Towards the design of reinforced concrete eccentric beam–column joints

R. L. Vollum* and J. B. Newman*

Imperial College

The authors have previously proposed a simplified method for the design of external beam–column joints. This paper extends that work to joints in which one of the beams is eccentric to the column. Such connections occur in practice owing to architectural and geometrical constraints. Results are presented for tests on ten specimens in which one of the two beams framing into the column was eccentric to the column. The authors are unaware of any previous tests on such specimens. The tests were designed to investigate the effects of eccentricity and reinforcement detailing on connection strength, cracking and deformation. The tests showed that such connections can be used in practice providing that the torsional capacity of the joint is not exceeded. An analytical model is developed for predicting the strength of such connections and preliminary design recommendations are made. The work is also relevant to the case of joints where the beam is wider than the column.

Notation

A	cross-sectional area of beam	h	depth of concentric beam
A_k	area enclosed by centre line of shear flow	h_c	column depth
A_{se}	area of flexural tension steel in the eccentric beam anchored in concentric beam	N	force in longitudinal bar of concentric beam, with subscripts as follows: b, bottom; t, top; i, internal; e, external; T, torsion; V, shear due to P_e
A_{total}	area of flexural tension steel in eccentric beam	P_c	load on concentric beam
A_{sj}	effective area of joint stirrups	P_{c0}	maximum possible value of P_c when $P_e = 0$
b^*	width of stress field parallel to axis of concentric beam	P_e	load on eccentric beam
b_1	width of overlap of eccentric and concentric beams	P_{e0}	maximum possible value of P_e when $P_c = 0$
b_c	column width	T	torsion
b_e	effective joint width	T_0	maximum possible torsional strength of concentric beam
b_{ec}	width of eccentric beam	T_{beam}	tensile force in beam reinforcement
c_1	cover to column stirrups	T_{clo}	tensile force in closing stirrup
d'	minimum distance from column face to centroid of nearest main column bars	T_{col}	resultant force in leg 1 (see Fig. 2(b)) of column stirrups at top and bottom of joint
d_{ec}	effective depth of eccentric beam	t_i	thickness of wall i
f	strength of concrete in inclined stress field	t_{max}	maximum permissible thickness of wall i
f'_c	concrete cylinder strength	U	external perimeter of cross-section
f_{cu}	concrete cube strength	V_c	joint shear strength without stirrups
		V_{col}	shear force in column
		V_{iT}	shear force induced in wall i by torsion
		V_j	joint shear force
		V_{j0}	uniaxial joint shear strength
		V_{jc}	joint shear force due to P_c at joint shear failure
		V_{je}	joint shear force due to P_e at joint shear failure

* Department of Civil and Environmental Engineering, Imperial College of Science, Technology and Medicine, Imperial College Road, London SW7 2BU, UK.

(MCR 765) Paper received 18 February 1999; last revised 30 June 1999; accepted 14 September 1999.

w	projection of inclined stress field onto axis of concentric beam
z	distance to centroid of tensile force in eccentric beam from adjacent column face
$z_{\min}$	minimum possible distance to centroid of tensile force in eccentric beam from adjacent column face
α	efficiency factor for V_c
β	constant representing influence of detailing of beam reinforcement on V_c
δt	thickness of fan in wall 1
θ	minimum angle of inclination of fan in wall 1 to horizontal
θ_c	angle of inclination of centre line of fan in wall 1 to horizontal
θ_i	angle of stress field in wall i
ν	strength reduction factor for cracked concrete
ϕ	diameter of column stirrups

Introduction

Situations arise in practice where geometrical constraints such as the architectural detailing of the facade lead to the introduction of eccentric beam–column joints. Typical examples of such connections are shown in Fig. 1. Raffaele and Wight¹ have tested a small number of joints of type 1 (see Fig. 1) but the authors are unaware of any previous tests on connections of type 2 (Fig. 1), which are the concern of this paper. A series of tests were carried out to investigate the behaviour of ten eccentric beam–column joints with two different geometries (see Fig. 2). In tests A1 to A8, the

Type 1

Type 2

Fig. 1. Plan view of typical eccentric beam–column joints

eccentric beam was adjacent to the column but did not intersect it (see Fig. 2(b)). This represents an extreme case of the spandrel beam problem, examined by Hsu and Hwang² among others. The offset of the eccentric beam was reduced by 50% in tests B1 and B2, making the centre line of the beam coincident with the internal column edge (see Fig. 2(c)). Tests of both geometries investigated the influence of varying the reinforcement in each beam on connection strength. Control of cracking under service loads was shown to be a key consideration in the design of eccentric beam–column joints. The authors' simplified design method^{3,4} for external beam–column joints is extended here to include type 2 eccentric joints with square columns. A strut and tie model is used to model the transfer of torsion into the column from the eccentric beam.

Test programme

The main aim of the test programme⁴ was to determine the strength and failure modes of eccentric beam–column joints. Importance was attached to determining the conditions under which a plastic hinge forms in the eccentric beam since hinge formation simplifies connection design by limiting the maximum torque in the concentric beam.

Details of specimens

The specimens are shown in Fig. 2 and details of the reinforcement are summarized in Table 1 (see Fig. 2(b) for description of stirrup details A and B). The area of longitudinal steel in the beams was varied to determine its effect on joint strength and stiffness. The effect of omitting the closing stirrup at the intersection of the beams (see Fig. 2(b)) was investigated in specimens A7 and A8, which were otherwise similar to specimens A3 and A4, respectively.

Material properties

High-tensile deformed reinforcement bars were used, with diameters ranging from 8 mm to 16 mm. Typical stress–strain diagrams for the reinforcement are shown in Fig. 3. The concrete was made from ordinary Portland cement, natural sand and 10 mm Thames Valley flint gravel. Three 150 mm cubes were cured in air alongside each specimen and tested on the same day as the specimens. Details of the cube strengths are given in Table 2.

Instrumentation

Displacements were measured at the loads and the joint by up to 15 displacement transducers. Reinforcement strains were measured within the joint by up to 40 surface-mounted electrical-resistance strain gauges with a gauge length of 10 mm. Full details of the strain and displacement data are reported elsewhere.⁴

Fig. 2. (a) Elevation of all specimens; (b) plan view of specimens of type A (dimensions in mm)

Test procedure and results

The specimens were tested with the column oriented vertically. Spherical seatings were provided at the top and bottom of the column. Lateral restraint was provided at the top of the column by orthogonal tie bars, which were pretensioned by applying an initial load of 5 kN to each beam. Subsequently the column load was increased to 300 kN in all the tests except A8 and B1, in which the column load was 500 kN. The aim of the

subsequent loading was to probe/trace the joint strength envelope. The sequences in which the loads on the eccentric and concentric beams (P_e and P_c , respectively) were increased in tests A1 to A8 are illustrated schematically in Fig. 4, which should be read in conjunction with Table 3. The lines BC, DC and FC in Fig. 4 follow the strength envelope of the connection as closely as possible. This was achieved by holding P_e at the maximum value that could be sustained, while increasing P_c . The actual load paths in tests A1 to A8 are

Fig. 2. (c) plan view of specimens of type B (dimensions in mm)

Table 1. Reinforcement details in eccentric beam-column tests

Specimen	Main reinforcement in eccentric beam	Main reinforcement in concentric beam	Stirrups at intersection of beams
A1	2 T16 U bars	3 T16 U bars	Detail A 2 T8 + 1 T12
A2	2 T12 U bars	3 T16 U bars	Detail A 2 T8 + 1 T12
A3	2 T10 U bars	2 T16 U bars	Detail B 1 T12
A4	2 T10 U bars	3 T16 U bars	Detail B 1 T12
A5	2 T10 U bars	2 T12 U bars	Detail B 1 T12
A6	2 T10 U bars	2 T16 U bars	Detail B 1 T12
A7	2 T10 U bars	2 T16 U bars	None
A8	2 T10 U bars	3 T16 U bars	None
B1	2 T16 U bars	3 T16 U bars	None
B2	1 T16 U bar and 1 T12 U bar	2 T16 U bars	None

Fig. 3. Typical stress-strain diagrams for reinforcement bars

compared with theoretical strength envelopes in Figs 5–7, in which P_e is plotted against P_c . Points A–F, which define the load paths in Fig. 4/Table 3, are also shown in Figs 5–7, in which the loads corresponding to joint failure in either shear or torsion are circled. Comparison of the load paths for tests A3 and A7 in Fig. 6 and tests A4 and A8 in Fig. 7 shows that the strength of specimens A7 and A8 was not adversely affected by the omission of the T12 closing stirrup from the joint (see Fig. 2(b)). Furthermore, Figs 6 and 7 show that connection strength is sensibly independent of loading sequence (P_e was reduced below the maximum that

Table 2. Concrete cube strengths in eccentric beam-column joint tests

Specimen	Cube strength: MPa
A1	36.3
A2	40.4
A3	42.8
A4	45.4
A5	45.4
A6	33.0
A7	35.5
A8	34.7
B1	30.1
B2	35.1

could be resisted in test A3 when P_c reached 80 kN). This is significant since it considerably simplifies design for strength. The load path in tests B1 (see Fig. 8, where the arrowheads indicate the sequence of loading) and B2 was designed to probe the joint strength envelope. In each test, the loads P_c and P_e were initially increased in a ratio of about 2:1 since this was considered representative of the loading in a framed structure. The points of interest in test B1 (see circles in Fig. 8) are those at which joint failure occurred or was imminent. In test B2, hinges formed in both beams, and joint shear failure did not occur.

Fig. 4. Schematic illustration of load paths

Table 3. Loading paths in tests A1 to A8

Specimen	Loading path
A1	(a) A/B/unload (b) A/E/F
A2–A5	A/B/C
A6	A/G
A7	(a) A/B/unload (b) A/E/F/C
A8	(a) A/D/C/unload (b) A/G

Fig. 5. Comparison between test load path and predicted load interaction diagram for tests A1 and A2

Fig. 6. Comparison between test load path and predicted load interaction diagram for tests A3, A5 and A7

Fig. 7. Comparison between test load path and predicted load interaction diagram for tests A4 and A8

Fig. 8. Loading sequence for test B1

Complete crack patterns were recorded in each test together with the widths of the most critical cracks. A clear indication of the crack pattern in tests A1 to A8 is given by Fig. 9, which shows the crack pattern in test A4. Crack widths generally remained small until the main reinforcement yielded in the beams. Consequently, crack widths were reduced by increasing the area of longitudinal reinforcement in the concentric beam (tests A1, A2, A4 and A8). A major crack formed across the eccentric beam at the column face in tests A3 and A4, in which a closing stirrup was provided at the intersection of the beams (see Fig. 2(b)). This crack was displaced into the top face of the concentric beam in tests A7 and A8, in which the closing stirrup was omitted. It is concluded that the closing stirrup had the beneficial effect of inducing the crack to form at the column face in preference to the top face of the concentric beam. Crack widths were smaller in tests B1 and B2 because torsion was less significant. In each test, a crack formed across the eccentric beam at the column face. The crack was significantly wider at the junction of the eccentric beam with the column than at the junction of the eccentric beam with the concentric beam because the inner longitudinal bar in the eccentric beam (which was anchored in the column) attracted considerably more load than the external bar. In test B2, the T16 bar in the eccentric beam, which was anchored in the column, yielded before the outer T12 bar.

Fig. 9. Crack pattern at failure in specimen A4 (formerly⁴ EBCJ4)

Analysis of specimens at ultimate limit state

The tests showed that the strength of eccentric beam–column joints is limited by the following failure modes:

- mode 1: column flexure
- mode 2: uniaxial joint shear failure (e.g. A6 and A8)
- mode 3: torsional failure of concentric beam (possibly leading to column failure) prior to yield of its longitudinal reinforcement (e.g. A1)
- mode 4: biaxial joint shear failure (e.g. A4 and B1)
- mode 5: yielding of reinforcement (e.g. A3).

The extreme loads sustained by each specimen and the corresponding failure modes are summarized in Table 4, where an asterisk indicates that failure was imminent. Failure modes 1 to 4 should be avoided in design since they involve column failure. Mode 1 is avoided by designing the upper and lower columns for the appropriate actions and is not considered further.

Failure mode 2

The uniaxial joint shear strength is predicted using a simplified method developed by the authors^{3,4} from a comprehensive analysis of test data.^{5–11} The uniaxial joint shear strength V_{j0} is taken as the greater of V_c and

Table 4. Extreme loads and failure modes in eccentric beam–column joint tests

Test	$P_{e \max}$: kN			$P_{c \max}$: kN		
	$P_{e \max}$	P_c	Mode	P_e	$P_{c \max}$	Mode
A1	48.0	4.2	3*	52.5	59.2	3
A2	50.9	3.8	5/3*	42.3	100.0	4
A3	39.5	4.5	5	12.6	111.2	4
A4	43.5	27.3	5	33.1	109.7	4
A5	38.0	4.05	5	12.85	80.5	5
A6	—	—	—	5.0	100.0	2
A7	36.9	4.5	5	25.5	92.1	5
A8	38.1	74.2	5	36.8	112.6	4*
A8	—	—	—	5.0	120.0	2
B1	69.2	43.0	4*	5.0	98.0	2*
B1	56.0	72.0	4*	55	82	4
B2	59.5	97.6	5	41.8	111.0	5

$$V_{j0} = V_c - \alpha b_e h_c \sqrt{f'_c} + A_{sj} f_y \quad (1)$$

where A_{sj} is the cross-sectional area of the stirrups below the main beam reinforcement within the top five-eighths of the beam depth, b_e is the effective joint width, which is taken as the column width in this paper, and h_c is the column depth, f_y is the yield strength of the stirrups; α is an efficiency factor, which lies between 0 and 0.2 depending on factors including column load, concrete strength, joint stirrups and joint aspect ratio. A value of 0.2 is recommended for design. V_c is the joint shear strength without stirrups, which is taken as

$$V_c = 0.642\beta[1 + 0.555(2 - h_b/h_c)]b_e h_c \sqrt{f'_c} \quad (2)$$

where β varies between 1 and 0.9 depending on the detailing of the beam reinforcement. The maximum joint shear strength is limited to

$$V_j \leq 0.97b_e h_c \sqrt{f'_c}[1 + 0.555(2 - h_b/h_c)] \leq 1.33b_e h_c \sqrt{f'_c} \quad (3)$$

The coefficients in equations (1) to (3) depend on the assumptions made in the calculation of joint shear force, which is given by

$$V_j = T_{\text{beam}} - V_{\text{col}} \quad (4)$$

where V_{col} is the shear force in the upper column and T_{beam} is the tensile force in the beam reinforcement, which depends on the design moment and the assumptions made in the section analysis. In the derivation of equations (1) to (3), the shear force in the beam is assumed to be transferred into the centroid of the nearest external layer of column bars. The tensile force in the beam reinforcement is calculated assuming that plane sections remain plane, using the rectangular-parabolic stress block of EC2.¹² The stress in the concrete is assumed to reach a maximum value of $0.8f_{cu}$ at a strain of -0.002 . Equations (1) and (2) were calibrated for the authors' tests with $\alpha = 0.14$ and $\beta = 1.0$ to give a lower bound to the joint shear strength of specimens A6, A8 and B1, which failed in essentially uniaxial joint shear.

Failure mode 3

In general, part of the moment in the eccentric beam is transferred directly into the column by way of flexure and the remainder by torsion. Tests B1 and B2 showed that the longitudinal bar in the eccentric beam that was anchored in the column attracted considerably more load than the outer bar before yield. Therefore a conservative estimate of the torque in the concentric beam, for the authors' tests, is given by

$$T_{\text{applied}} = P_c(650 + d')A_{se}/A_s \quad (5)$$

where d' is the minimum distance to the centroid of the column bar from the column face, A_{se} is the area of flexural tension steel in the eccentric beam anchored in the concentric beam and A_s is the total area of flexural

tension steel in the eccentric beam. Equation (5) is based on the assumption that the vertical shear force in the eccentric beam is transferred into the column bar at the re-entrant corner. Torsional failure occurs if T_{applied} exceeds the maximum possible torsional strength of the concentric beam T_0 , which is assumed to depend solely on the section dimensions and concrete strength. Analysis of the authors' test data shows that a lower-bound estimate of T_0 is given by

$$T_0 = \nu f'_c A A_k / U \quad (6)$$

where A is the cross-sectional area of the concentric beam, U is the perimeter of the concentric beam, A_k is the area enclosed by the centre line of the thin-walled cross-section of wall thickness $t = A/U$ and ν is the strength reduction factor $0.6(1 - f'_c/250)$, as recommended by the CEB-FIP Model Code 1990¹³ (MC90) for cracked concrete. Equation (6) is based on the thin-walled-tube analogy of MC90 with $\cot \theta = 1$. T_0 is the maximum strength in pure torsion. The reduction in torsional strength due to shear can be estimated using the interaction equation of EC2¹² or similar. In the present tests, the reduction in torsional strength due to shear is small and is therefore neglected.

Failure in mode 4

The effect of biaxial loading and torsion on joint shear strength is clearly demonstrated in Fig. 10, in which (V_{je}/V_{j0}) is plotted against (V_{jc}/V_{j0}) at failure or imminent failure in modes 2 to 4 (see Table 4 for corresponding loads) for all the authors' specimens. The loads corresponding to the points marked 'imminent joint failure' in Fig. 10 are asterisked in Table 4. The line for V_{je}/V_{j0} corresponding to T_0 for specimen A1 is plotted in Fig 10 (V_{je}/V_{j0} corresponding to T_0 varies between 0.59 and 0.67 for tests A1 to A8 owing to variations in concrete strength). Fig. 10 shows that if $T_{\text{applied}} < T_0$ (where a conservative estimate of T_{applied}

Fig. 10. Influence of biaxial loading and torsion on joint shear strength

is given by equation (5)) a lower bound to the biaxial joint shear strength of all the specimens is given by

$$(V_{jc}/V_{j0})^2 + (V_{je}/V_{j0})^2 \leq 1 \quad (7)$$

where V_{j0} is the uniaxial joint shear strength, V_{jc} is the joint shear force due to P_c at failure and V_{je} is the joint shear force due to P_e at failure. The failure envelope corresponding to equation (7) (modes 2 and 4) and the requirement that $T_{\text{applied}} < T_0$ (mode 3 is not critical for tests B1 and B2) is plotted in Figs 5–8. It can be seen that the test points corresponding to joint failure (circled and labelled with failure mode) lie outside the failure envelope as required.

Only a few relevant tests have been carried out by others. Raffaele and Wight¹ investigated the behaviour of eccentric beam–column connections of type 1 (see Fig. 1) under earthquake-type loading. All the tests considered by Raffaele and Wight¹ were on specimens with square columns. They found that eccentricity reduced the joint shear strength below that given by ACI–ASCE Committee 352¹⁴ for uniaxially loaded joints and proposed that the effective joint width should be reduced to compensate for the effect of eccentricity. Their recommendations are very conservative for the current tests. Fujiwara *et al.*¹⁵ tested three-member beam–column specimens (with concentric beams) with load reversals in an inclined direction. They found that the joint shear resistance (defined as the vectorial sum of the longitudinal and transverse shear resistances) was fairly constant, which is consistent with equation (7).

Failure in mode 5

A three-dimensional strut and tie model has been developed to determine the reduction in torsional strength as P_c is increased (see Figs 5–7) due to yield of reinforcement. The model is necessarily complex but can be incorporated in a spreadsheet. The strut and tie model is an extension of the space truss model that is widely used to model torsion.^{12,13} The philosophy

adopted in the modelling is consistent with that recommended in MC90¹³ for design for combined shear, flexure and torsion. The layout of the strut and tie model is shown in Figs 11–13 for a type A joint (see Fig. 2(b) but equations (8) to (19) below are also valid for type B joints (see Fig. 2(c)). The orientation of the inclined stress field is shown from outside the section. The inclined strut marked I* in Fig. 11 is required to distribute the torque into walls 1 to 4 (of thickness t_1 to t_4 , respectively). This strut was not critical in any of the tests and is not checked in the model. Furthermore, no analysis is made of the stress state in the column, since the model is for failure in mode 5. The longitudinal reinforcement in the concentric beam is assumed to be concentrated at the corners of the section. The shear force in the concentric beam is assumed to be resisted by a central core of width $b_c - 2t_1 - 2\delta t$ (where b_c is the width of the concentric beam and the dimensions t_1 and δt are defined in Fig. 11). Failure in mode 5 is due to yielding of reinforcement. Tension is induced in the longitudinal reinforcement of the concentric beam by flexure, shear and torsion. These actions are considered separately and the resultant tensile force is found by superposition. The moment used to find the tensile force in the reinforcement due to flexure in the concentric beam is taken as

$$M = P_c(L + 0.5\delta L) \quad (8)$$

where L is the distance from P_c to the column face, equal to 450 mm, and δL is the bearing width, equal to the lesser of $P_c/[f'_c(b - 2t_1 - 2\delta t)]$ and $2d'$, where d' is the distance to the centroid of the nearest column bar. The shear force in the eccentric beam is assumed to be transferred into the column through a fan-shaped inclined stress field in wall 1 (see Figs 11 and 13). The consequence of this is that the tensile force in the longitudinal steel of the concentric beam is greatest at its top internal corner, which is consistent with the strain measurements. Therefore, failure is predicted in mode

Fig. 11. Strut and tie model for eccentric beam–column joints

Fig. 12. Details of strut and tie model for torsion (A, B, C and D are setting-out points for stress field)

 Fig. 13. Transfer of P_e into column

5, owing to yielding of either the longitudinal bar at the top internal corner of the concentric beam or the joint stirrups. The tensile force in the bar at the top internal corner of the concentric beam due to shear from P_e is given by

$$N_{tiV} = P_e \cot \theta_c b_1 / b_{ec} \quad (9)$$

where θ_c is defined in Fig. 13, b_1 is the overlap of the beams (see Fig. 2(c)) and b_{ec} is the width of the eccentric beam. The thickness δt of the fan-shaped stress field is taken as

$$\delta t = P_e b_1 / [0.5 f b_{ec} (h - t_2) \sin 2\theta] \quad (10)$$

where f is the concrete strength in the inclined stress field and $\tan \theta = d_{ec} / b_1$, where d_{ec} is the effective depth of the eccentric beam. Equation (10) is based on

the assumption that δt equals the wall thickness corresponding to a uniform stress field of magnitude f at angle θ , where θ is the smallest angle subtended by the fan to the horizontal (see Fig. 13).

Consideration of horizontal and vertical equilibrium in the thin-walled tube shown in Fig. 11 shows that

$$V_{1T} = V_{3T} \quad \text{and} \quad V_{2T} = V_{4T} \quad (11)$$

where V_{iT} is the shear force induced in wall i (where $i = 1$ to 4) by torsion.

Consideration of longitudinal equilibrium shows that $\theta_1 = \theta_3$ and $\theta_2 = \theta_4$, where the angles are defined in Fig. 12, $t_1 = t_3$, $t_2 = t_4$ and

$$N_{teT} = N_{tiT} = N_{beT} = N_{biT} = V_{1T} \cot \theta_1 = V_{2T} \cot \theta_2 \quad (12)$$

where N_{biT} is the tensile force induced in the bar at the bottom internal corner of the concentric beam by torsion; N_{tiT} , N_{teT} and N_{beT} are defined similarly with subscripts as follows: b, bottom; t, top; e, exterior; i, internal; and T, torsion.

V_{1T} and V_{2T} can be expressed in terms of the wall thickness t and the concrete strength as follows:

$$V_{1T} = f b^* t_1 \sin^2 \theta_1 \quad (13)$$

$$V_{2T} = f b^* t_2 \sin^2 \theta_2 \quad (14)$$

where f is the concrete strength in the inclined stress field and b^* is the width of the inclined stress field parallel to the axis of the concentric beam, equal to $2(w - z) \leq \min(b_{ec}, h_c)$ (see Fig. 12); b^* depends on

z , which can be varied by adjusting the proportion of the resultant force in the tensile reinforcement of the eccentric beam resisted by the bar at its top internal corner. Substituting for V_{1T} and V_{2T} (from equations (13) and (14), respectively) into equation (12) and rearranging gives

$$t_2 = t_1 \sin 2\theta_1 / \sin 2\theta_2 \quad (15)$$

The angles of the inclined stress field θ_1 and θ_2 are given by

$$\tan \theta_1 = (h - t_2)/w \quad (16)$$

$$\tan \theta_2 = (b - t_1 - \delta t)/w \quad (17)$$

The maximum wall thickness is limited to $t_{\max} = A/U$ in accordance with the recommendations of MC90.¹³ The strength of the concrete in the inclined stress field is taken as $0.745f'_c(1 - f'_c/250)$ on the basis of analysis of tests A1 to A8. The torque resisted by the joint block is given by

$$T = V_{1T}(b - \delta t - t_1) + V_{2T}(h - t_2) \quad (18)$$

The applied torque in the authors' tests is given by

$$T_{\text{applied}} = P_e(650 + 0.5\delta t)A_{se}/A_s \quad (19)$$

The load P_e (for a given value of P_c) at which the longitudinal steel in the concentric beam yields (i.e. mode 5 failure) can be found using the following procedure, which is readily incorporated into a spreadsheet.

- Select initial values for the unknowns P_e , w , t_1 and t_2 .
- Calculate δt using equation (10).
- Calculate T_{applied} using equation (19).
- Put $z = z_{\min}$ to minimize N_{ti} . To calculate $z_{\min}$, it is necessary to find the tensile force in the main steel of the eccentric beam from section analysis.
- Calculate $\tan \theta_1$ and $\tan \theta_2$ using equations (16) and (17), respectively.
- Calculate V_{1T} and V_{2T} using equations (13) and (14), respectively.
- Calculate the torque resisted by the joint block T using equation (18).
- Calculate the moment at the support of the concentric beam using equation (8).
- Calculate the resultant force in the reinforcement bar at the top inner corner of the concentric beam N_{ti} by superposition.
- Maximize P_e by varying P_e , w , t_1 and t_2 subject to the constraints that t_1 , $t_2 \leq t_{\max}$, $V_{1T} \cot \theta_1 = V_{2T} \cot \theta_2$, $T = T_{\text{applied}}$ and $N_{ti} = \text{yield capacity of bar}$.

The components of the inclined stress field in Fig. 12 normal to the axis of the concentric beam are balanced at the corners of the thin-walled tube by (a) U bars in the eccentric beam and the closing stirrup at the intersection of the beams and (b) stirrups in the column

at the top or bottom of the beams. Therefore, failure is also predicted to occur owing to yielding of these bars. Theoretically, torsion induces a resultant tensile force in the inner legs of the column stirrups parallel to the eccentric beam (leg 1 in Fig. 2(b)) at the top and bottom of the joint, given by

$$T_{\text{col}} = V_{T2}(h_c - c_1 - 0.5\phi - 0.5b^*)/(h - 2c_1 - \phi) \quad (20)$$

where c_1 is the cover to the column stirrups and ϕ is the stirrup diameter. Strain measurements showed that (a) strains were greater at the top of the joint than at the bottom and (b) strains in leg 2 (see Fig. 2(b)) were significantly less than in leg 1. Strains in leg 1 were similar to those predicted at the top of the joint at load point B (see Fig. 4) in tests A2 to A4. Strains in legs 1 and 2 increased as P_c was increased and at least the lower of the upper two stirrups yielded in leg 1 towards the end of tests A2 and A4. To control cracking, it is recommended that T_{col} should be taken as V_{T2} at the top of the joint and be derived from equation (20) at the bottom of the joint. The stirrups at the top of the joint are additional to those required for joint shear.

Theoretically, the force in the closing stirrup is approximately

$$T_{\text{clo}} = V_{T1}z/b_{ec} + 0.5P_e b_1/b_{ec} \quad (21)$$

In practice, tests A7 and A8 (in which the stirrup was omitted) indicate that the closing stirrup is less important than implied by the strut and tie model. Nevertheless it is recommended that closing stirrups are provided in practice because the tests indicate that they improve joint performance at the serviceability and possibly ultimate limit states.

The model has been used to analyse the authors' test results. In the analysis, the yield strength of the reinforcement was taken as follows: T8, 450 MPa; T10, 640 MPa; T12, 550 MPa; and T16, 500 MPa. Predicted (for modes 2 to 4 and mode 5) and experimental load interaction diagrams are compared in Figs 6–8. The least of the predicted failure loads corresponding to either modes 2 to 4 or mode 5 is critical. The figures show that the strut and tie model predicts the strength in mode 5 reasonably well and is conservative in that it overestimates the reduction in torsional capacity due to P_c . The theoretical maximum value of P_e was limited by T_0 (see equation (5)) in tests A1 and A2 and by the flexural capacity of the eccentric beam in tests A3, A4, A7 and A8. The model significantly overestimates the reduction in P_e in tests A3 and A7 because the torsional resistance of the joint does not reduce as rapidly as predicted on yielding of the longitudinal bar at the top inner corner of the concentric beam. This is thought to be the case because dowel action and aggregate interlock are neglected. Analysis showed that the strength of specimen B1 was not governed by the transfer of torsion into the joint block. In the case of

test B2, the model underestimates the torsion that can be transferred into the column from the outer bar in the eccentric beam. In test B2, a wide crack formed across the concentric beam at the junction with the column when the longitudinal steel in the concentric beam yielded. This indicates that loads were transferred across the crack in wall 2 (see Fig. 11) by dowel action rather than through an inclined stress field as assumed.

Practical design of eccentric beam–column joints

Failure in modes 1 to 4 can be avoided as described earlier. It is recommended that V_{j0} is limited to V_c in equation (7) since V_{j0} was not significantly greater than V_c in the authors' tests. It will seldom be necessary to derive the strength envelope for failure in mode 5. The longitudinal reinforcement required for torsion in the concentric beam can be designed using Code methods (e.g. EC2¹² with $\cot \theta = 1$). Longitudinal steel should also be placed at the top internal corner of the concentric beam to resist the tensile force induced by the transfer of shear from the eccentric beam into the column (see equation (9)). Stirrups should be placed at the top and bottom of the joint to resist T_{col} (see equation (20) and the following discussion). It is recommended that closing stirrups are provided at the intersection of the beams (see Fig. 2(b)) in accordance with equation (21) with $z = 0.5b_{ec}$.

Conclusions

The authors have extended their simplified design method^{3,4} for external beam–column joints to include eccentric joints of type 2 (see Fig. 1). Ten three-member eccentric beam–column joint specimens were tested with the primary aims of (a) determining the strength of the joints under combined loading and (b) developing an analytical method to predict connection strength. In the first series of tests (A1 to A8), the eccentric beam was adjacent to the column but did not intersect it. The effect of reducing the eccentricity by 50% was investigated in tests B1 and B2. The main conclusions are as follows.

- (a) Eccentric beam–column joints can fail in five modes, namely mode 1, column flexure; mode 2, uniaxial joint shear; mode 3, torsion in the concentric beam without yielding of its longitudinal steel; mode 4, biaxial joint shear; and mode 5, yielding of reinforcement.
- (b) Failure modes 2 to 4 are undesirable because they involve column failure. Such failures can be avoided if $T_{applied} < T_0$ and the design actions lie within the strength envelope defined by equation (7). Mode 5 failure can be investigated using the proposed strut and tie model. Practical design recommendations are also made.
- (c) Cracking was more severe than in conventional

joints but crack widths were small until the reinforcement yielded.

- (d) The performance of the beam–column joint improved significantly, in terms of crack control and strength, when the eccentricity was reduced.

It should be noted that all the tests were carried out on specimens with square columns. Further tests are required to determine (a) the effect of varying the aspect ratio of the column cross-section, (b) the strength envelope for biaxial joint failure in the absence of eccentricity and (c) the influence of slabs. The work is also relevant to joints where the beam is wider than the column and load is transferred into the column through torsion.

References

1. RAFFAELLE G. S. and WIGHT J. K. Reinforced concrete eccentric beam–column connections subjected to earthquake type loading. *ACI Structural Journal*, 1995, **92**, No. 1, 45–55.
2. HSU T. C. and HWANG C. Torsional limit design of spandrel beams. *ACI Structural Journal*, 1977, **74**, 71–79.
3. VOLLUM R. L. and NEWMAN J. B. The design of reinforced concrete external beam–column joints. *The Structural Engineer*, in press.
4. VOLLUM R. L. *Design and Analysis of Reinforced Concrete Beam–Column Joints*. PhD thesis, University of London, 1998.
5. REYS DE ORTIZ I. *Strut and Tie Modelling of Reinforced Concrete Short Beams and Beam–Column Joints*. PhD thesis, University of Westminster, 1993.
6. SCOTT R. H. The effects of detailing on RC beam/column connection behaviour. *The Structural Engineer*, 1992, **70**, No. 18, 318–324.
7. SCOTT R. H. and HAMIL S. J. Private communications, 1997.
8. KORDINA K. Bewehrungsführung in Ecken und Rahmenendknoten. *Deutscher Ausschuss für Stahlbeton*, 1984, Part 354.
9. PARKER D. E. and BULLMAN P. J. M. Edge–column connections in flat slab construction. *The Structural Engineer*, 1998, **76**, No. 13, 253–257.
10. MOEHLE J. Strength of slab–column edge connections. *ACI Structural Journal*, 1988, **85**, 89–98.
11. LUO Y. H. and DURRANI A. J. Equivalent beam model for flat-slab buildings—part II: exterior connections. *ACI Structural Journal*, 1995, **92**, 250–257.
12. BRITISH STANDARDS INSTITUTION. *Eurocode 2: Design of Concrete Structures. Part 1, General Rules and Rules for Buildings*. BSI, Milton Keynes, 1992, DD ENV 1992-1-1.
13. COMITÉ EURO-INTERNATIONAL DU BÉTON. *CEB-FIP Model Code for Concrete Structures. CEB-FIP International Recommendations, 1990*. Thomas Telford, London, 1993.
14. ACI-ASCE COMMITTEE 352R-91. Recommendations for the design of beam–column joints in monolithic reinforced concrete structures. In *ACI Manual of Concrete Practice*, American Concrete Institute, Detroit, 1998, Part 3.
15. FUJIWARA M., NISHIMURA Y. and MINAMI K. Behaviour of three-dimensional beam–column subassemblages under bi-directional loading. *Proceedings of the Japan Concrete Institute*, 1988, **10**, No. 3, 525–530 [in Japanese].

Discussion contributions on this paper should reach the editor by 31 May 2000