping the elements in the FE model into horizontal and vertical bunches. Design constraints are specified in terms of allowable mean strains in the reinforcement and concrete, as well as maximum allowable stresses in the reinforcement at cracks. The design procedure is iterative owing to the dependence of the NLFEA results on the reinforcement ratios. The equations of the MCFT are used in both the NLFEA and the design of the reinforcement. The design procedure is implemented in NonOpt, which is a Fortran program that works in conjunction with the program DIANA used to carry out the NLFEA. The entire design is performed automatically once the designer has generated the FE model, and specified the design constraints and solution procedure. This paper considers three alternative solution strategies for designing the reinforcement, which are denoted A, C_A and C_B. Strategy A (Amini Najafian, 2011) is the least computationally demanding as it sets the reinforcement ratios in each VB and HB equal to the maximum of the ratios calculated in the linked VHB. Strategy C_A simultaneously minimises the sum of the reinforcement ratios in the VB and HB, whereas strategy C_B finds the minimum weight of reinforcement required to resist stresses calculated in the NLFEA. The resulting reinforcement design is an absolute minimum weight solution for the design stresses and design constraints, but not in general owing to the dependence of the stress distribution on the reinforcement arrangement. The case study demonstrates that the proposed design procedure is a practical alternative to STM, which can be complex to develop for reasons discussed earlier. There appears to be relatively little benefit in seeking a minimum weight solution as strategy A gives only slightly greater reinforcement weights than strategy C with considerably less computation.

Figure 11. Deep beam with opening: (a) strut-and-tie model of Schlaich *et al.* (1987); (b) reinforcement arrangement as rationalised by Fernández Ruiz and Muttoni (2007)

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