Numerical modelling of hydrodynamic pressures on dams

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Abstract

This paper discusses several considerations related to appropriate numerical modelling of the reservoir hydrodynamic pressures on dams. The reservoir is modelled with 8-noded isoparametric displacement based solid finite elements. The study includes both stiff and flexible dams with vertical and sloped upstream faces under ramp, harmonic and random acceleration loads. The numerical results were compared and found to be in good agreement with available closed-form solutions. The same approach may be used in analyses of other waterfront structures such as quay walls.

Keywords: hydrodynamic pressures, dams, solid finite elements, reservoir, dynamic analysis

1. Introduction

The seismic behaviour of dams has long been studied [14] and several methods of analysis were developed such as the pseudo-static [36] [32], the shear beam [1] and the dynamic finite element [4, 5] methods. Significant progress was made regarding the dynamic characteristics of inhomogeneous visco-elastic dams [10, 11] and the seismic response of nonlinear elasto-plastic dams [42, 13]. However, most of the analyses neglect the hydrodynamic pressures from the upstream reservoir and consider only the hydrostatic part of the reservoir pressures.

The hydrodynamic pressures on dams during earthquakes were first investigated by Westergaard [40] who considered a stiff dam with a vertical upstream face under harmonic loading. Zangar [43] [44] and Chwang [9] considered dams with a sloped upstream face whereas Liu [25] considered a

sloped reservoir base. Moreover, the effects of the reservoir water compressibility were studied by Chopra [6] who showed that the magnitude of the total hydrodynamic force on the dam from a compressible reservoir depends on the frequency of the applied loading. In addition, the work of Chopra [6] was extended to investigate the hydrodynamic pressures due to random acceleration loading and a closed-form solution was developed.

A number of analytical solutions were developed [40] [44] [6] [3] [27] covering several aspects of the problem (such as slope angle, water compressibility, nature of load etc.) and provided various solutions for the magnitude and distribution of the hydrodynamic pressures on dams. However, they suffered from several assumptions and limitations (such as simple geometries, undeformable dams etc.). More elaborate problems involving complicated geometries, deformable and inhomogeneous dams required discretisation of the reservoir domain [16], so that reservoir-dam interaction can be fully analysed. To model the reservoir, various techniques have been developed, such as Eulerian [30] and Lagrangian [41] fluid finite elements, coupling of Finite Element and Boundary Element methodologies [38] [33] and development of advanced boundary conditions [34] [19] [15] [39] [31].

This paper investigates the numerical modelling of the hydrodynamic pressures by discretising the reservoir domain. The reservoir is modelled with two-dimensional plane-strain displacement-based isoparametric quadrilateral solid finite elements (FEs) which are the same as those used to model the dam structure. The investigation includes the upstream reservoir boundary condition (BC) and its distance from the dam, the dam-reservoir and foundation-reservoir interfaces, the material properties of the reservoir, the angle of the dam slope and various types of loading.

2. Statement of the problem

The problem under study is shown schematically in Figure 1. It consists of a dam (1) which retains a large volume of water in the reservoir (2) and rests on the ground (3) which serves as the foundation of the dam. Under seismic or general dynamic conditions, the reservoir induces hydrodynamic pressures on the upstream (US) dam face, A-B. The hydrodynamic pressures induced on the dam depend on the magnitude and the frequency characteristics of the load as well as the properties of the dam, the foundation and the reservoir.

In order to model efficiently and economically the hydrodynamic pressures, the US reservoir is truncated at some distance from the dam, (C-D).

Likewise, the foundation soil is truncated at some distance from the dam and the reservoir (E-F-G-C). On both soil and reservoir boundaries, special absorbing boundary conditions (BCs) need to be applied so that reflection of outgoing waves is avoided.

Therefore, issues that need to be considered for appropriate modelling of the hydrodynamic pressures include: the type of the absorbing BC on the upstream reservoir boundary (C-D), the distance of this BC (B-C), the dam-reservoir and foundation-reservoir interfaces (A-B-C) and the geometric and material properties of the reservoir (A-B-C-D).

3. Ramp Acceleration

This section concentrates on the evaluation of the hydrodynamic pressures on a stiff (undeformable) dam with a vertical upstream face under a ramp acceleration load. The modelling of hydrodynamic pressures on dams was recently investigated by Küçükarslan et al. [23] who represented the reservoir in their analyses with Eulerian fluid elements. Their study considered a stiff dam with a vertical upstream face of height $H=180\text{m}$, following the work of Tsai et al. [37], and examined the evolution of hydrodynamic pressures at the base of the dam (Point B in Figure 1) with time, under the ramp acceleration load shown in Figure 2.

3.1. Upstream boundary condition

Following the work of Küçükarslan et al. [23], analyses were carried out using the dam-reservoir system shown in Figure 3 under the above-mentioned ramp acceleration. The reservoir was modelled with 8-noded isoparametric displacement based solid elements (Figure 19). All the analyses carried out were two-dimensional plane-strain dynamic in the time-domain using the FE software ICFEP (Imperial College Finite Element Program) [29]. The time-integration scheme employed was the generalised α -method of Chung & Hulbert [8] which is able to use numerical damping to selectively filter high frequencies (spectral radius at infinity, $\rho_\infty = 9/11$, see Kontoe et al. [21]) and the time step used was $\Delta t = 0.002\text{sec}$.

In Figure 3, A-B-F-E represents a stiff dam with a vertical upstream face (A-B), A-B-C-D represents the reservoir and F-C-H-G represents a stiff rock underneath the reservoir. The height of the reservoir was $H=180\text{m}$, whereas five values were adopted for the length of the reservoir, $L=540, 960, 1260, 1800$ and 2400m (i.e. $L/H = 3, 5.33, 7, 10$ and 13.33). The width of the stiff

Figure 1: Geometry of the dam-reservoir-foundation system: (1) dam, (2) reservoir, (3) foundation soil, US dam face A-B, US reservoir boundary C-D and foundation boundary E-F-G-C.

Figure 2: The ramp acceleration load used by Küçükarslan et al. [23]

Figure 3: Geometry of the dam-reservoir system considered

dam, W and the thickness of the foundation, T were 18m. The maximum element dimension, d used was 4.5m ($= H/40$). The load was applied as prescribed values of acceleration in the horizontal direction along the bottom and left vertical boundaries, while the displacements in the vertical direction were restricted to be equal to zero. Four boundary conditions (BCs) were considered for the upstream reservoir boundary (CD):

- Free, i.e. zero stress and displacement in the horizontal and vertical directions respectively
- Viscous [26], i.e. dashpots in the horizontal and zero displacements in the vertical direction
- Cone [20], i.e. dashpots and springs in the horizontal and zero displacements in the vertical direction
- Viscous with a constant hydrostatic pressure

As far as the last BC is concerned, it should be noted that, according to Parrinello & Borino [28] (who followed the Lagrangian approach to discretise the reservoir domain), the Sommerfeld radiation condition [34] [35] may be represented by a series of dashpots and a hydrostatic stress at the boundary.

The reservoir water is modelled as a linear elastic material and its properties are the bulk modulus, $K_w = 2.2 \cdot 10^6$ kPa and the shear modulus, $G_w = 100$ kPa. A nominal value of the shear modulus was adopted to avoid numerical instability without causing unrealistic shear wave propagation in the water. The dam and foundation (A-B-C-H-G-F-E in Figure 3) were considered as rigid, therefore the bulk modulus assigned was $K_d = 10^8 \cdot K_w = 2.2 \cdot 10^{14}$ kPa. No material (viscous Rayleigh) damping was specified in the reservoir, dam or foundation domains. The dashpots and the springs are defined by Equations 1 and 2 respectively.

$$\sigma = \rho V_p \dot{u} \quad (1)$$

$$\sigma = \rho \frac{V_p^2}{2r} u \quad (2)$$

where σ is a normal stress on the boundary, ρ is the density of the material the BC has been applied to (i.e. water in this case), V_p is the p-wave velocity of the water ($=1483$ m/s), r is the distance of the boundary from

Figure 4: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Free BC

Figure 5: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Viscous BC

Figure 6: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Cone BC

Figure 7: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Viscous + hydrostatic stress BC

the excitation source (taken as equal to the length, L of the reservoir), u and $\dot{u}$ are the displacement and velocity in the horizontal direction respectively. More details about these BCs may be found in Kontoe et al. [22].

Figures 4 - 7 show the hydrodynamic pressure at the base of the dam (Point B) with time for the four different BCs considered. The hydrodynamic pressures, P_{dyn} , are normalised with respect to $\rho a H$, where ρ is the mass density of water, $a = 1\text{m/s}^2$ is the maximum value of the ramp acceleration load (see Figure 2) and H is the height of the reservoir. The numerical results are compared to the analytical solution which was calculated using the relation of Chopra [6], given by Equation 3.

$$p(y, t) = \frac{4\gamma_w V_p}{\pi g} \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{2n-1} \cos(\lambda_n y) \int_0^t \ddot{u}_g(\tau) J_0[\lambda_n V_p(t-\tau)] d\tau \quad (3)$$

where, y is the vertical distance from the base of the dam, t is the time, γ_w is the unit weight of water, V_p is the p-wave velocity of water, λ_n is the n^{th} wavelength, $\ddot{u}_g(t)$ is the ground acceleration and $J_0(\cdot)$ is a Bessel Function of the first kind of order 0. Equation 3 refers to the hydrodynamic pressures on a rigid dam with a vertical upstream face from a rectangular reservoir under general horizontal ground acceleration.

As it may be observed from Figures 4 - 7, none of the four BCs satisfactorily captured the analytical solution for the hydrodynamic pressures, regardless of the L/H ratio. In all four cases the first half cycle is predicted correctly, whereas the hydrodynamic pressures are generally increasing or decreasing with time for shorter and longer meshes respectively. Moreover, the amplitude of the pressure fluctuations for the Free BC is significantly larger than that of the pressures from the analytical solution. The amplitude of the fluctuations for both the Viscous and the Cone BCs is more comparable to the analytical solution. The results for the Viscous + hydrostatic pressure BC show some spurious peak values of the pressure which occur at different times according to the length of the mesh. The longer the FE mesh, the later the peaks appear. This is believed to be due to the specification of a constant stress at the boundary which causes reflection of the waves back towards the dam.

3.2. Reservoir-dam and reservoir-foundation interface

The analyses were repeated after the introduction of zero-thickness isoparametric interface elements [12] [29] at the reservoir-dam and reservoir-foundation

147 interfaces (ABC in Figure 3). The reason for introducing these elements is
 148 to allow relative movement between the reservoir and the solid material. For
 149 this reason, a very high normal ($k_N = 10^8$ kN/m) and very low shear stiffness
 150 ($k_S = 1$ kN/m) were assigned to these elements so that only shear relative
 151 movement occurs.

152 Figures 8 - 11 show the evolution of the hydrodynamic pressures at the
 153 base of the dam (Point B) with time for the all four BCs. As it may be
 154 observed, the results are now in better agreement with the analytical solution
 155 as the general increase or decrease of the pressures with time disappeared.
 156 However, the amplitude of the fluctuations for the Free BC (Figure 8) is still
 157 larger than that of the analytical solution and the pressures for the Viscous
 158 + hydrostatic pressure BC (Figure 11) still show some spurious peaks. As
 159 far as the Viscous (Figure 9) and the Cone (Figure 10) BCs are concerned,
 160 a good agreement is obtained although there are still some differences and
 161 these are mainly a slight increase of the amplitude and a small shortening of
 162 the period of the pressure fluctuations, which seem to be more severe for the
 163 shorter FE meshes.

164 Moreover, Figures 12 and 13 show the comparison between the analyti-
 165 cal and the numerical (with the Viscous BC) hydrodynamic pressure time-
 166 histories at the base of the dam for $L/H = 3$ and 13.33 respectively. It is
 167 shown that for the shorter mesh, the increase in the amplitude of the pres-
 168 sure fluctuations occurs much earlier than that for the longer mesh and also
 169 the shortening of the period is more pronounced. Figure 14 compares the
 170 computed hydrodynamic pressures for $L/H = 7$ with both the analytical
 171 solution and the numerical solution of Küçükarslan et al. [23] (obtained by
 172 adopting the Eulerian approach). It is shown that the agreement of both
 173 numerical approaches (Eulerian fluid FEs and elastic solid FEs) with the
 174 analytical approach of Chopra is acceptable. Finally, Figure 15 compares
 175 the distribution of the peak hydrodynamic pressure on the upstream face of
 176 the dam for all 5 lengths computed with the Viscous BC with the analytical
 177 solution, showing an excellent agreement (as it occurs during the first half
 178 cycle). It should be noted that the results for the case of the Cone BC with
 179 interface elements are almost identical to the results of the Viscous BC and
 180 are not shown here for brevity.

181 Of course, the ramp acceleration may not be the most suitable load to
 182 determine an appropriate length of the reservoir and consequently the dis-
 183 tance of the boundary from the problem of interest. However, so far it may
 184 be concluded that the Viscous and the Cone BCs perform well as absorbing

Figure 8: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Free BC with interface elements

Figure 9: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Viscous BC with interface elements

Figure 10: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Cone BC with interface elements

Figure 11: Hydrodynamic pressures at the base of the dam due to the ramp acceleration for the Viscous + hydrostatic stress BC with interface elements

Figure 12: Hydrodynamic pressures at the base of the dam due to the ramp acceleration with the Viscous BC for $L/H = 3$

Figure 13: Hydrodynamic pressures at the base of the dam due to the ramp acceleration with the Viscous BC for $L/H = 13.33$

Figure 14: Comparison of the hydrodynamic pressures at the base of the dam due to the ramp acceleration with the Viscous BC for $L/H = 7$ against the analytical solution of Chopra [6] and the numerical results of Küçükarslan et al. [23].

Figure 15: Distribution of the peak hydrodynamic pressure on the face of the dam due to the ramp acceleration for the Viscous BC

BCs on the reservoir and that interface elements with appropriate stiffness should be placed at the interface between the reservoir and the solid domains (dam and foundation). It is therefore concluded that both the “Free” and “Viscous + hydrostatic pressure” BCs are not appropriate absorbing BCs and will not be considered further in this study.

4. Harmonic Acceleration

4.1. Frequency Response

As mentioned earlier, Chopra [6] investigated the hydrodynamic pressures on dams under harmonic loading and expressed the results in terms of a spectrum of forces. In order to examine the performance of the Viscous and Cone BCs under a wide range of loading frequencies, the same analyses were repeated for different loading frequencies according to Equation 4, with amplitude $a_0 = 1 \text{ m/s}^2$ for 40 cycles.

$$a(t) = a_0 \cos \omega t \quad (4)$$

where, $a(t)$ is acceleration, a_0 is the amplitude of the harmonic acceleration, ω is the circular frequency of the load and t is time. The time step used was $\Delta t = T/40$, where $T = 2\pi/\omega$.

Figures 16 and 17 show the ratio of the total hydrodynamic force, F_{dyn} (hydrodynamic pressures integrated over the upstream dam face at steady state conditions, i.e. when the hydrodynamic pressures take the form of a periodic function), over the hydrostatic force, F_{st} , multiplied by the acceleration of gravity, g , against the frequency ratio, $\Omega = \omega/\omega_1$. The parameter ω refers to the circular frequency of the load and ω_1 to the fundamental circular frequency of the reservoir, given by Equation 5 [6].

$$\omega_1 = \frac{\pi V_p}{2H} \quad (5)$$

where H is the height of the reservoir and V_p is the p-wave velocity of water. The values of the ratio $g \cdot F_{dyn}/F_{st}$ at the natural frequencies of the reservoir ($\omega/\omega_1 = 1, 3, 5, \dots$) are not included as the hydrodynamic pressures, F_{dyn} , from the reservoir become infinite due to resonance.

As it may be observed, a generally good agreement is obtained for the whole spectrum of frequencies for both BCs and in fact their performance is almost identical. This suggests that both of these BCs can be applied on

Figure 16: Frequency response of the total hydrodynamic induced by a compressible reservoir due to horizontal harmonic loading with the Viscous BC

Figure 17: Frequency response of the total hydrodynamic induced by a compressible reservoir due to horizontal harmonic loading with the Cone BC

the upstream boundary of the reservoir in order to model the hydrodynamic pressures on dams due to a dynamic load of a wide range of frequencies.

4.2. Water compressibility

The significance of considering the true compressibility of water was highlighted by Chopra [6]. In order to examine the behaviour of the numerical model if water is considered incompressible, the analyses of the previous section with the Viscous BC were repeated for the same loading frequencies. The compressibility of the water, K_w was taken as 100000 times the real value (i.e. $K_w = 2.2 \cdot 10^{11}$ kPa). The resulting force spectrum is shown in Figure 18.

The numerical results for incompressible water agree with the analytical result of Westergaard [40] (Equation 6), which was also confirmed by Chopra [6].

$$\frac{g \cdot F_{dyn}}{F_{st}} = \frac{g \cdot 0.543(a_0/g)\gamma_w H^2}{1/2\gamma_w H^2} = 1.09 \quad (6)$$

The higher hydrodynamic pressures due to resonance for loading frequencies close to the natural frequency of the reservoir cannot be predicted, whereas the pressures for higher frequency loads are overestimated. It is therefore suggested that water compressibility should be taken into account by using the real value of bulk modulus of the water, K_w in order not to underestimate or overestimate the pressures for different loading frequencies.

4.3. Effect of element size

A further investigation was carried out in order to examine the effect of the size of the reservoir elements on the hydrodynamic pressures. When modelling dynamic problems, care needs to be taken in order to appropriately model wave propagation and therefore a sufficient number of nodes should be provided to model the length of the propagating wave [26] [24] [2]. A very fine mesh could in one hand provide sufficient nodes; on the other hand, it would increase the computational cost. The aim is therefore to identify the largest element size that provides accurate results.

For this investigation, a FE mesh of height, $H=90\text{m}$ and length, $L=900\text{m}$ was employed. Eight-noded isoparametric quadrilateral elements [29] were used as shown in Figure 19. The Viscous BC was applied on the upstream reservoir boundary. The harmonic load is described by Equation 4 with a single value of ω so that the ratio $\omega/\omega_1 = 4$. The various element sizes used

Figure 18: Frequency response of the total hydrodynamic induced by an incompressible reservoir due to horizontal harmonic loading

Table 1: Size of the reservoir finite elements considered

CASE	Element size, d [m]	$H/d = \lambda/d$
1	90	1
2	45	2
3	30	3
4	22.5	4
5	18	5
6	15	6
7	11.25	8
8	9	10
9	7.5	12
10	6	15
11	4.5	20

Figure 19: Eight-noded isoparametric quadrilateral element [29]

Figure 20: Distribution of the peak hydrodynamic pressure on the dam face for different element sizes

are listed in Table 1 and are expressed as a fraction of the wavelength, λ . For a monochromatic load (i.e. single frequency, $f = \omega/2\pi$), the wavelength of a p-wave (with velocity, V_p) is given by Equation 7.

$$\lambda = \frac{V_p}{f} \quad (7)$$

Considering the fundamental frequency of vibration of a reservoir, $f_1 = \omega_1/2\pi$ (ω_1 from Equation 5), then the wavelength is given by Equation 8.

$$\lambda = \frac{4H}{f/f_1} = \frac{4H}{\omega/\omega_1} \quad (8)$$

Figure 20 shows the distribution of the peak hydrodynamic pressure on the upstream face of the dam for all 11 values of the element dimension, d . The results from the finest mesh ($\lambda/d = 20$) are considered to be the most accurate because a large number of nodes has been provided to model the wavelength. As it may be observed from that figure, the peak hydrodynamic pressure obtained using elements of size equal or smaller than a fifth of the wavelength deviate significantly from the most accurate prediction, whereas the results for an element dimension equal to a sixth and an eighth of the wavelength are close. It is therefore suggested that if the reservoir is modelled with eight-noded quadrilateral solid elements, the side of these elements should be smaller than a fifth of the wavelength. This is in agreement with earlier work [2] which suggests that the size of 8-noded quadrilateral elements should be between a fifth and a quarter of the wavelength.

5. Random Acceleration

The final part of the investigation considered the behaviour of the reservoir under random acceleration, i.e. under seismic loading. The acceleration record of the 15/11/1975 Mexico earthquake recorded at the rock canyon of La Villita dam (maximum value, $a_m = \text{PGA} = 0.04g$) was used as the input motion and it is shown in Figure 21. This section examines the distance of the upstream reservoir BC from the dam for both stiff and flexible dams with vertical and sloped upstream faces. Also, the value of the reservoir water shear modulus, G_w is investigated.

Figure 21: Accelerations recorded at the rock canyon of La Villita dam in Mexico during the 15/11/1975 Mexico earthquake

Figure 22: Geometry of the dam-reservoir system considered for the random acceleration load

Table 2: Values of the length of the mesh considered

MESH	Length, L/H
1	0.5
2	1
3	2
4	3
5	4
6	5
7	8
8	10
9	12
10	15

Figure 23: Distribution of the peak hydrodynamic pressure on the face of the dam due to the earthquake load - Stiff dam with vertical face ($B/H = 0$)

Figure 24: Distribution of the peak hydrodynamic pressure on the face of the dam due to the earthquake load - Stiff dam with sloped face ($B/H = 3$)

Figure 25: Distribution of the peak hydrodynamic pressure on the face of the dam due to the earthquake load - Flexible dam with vertical face

Figure 26: Distribution of the peak hydrodynamic pressure on the face of the dam due to the earthquake load - Flexible dam with sloped face

275 5.1. Sloped dam

276 The investigations so far have been restricted to stiff (undeformable) dams
277 with a vertical upstream face. However, earth dams are flexible compared
278 to concrete gravity dams and have a sloped upstream face. This section
279 investigates the effect of the slope and the flexibility of the dam on the
280 distance of the upstream reservoir boundary, i.e. the length of the reservoir
281 domain of the FE mesh. Analyses were performed with the acceleration
282 record shown in Figure 21 for a vertical ($B/H = 0$) and a sloped dam (3:1
283 horizontal:vertical, i.e. $B/H = 3$) for various values of the length of the
284 reservoir, L , as listed in Table 2 and shown in Figure 22. As it may be
285 observed from Figure 22, B is the breadth of the dam face and H is the height
286 of the reservoir. For all the analyses, the height of the reservoir, H was equal
287 to 60m and both stiff (undeformable) and flexible dams were considered. For
288 the analyses assuming a flexible dam, material properties have been assigned
289 to the dam that correspond to a shear wave velocity $V_s = 300\text{m/s}$, which is
290 considered a typical value for earth materials. The time step used was $\Delta t =$
291 0.02sec.

292 Figures 23 - 26 show the distribution of the peak hydrodynamic pressure
293 on the upstream face of the dam for a (i) stiff and vertical, (ii) stiff and sloped,
294 (iii) flexible and vertical and (iv) flexible and sloped dam respectively. The
295 hydrodynamic pressures are normalised with respect to $\rho a_m H$, where ρ is
296 the mass density of water, $a_m = 0.04g$ is the maximum value of the input
297 acceleration load (see Figure 21) and H is the height of the reservoir. The
298 analyses for the longest mesh, i.e. $L/H = 15$ are considered as the most
299 accurate because the longer the distance, the smaller the boundary effect
300 on the hydrodynamic pressures. As it may be observed from Figure 23,
301 for a stiff and vertical dam (which is generally the case for concrete gravity
302 dams), the results for $L/H < 5$ differ significantly from the most accurate
303 solution, whereas the rest are reasonably close. Similarly, it can be seen from
304 Figure 26 that for a flexible and sloped dam (which is generally the case for
305 earth dams), the results for $L/H < 1$ differ from the most accurate solution.
306 Consequently, it is suggested that for a stiff dam with a vertical face (such
307 as concrete gravity dams), the upstream boundary should be placed at a
308 distance of more than five times the height of the reservoir. On the other
309 hand, for a flexible and sloped dam (such as earth dams), the upstream
310 boundary should be placed at a distance equal to two to three times the
311 height of the reservoir.

It should be noted that the negative values of the normalised hydrodynamic pressure on the flexible dam in Figures 25 and 26 are a result of the deformation of the dam. They are smaller than the hydrostatic pressure and therefore the total (hydrostatic + hydrodynamic) reservoir pressure is positive (compressive). It should also be noted that for the case of the stiff dam (Figures 23 & 24), the hydrodynamic pressures are smaller for the sloped dam than for the vertical dam which is in agreement with the results of Zangar [44]. However, for the flexible dam (Figures 25 & 26) the hydrodynamic pressures on the sloped dam seem to be larger than on the vertical dam. This is believed to be due to the vibration of the dam body which gives rise to dam-reservoir interaction effects which may amplify the response of the dam. The latter issue has been addressed to a great extent by other researchers [7] [17] [18] and it is beyond the scope of this work.

5.2. Shear stiffness of water

Although the shear stresses in the water are considered negligible, a nominal value of shear modulus is usually employed in numerical analysis to avoid instability of the numerical solution. The critical question is what is the maximum value of shear modulus that can be assigned to water, G_w , without affecting the results significantly and without causing propagation of shear waves. In order to identify this value, further analyses were undertaken for different values of the shear modulus, which was expressed in terms of the bulk modulus of the water, K_w , as listed in Table 3. A stiff dam with a vertical upstream face and $L/H=15$ was considered.

Figure 27 shows the distribution of the peak normalised hydrodynamic pressure on the upstream face of the dam. It is considered that the case with the smallest G_w ($= 0.00005\% K_w$) is the most accurate because it does not allow propagation of shear waves. It is clear from Figure 27 that the results corresponding to a value of the shear modulus larger than $0.5\%K_w$ deviate from the most accurate solution, whereas the results for 0.1% and below are quite close. It is therefore recommended that the value of the shear modulus assigned to reservoir water should be kept below 0.1% of the bulk modulus, K_w , i.e. less than 2000kPa.

6. Conclusions

Several issues related to the numerical modelling of the hydrodynamic pressures on dams by discretising the reservoir domain with displacement-

Table 3: Values of the Shear Modulus, G_w assigned to the reservoir water elements

CASE	Shear Modulus, G_w [kPa]	% K_w
1	220000	10
2	110000	5
3	22000	1
4	11000	0.5
5	2200	0.1
6	1100	0.05
7	220	0.01
8	110	0.005
9	22	0.001
10	11	0.0005
11	2.2	0.0001
12	1.1	0.00005

based isoparametric solid finite elements were explored. Both stiff and flexible dams with vertical and sloped upstream faces were investigated under ramp, harmonic and random acceleration loads. The results have been compared to analytical solutions where available and showed a good agreement. The same approach may be used in analyses of other waterfront structures such as quay walls. The conclusions of this study may be summarised as follows:

- As far as the truncated upstream boundary of the reservoir is concerned, the Viscous and Cone BCs can be employed to absorb the radiating waves. Their performance in a wide range of loading frequencies has been confirmed and it is almost identical. The Free and Viscous + hydrostatic pressures BCs were also examined and did not perform satisfactorily.
- At the interface between the reservoir and the dam and the reservoir and the foundation, interface elements should be used in order to allow relative movement between the water and the solid materials. Appropriate values should be used for the shear and normal stiffness of these elements.
- The reservoir water should not be treated as incompressible in order not to underestimate possible resonance for loading frequencies close

Figure 27: Distribution of the peak hydrodynamic pressure on a stiff dam for different values of the shear modulus of the water, G_w

to the fundamental frequencies of the reservoir and not to overestimate pressures for higher frequency loads.

- The size of the reservoir elements should be smaller than a fifth of the acoustic (p-wave) wavelength of the water.
- For a stiff dam with a vertical upstream face (such as concrete gravity dams), the upstream reservoir boundary should be placed at a distance of five times the height of the reservoir from the upstream face of the dam. This distance may be reduced for a flexible dam with a sloped upstream face (such as earth dams) down to two to three times the height of the reservoir.
- The value of the shear stiffness assigned to water, G_w , should be smaller than 0.1% of the real value of the bulk modulus of the water, K_w .

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496 **Appendix A. Nomenclature**

a	maximum value of ramp acceleration load ($= 1\text{m/s}^2$)
a_o	amplitude of harmonic acceleration load ($= 1\text{m/s}^2$)
a_m	maximum value of random acceleration load ($= 0.04g$)
$a(t)$	acceleration load time history
B	breadth of dam slope face
d	maximum element size

f	frequency of harmonic load
f_1	fundamental frequency of the reservoir
F_{dyn}	total hydrodynamic force on the upstream reservoir face
F_{st}	total hydrostatic force on the upstream reservoir face
g	acceleration of gravity ($= 9.81\text{m/s}^2$)
G_w	shear modulus of water
H	height of the dam
K_d	bulk modulus of the dam materials
k_N	normal stiffness of interface elements
k_S	shear stiffness of interface elements
K_w	bulk modulus of water ($= 2.2 \cdot 10^6 \text{kPa}$)
L	length of the reservoir
P_{dyn}	hydrodynamic pressure at the base of the dam
r	distance of the upstream reservoir boundary from the excitation source
t	time
T	thickness of the dam foundation
$\dot{u}$	velocity of water in the horizontal direction
$\ddot{u}_g$	horizontal ground acceleration
V_p	p-wave velocity of water ($= 1483\text{m/s}$)
V_s	shear wave velocity of the dam materials
W	width of the dam crest
y	vertical distance from the base of the dam
γ_w	unit weight of water ($= 9.81\text{kN/m}^3$)
λ	wavelength
λ_n	n^{th} wavelength
ρ	mass density of water ($= 1000\text{kg/m}^3$)
σ	normal stress on the upstream reservoir boundary
ω	circular frequency of harmonic load
ω_1	fundamental circular frequency of the reservoir
Ω	frequency ratio ($= \omega/\omega_1$)