Stability of shocks relating to the shock ignition inertial fusion energy scheme

C. J. Davie, I. A. Bush, and R. G. Evans

Citation: [Physics of Plasmas \(1994-present\)](#) **21**, 082701 (2014); doi: 10.1063/1.4891666

View online: <http://dx.doi.org/10.1063/1.4891666>

View Table of Contents: <http://scitation.aip.org/content/aip/journal/pop/21/8?ver=pdfcov>

Published by the [AIP Publishing](#)

Articles you may be interested in

[Metrics for long wavelength asymmetries in inertial confinement fusion implosions on the National Ignition Facility](#)

Phys. Plasmas **21**, 042708 (2014); 10.1063/1.4871718

[Mode 1 drive asymmetry in inertial confinement fusion implosions on the National Ignition Facility](#)

Phys. Plasmas **21**, 042702 (2014); 10.1063/1.4870390

[Shock-ignition relevant experiments with planar targets on OMEGA](#)

Phys. Plasmas **21**, 022702 (2014); 10.1063/1.4865373

[Shock ignition target design for inertial fusion energy](#)

Phys. Plasmas **17**, 042701 (2010); 10.1063/1.3385443

[Theory of laser-induced adiabat shaping in inertial fusion implosions: The relaxation method](#)

Phys. Plasmas **12**, 042703 (2005); 10.1063/1.1856481

A collection of five different vacuum equipment units from Pfeiffer Vacuum, including a red turbopump, a silver backing pump, a red leak detector, a red measurement and analysis equipment unit, and a silver chamber and component unit.

Vacuum Solutions from a Single Source

- Turbopumps
- Backing pumps
- Leak detectors
- Measurement and analysis equipment
- Chambers and components

PFEIFFER **VACUUM**

Stability of shocks relating to the shock ignition inertial fusion energy scheme

C. J. Davie,^{a)} I. A. Bush, and R. G. Evans
Imperial College London, London SW7 2AZ, United Kingdom

(Received 23 May 2014; accepted 18 July 2014; published online 4 August 2014)

Motivated by the shock ignition approach to improve the performance of inertial fusion targets, we make a series of studies of the stability of shock waves in planar and converging geometries. We examine stability of shocks moving through distorted material and driving shocks with non-uniform pressure profiles. We then apply a fully 3D perturbation, following this spherically converging shock through collapse to a distorted plane, bounce and reflection into an outgoing perturbed, broadly spherical shock wave. We find broad shock stability even under quite extreme perturbation. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4891666>]

I. INTRODUCTION

Shock Ignition¹ (SI) is a recent variant on fast ignition² of inertial fusion targets which improves target performance by compressing the deuterium-tritium fuel on a lower adiabat (and therefore with less input energy) than is possible using central ignition.¹ Shock ignition occurs as two shock waves heat a central hotspot; the ingoing “ignition” shock is the result of a rapid, late time increase in driver power and the outgoing shock is the rebound of the earlier “compression” shock. This “compression” shock converges, reflects at its minimum radius, and then expands.

Full radiation hydrodynamics calculations,³ in 2D, show that SI is reasonably tolerant of departures from spherical symmetry. This stability of SI has been shown to be present in the underlying hydrodynamics, within the limits of 2D hydrodynamics simulation.⁴

In addition to the symmetry issues present in conventional Inertial Fusion Energy (IFE), the late-time ignition shock also influences stability. The reflected “compression” shock front is subject to shock front perturbation growth through reflection, including the potential collapse of the shock front away from the centre of convergence. The “ignition” shock moves through ablated, Rayleigh-Taylor unstable material, the Rayleigh-Taylor instability being the fastest growing, most disruptive instability in IFE. The uniquely late time of this shock in SI allows perturbations in the ablated material additional time to grow.

This paper examines various idealised symmetry problems related to these unique asymmetries, focusing on both the “compression” and “ignition” shocks. We first examine the behaviour of hydrodynamic shocks moving through perturbations in background material with 2D planar and 2D spherically converging shocks in Secs. II and IV A, respectively, representing the “ignition” shock moving through strongly perturbed material. We then examine pressure perturbations in the material driving 2D spherically converging shocks in Sec. IV B, representing asymmetries in the drive profile. The basic stability against 2D perturbation of shock

shape found by Davie and Evans,⁴ broad conservation of size and shape of perturbation on a converging shock through collapse, bounce, and development into an outgoing shock wave is extended to 3D in Sec. V B. An asymmetry that can only be represented in 3D is then examined in Sec. V C—collapse and reflection of perturbed shock fronts without axial symmetry.

We examine the underlying hydrodynamics using an ideal gas, in the absence of heat conduction and other sources and sinks of energy. Pure hydrodynamics is something of a worst case for these asymmetries—much of the additional physics will be dissipative and favour greater stability against perturbation growth.

We show that shock fronts are robust in the presence of a range of asymmetries; the shock front is broadly stable against moderate perturbation, with only minor deviations from the symmetric behaviour. As asymmetries are added in further fluid variables and in increasing magnitude, the behaviour deviates from the symmetric case. Even under fairly extreme, 3D perturbations in multiple parameters in convergent geometry, the shock front remains robust and transfers with little change through convergence into expansion and recovers its approximate ingoing form.

Our validation for our numerical models begins with comparison with existing solutions for perturbed, 2D planar shocks in Sec. II B. We extend our earlier “piston” verification⁴ for convergent shocks to fully 3D geometries, additionally including 1D spherical and 2D axial geometries, in Sec. V A. The 3D model then has small perturbations applied and are validated against linear solutions in Sec. V B.

Our primary fluid model is the *ZeusMP*⁵ code, developed and widely used in the astrophysics community, set to follow the behaviour of an ideal fluid. We also present specific results from *Odin*, a multi-material, arbitrary equation of state, 2D radiation magnetohydrodynamics ALE code, set to follow an ideal fluid here, recently developed in collaboration between the Centre for Fusion, Space and Astrophysics at the University of Warwick and the Plasma Physics Research Group at Imperial College London. *Odin* has a more sophisticated hydrodynamics solver that shows more subtle features of the flow, particularly for very small perturbations.

^{a)}Electronic mail: c.davie10@imperial.ac.uk

Although our simulations use specific length and time scales, these results can be rescaled using Euler and Mach rescaling; the Euler equations of hydrodynamics are scale invariant, directly exploited by the analytic solutions used to verify these models. The behaviours will be consistent when appropriately rescaled, as in a wind tunnel, maintaining the same Mach and Euler numbers.

II. BACKGROUND DENSITY VARIATIONS WITH 2D PLANAR SHOCKS

Perturbed material ahead of a strong shock is of special interest for shock ignition—the strong, late-time “ignition” shock, unique to SI, travels through ablated material, in the wake of a Rayleigh-Taylor unstable configuration.⁶ The late time of the final shock allows the instabilities in the ablated material additional time to evolve and grow; these further seeding RTI growth in the hotspot.

This shock must thus traverse a larger distance through the rising density gradient of ablated material from the critical density to the density of the assembled capsule. Material perturbations will lead to locally varying, perhaps negative, density gradients. These density gradients alter the shock propagation speed and perturbation growth rates.

To examine the behaviour of shock fronts in the presence of background perturbations, we perturb the density profile in front of a planar shock.

A. Background density slope

Shocks in uniform media with well-behaved equations of state are known to be stable. However, shocks going up and down density gradients can be subject to refraction for non-normal incidence, and stability is also changed. In SI, the need for shock collision to heat the fuel for ignition requires the shock to be transmitted intact. Hence, we examine density jumps small enough that the shock is transmitted—sufficiently large density increases cause the shock front to reflect and a rarefaction wave to be transmitted, while sufficiently large density decreases cause the shock front to become a rarefaction wave.⁷ We choose an exponential slope for the density gradient to preserve the self-similarity of the shocks, in line with the results of Chevalier and Gardner.⁸

Figure 1 shows the initial density profile of the simulation. The shock has Mach number $M=12$ to visibly show the change in stability and to propagate up a significant density gradient. The perturbation is sinusoidal with initial amplitude of 2.4×10^{-5} cm. We then introduce a density ramp in front of the shock front of the form

$$\rho(x) = \rho_0 e^{x/x_p}, \quad x > 0. \quad (1)$$

The shock is launched at $x = -x_0$, travelling towards $x = 0$, the density gradient being either positive or negative depending on the sign of the scale length x_p . The background density is initialised as $\rho_0 = 1.0$ g/cc, $x_0 = 1.9 \times 10^{-3}$ cm, and the scale length $x_p = \pm 1.3 \times 10^{-3}$ cm for positive and negative gradients, respectively.

Figure 2 shows the change in growth rate due to density gradients. Shocks moving down sufficiently negative density

FIG. 1. Initialised density ramp along a single y line, with a negative gradient and scale length $x_p = 1.3 \times 10^{-3}$ cm, Eq. (1). The position of the shock front varies sinusoidally with y .

gradients are less stable; the crests of the perturbations move through lower density regions ahead of the rest of the shock, accelerating, enhancing the perturbation. Shocks moving up positive density gradients decelerate, increasing stability; the crests of the perturbation, ahead of the rest of the shock, move through higher density regions, and thus decelerate first, further decreasing the perturbation size.⁹

This mechanism for increased damping gives the “shape fitting” feature for spherical shock fronts converging on non-spherical cores in Sec. IV A.

B. Richtmyer-Meshkov instability

Extreme density gradients become discontinuities and can give rise to the Richtmyer-Meshkov instability (RMI). We reproduce the results of Brouillette¹⁰ for illustration and code validation.

The initial shock front collides with this density discontinuity or “interface,” a shock then being transmitted into the high density material and a separate shock reflected into the

FIG. 2. RMS amplitude of a shock front perturbation. Shocks moving up positive density gradients are more stable, shocks moving down negative density gradients less so.

FIG. 3. Initial conditions for the Richtmyer-Meshkov instability. The arrow indicates the movement of the shock front at $y \approx 2.4$ cm, colliding with the perturbed interface at $y = 0$, $t = 0$.

low density material. In RMI, this density interface is perturbed, shown in Figure 3. RMI occurs as the interface is accelerated by the shock front and the perturbation on the interface grows, “bubbles” of the low density material protruding into the high density material, while “spikes” of the high density material move into the low density material. The RMI growth rate increases then slows, the initial most rapid growth partially stabilised by the onset of the Kelvin-Helmholtz instability on the interface, due to the shear between the flow of bubbles and spikes, shown in Figure 4.

The transmitted shock front, of interest to this study, initially takes on the shape of the perturbation, the shock front then stabilises, the size of this perturbation decaying. The perturbation size is supported by sonic pressure waves transmitted between the interface and the shocks,¹¹ diminishing less rapidly than a shock front perturbation alone. The

FIG. 4. Late time behaviour of the Richtmyer-Meshkov instability on the interface between the regions of different density. The vortices indicate the onset of Kelvin Helmholtz instability on interface.

FIG. 5. Amplitude of the perturbations on the shock front in RMI.

amplitude of the perturbed shock front over the simulation time is shown in Figure 5.

To verify the behaviour of this instability, the amplitude and growth rate of both the shock front and interface are found. The initial shock has Mach number $M_s = 1.2$, amplitude size $\nu_0 = 0.24$ cm, wavelength $\lambda = 3.75$ cm, and the initial densities either side of the interface are of air and SF_6 at atmospheric pressure.

The interfacial growth rate is shown in Figure 6 and is in close agreement with results presented by Brouillette,¹⁰ specifically Figure 6 (right), with the position of the initial peaks and the final growth rate matching closely. Since we are reproducing an earlier calculation, these parameters do not match SI but can be Euler scaled to more relevant conditions.

The size of perturbations on the shock front oscillates and decays to a stable limit, shocks being generally stable. The perturbations on the interface grow monotonically; the growth rate decreases with time. This is somewhat representative of the general behaviour of our results—the shock fronts stabilise and the material behind the shock front is distorted, this perturbation growing monotonically, usually slowly relative to the time scales and sizes of the systems.

FIG. 6. Growth rate of the interfacial Richtmyer-Meshkov instability.

C. Background density modulation

To examine this behaviour of shocks moving through perturbed material, we initialise a shock of Mach number $M=4$ moving through a background density of $\rho=1\text{ g/cc}$ towards density modulations normal to the shock front, shown in Figure 7. This is an idealised situation related to RTI; the bubbles and spikes in RTI leave adjacent tracks of modulated density material during RTI growth; here, these perturbations are exaggerated for maximum effect.

The region of the shock front that first strikes the low density material accelerates ahead of the shock front moving through the high density region. This difference in position quickly saturates, reaching a stable limit (Fig. 8).

This surprising result is a hydrodynamic effect—the regions of different density are shock accelerated to different velocities, the Kelvin-Helmholtz instability forms on the interface between these regions, redirecting the flow of material and, in the case of square modulation of the interfaces, secondary shocks form behind the shock front. Additionally, the stability of perturbed shock fronts (Sec. II B) acts against continued growth, increasing the velocity of convex troughs and reducing the velocity of concave crests.

This stable limit varies strongly with the depth of the density modulation, shown in Figure 9, and weakly with shape. The perturbation size, $\Delta y = [y_s] - [y_s]$, is almost directly proportional to $\Delta\rho/\rho = ([\rho_{\text{mod}}] - [\rho_{\text{mod}}])/\rho_0$, y_s being the position of the shock front, ρ_{mod} is the modulated density profile, and ρ_0 is the background density. Δy is given in units of wavelength of the perturbation λ as this should be subject to appropriate Euler scaling for a given shock strength.

Once the shock front leaves the perturbed material, the perturbations oscillate and decay, while the material behind the shock remains significantly perturbed, in analogy with the shock fronts in RMI.

FIG. 7. Initial conditions of the background density modulation. The arrow indicates direction of movement of the shock front at $y=0$, moving towards the modulated material.

FIG. 8. Shock front moving through the modulated material. The arrow indicates direction of movement of the shock front. The onset of the vortex is a result of shear flow due to pressure and density gradients behind the shock.

III. CODE INITIALISATION FOR SPHERICALLY CONVERGING SHOCKS

The scale invariant self-similar solution of Guderley¹² for strong shocks passing through collapse cannot be transferred to a computational model with a finite grid since it relies on an infinite medium behind the collapsing shock. We construct an appropriate piston to drive the shock, closely recreating the Guderley solution and then perturb this piston and examine the evolving behaviour.⁴

We provide an outline here of the construction of the symmetric piston, a more complete explanation can be found in Davie and Evans.⁴ We first make use of a result by Ribeyre *et al.*¹³ who give an approximation to the velocity of a Lagrangian fluid element following the passage of a strong, spherically converging shock wave. This is used as a boundary condition in a 1D spherical fluid model,¹⁴ set to follow an ideal fluid with no dissipation apart from the shock itself. We use this 1D model to establish the evolved behaviour of an imploding spherical shell of finite thickness, giving us an initial condition, the piston, easily transferred to other

FIG. 9. Shock front perturbation size Δy in units of wavelength of the perturbation $\lambda = 5.43 \times 10^{-4}\text{ cm}$ compared to $\Delta\rho/\rho$ the peak-trough density difference, for square-wave and sinusoidal perturbations.

models. The inner surface of the spherical piston approximates the convergent shock—the piston thickness must be large enough such that the rarefaction wave from the outer surface of the piston does not affect the shock dynamics. Even if the model is not initialised precisely with the Guderley self-similar solution, the evolution of the shock front will asymptotically approach the self-similar result at later times.⁶

For our 2D converging simulations, we return solely to *ZeusMP*. We use a cylindrical $R - z$ grid to follow the implosion instead of an $R - \theta$ grid; the $R - \theta$ mesh gives rise to a very large interchange of the radial and angular components of velocity, as material passes close to the origin. Similarly for our 3D simulations, we use an $x - y - z$ rather than an $R - \theta - \phi$ or $R - z - \phi$ grid. It is a clear requirement that all results presented here are “grid converged” in the sense that our conclusions are unchanged if the resolution of the mesh is doubled.

ZeusMP is initialised with the imploding piston of material described above and, in the spherically symmetric case (within the limitations of mapping onto $R - z$ and $x - y - z$ meshes), we reproduce the results of the equivalent 1D spherical model. A least squares fit is calculated for the radial profile of fluid variables $f(r)$ in the 1D model and is then applied to *ZeusMP* from the shock front position r_s , to the position of the outer Lagrangian element r_L

$$f(r) = \sum_{n=0}^N a_{f,n} r^n, \quad (2)$$

where $a_{f,n}$ is a fitting parameter of order n . f represents the fluid variables, the mass density ρ , radial velocity v_r , and energy density e . A quadratic fit, $N=2$, was found to be sufficiently good. Here, r has its usual meaning, the radial coordinate; in 2D $r^2 = R^2 + z^2$, in 3D $r^2 = x^2 + y^2 + z^2$. The regions in front of and behind the piston, $r < r_s$ and $r > r_L$, are initialised with the background density $\rho(r) = \rho_0$ and energy $e(r) = e_0$, and zero velocity $v_r(r) = 0$.

The perturbations vary depending on the scenario of interest—in this paper, we present the results for 2D background density perturbations in Sec. IV A and pressure perturbations in Sec. IV B. In Sec. V B, we present spatial shock front perturbations in 3D. We then present a further 3D effect, non-central collapse with the initialisation of non-central momentum vector perturbations in Sec. V C.

IV. 2D SPHERICALLY CONVERGING SHOCKS

A. Perturbed background

Shocks driven through a perturbed background are more significant in shock ignition than in conventional IFE—the final “ignition” shock arrives much later, allowing additional time for perturbation growth in the imploding material.

Additionally, the “compression” shock, after collapse and reflection, travels through material previously compressed. 2D simulations show reflected shocks moving through material previously compressed,⁴ as do the 3D results in Sec. V.

We examine a converging shock moving through non-uniform background, in 2D initialising the spherically converging piston and perturbing the background density around the centre of convergence. The piston is in the spherically symmetric configuration from Sec. III. The density is perturbed with an elongated Gaussian profile in order to deliberately destroy the spherical collapse

$$\rho(r < r_s) = \left(1 + \Delta_\rho e^{-\left(\frac{r'(R,z)}{r_\rho}\right)^2}\right) \rho_0, \quad (3)$$

$$r'(R, z) = \begin{cases} \sqrt{R^2 + (z - \Delta z)^2}, & z > \Delta z \\ R, & \Delta z > z > -\Delta z \\ \sqrt{R^2 + (z + \Delta z)^2}, & -\Delta z > z, \end{cases} \quad (4)$$

where Δ_ρ describes the peak of the density perturbation, r_ρ is the width of the perturbation, and Δz is the z elongation of the perturbation.

This background perturbation and subsequent behaviour are shown in Figure 10. The region of the shock that first

FIG. 10. Density profile of a shock front, highlighted with the green dashed lines and direction shown with arrows, converging on a perturbed background described by Eq. (3), with $\Delta_\rho = 10$, $\Delta z = 0.12r_i$, $r_\rho = 0.2r_i$ and r_i is the initial radius of the shock front.

FIG. 11. Perturbed pressure profiles, with the shock front shown by the green dashed line and direction shown with arrows, initialised at $t=0$, described in Eq. (5) with $\Delta_\epsilon=0.3$ and $l=5$. At the later time, the higher pressure regions have accelerated the shock front, causing the shock front to acquire the shape of the pressure perturbation.

strikes the density ramp is accelerated less strongly. The rest of the shock accelerates ahead, until it too hits the density ramp, similarly reducing the acceleration. The shock front thus conforms to the shape of the density perturbation.

Broadly, shock fronts conform to the shape of a distorted background with increasing density—this behaviour is likely to be seen in the converging “ignition” shock. The counter to this, that shocks travelling down distortions with decreasing density will form counter to the shape of the background, could perhaps be a source of instability in SI—approaching collision, the reflected “compression” and converging “ignition” shocks will be at similar locations and as such have opposite density gradients.

B. Pressure perturbations

Both direct drive and indirect (X-ray) drive have imperfect symmetry, either due to the finite number and imbalance of laser beams or, possibly to a lesser extent, due to the finite number of hotspots within a hohlraum. Thus, different energies and pressures drive the implosion as a function of angle, with more rapid shock speeds in the regions where greater energy is deposited.

This pressure perturbation is most severe at early times—the shock wave moves away from the region of energy deposition, losing the additional acceleration from drive asymmetry as this energy variation diffuses away. As the shock converges, the energy density directly behind the shock also increases.

To examine the underlying physics, we perturb the energy profile of the piston described in Sec. III at initialisation. We modify Eq. (2), such that $a_{\epsilon,n} \rightarrow a_{\epsilon,n,l}(\theta)$

$$a_{\epsilon,n,l}(\theta) = (1 + \Delta_\epsilon Y_l(\theta)/N) a_{\epsilon,n,0}, \quad (5)$$

where Δ_ϵ defines the size of the energy perturbation, $Y_l(\theta)/N$ is the azimuthally symmetric spherical harmonic of mode number l , rescaled such that the poles are at ± 1 , and $a_{\epsilon,n,0}$ is the unperturbed energy fit parameter.

Figure 11 shows this energy perturbation. The shock moves more rapidly in regions of high pressure, the dependence on angle defined by $Y_l(\theta)/N$, causing the shock front to acquire this shape. The spatial perturbation in turn sets up pressure perturbations, and after a certain convergence the growth of the perturbation matches closely the behaviour of purely spatial perturbations.⁴ The energy driving the perturbation oscillation increases with convergence, tending to dominate over the initial conditions. The proportional size of the perturbation $\delta r/r$ tends to a limit that depends on the initial mode l of the perturbation, shown in Figure 12, with $\Delta_\epsilon=0.3$ and $l=8, 12$, and 16 .

V. 3D SPHERICALLY CONVERGING SHOCKS

Simulations of IFE targets are traditionally run in two spatial dimensions, in cylindrical $R-z$ geometry to exploit the natural symmetry of hohlraum and target, avoiding the additional computational cost in 3D. Some behaviours are only accurately captured in 3D, including some asymmetries

FIG. 12. Amplitude of the perturbation for a converging shock front initialised with a pressure perturbation of mode l . The straight lines on the right are the stable limits for the spatial perturbations from our previous work.⁴

growing more rapidly in full 3D,¹⁵ and significant, detrimental effects in IFE can only be captured using full 3D¹⁶ due to their non-axisymmetric nature.

After benchmarking *ZeusMP* in 3D $x - y - z$ geometry against perturbed shock fronts in 2D $R - z$ geometry and validating *ZeusMP* in 3D against the analytic spherically symmetric Guderley solution,¹² we extend our earlier results⁴ to fully 3D perturbations. We confirm the reproduction of the linear theory for small perturbations and the conservation of shape and size of the perturbations through reflection, in azimuthally asymmetric initial configurations.

We then examine non-axially symmetric collapse of the shock front—a 3D effect of considerable interest but impossible to tackle with 2D simulations. NIF diagnostics appear to show non-axially symmetric compression of the target,¹⁷ likely contributing to the lower than expected neutron yield and ρr . This 3D effect, momentum pointing away from the centre of collapse and away from the z -axis, cannot be captured in 2D simulations.

ZeusMP is initialised with relatively low mode and large size perturbations due to the significant restriction in resolution with equivalent computing resources in 3D from 2D.

We find that the main results extend to 3D—broad conservation of shape and size of perturbations on a shock front through collapse, bounce and development into an outgoing front, even with large 3D perturbations in multiple parameters resulting in non-axially symmetric collapse.

A. Verification of *ZeusMP* in 3D against 2D

ZeusMP is initialised in 3D with a spherically symmetric piston within the limits of a finite grid, as described in Sec. III. The results overlay the analytic Guderley solution within plotting accuracy, shown in Figure 13, only diverging after the shock front moves into rarefied material.

Spatial perturbations are then applied. Similarly to 2D,⁴ these perturbations are applied by modifying Eq. (2), the method for transferring the fluid variables, such that $r \rightarrow r_l(\theta)$

FIG. 13. The position of the shock front at a given time for the analytical solution and a spherically symmetric piston in *ZeusMP* in 2 and 3D.

$$r_l(\theta) = r_0(1 + \epsilon_0 Y_l(\theta)/N), \quad (6)$$

where r_0 is the unperturbed radial co-ordinate, ϵ_0 is the size of perturbation. $Y_l(\theta)/N$ is the azimuthally symmetric spherical harmonic of mode number l , rescaled such that the poles are at ± 1 .

Figure 14 shows the perturbation growth for an azimuthally symmetric perturbation in *ZeusMP* in 2D $R - z$ geometry and in 3D $x - y - z$ geometry for equivalent grid resolutions. The perturbation growth is similar in both, only significantly diverging at high convergence during reflection, the region where shock front finding is most challenging, and then returns to similar behaviour as the shock front expands.

B. 3D spatial shock front perturbations

The spatial perturbations are extended to 3D, to include perturbations in the azimuth ϕ . We confirm that results found in 2D⁴ extend to 3D, most importantly the preservation of shape and size of the perturbation through reflection, and that the 3D behaviours also match the linear theory of K. Evans¹⁸ for small perturbations and establish a limit for perturbation growth $\delta r/r|_{l,m}$ that depends on polar mode l number and seek to show a dependence on azimuthal mode number m , but do not conclusively show any dependence—at most it is a weak effect.

Similarly to Eq. (6), these perturbations are applied by modifying Eq. (2), the method for initialising the piston, such that $r \rightarrow r_{l,m}(\theta, \phi)$

$$r_{l,m}(\theta, \phi) = r_0(1 + \epsilon_0 Y_{l,m}(\theta, \phi)/N), \quad (7)$$

$Y_{l,m}(\theta, \phi)/N$ is the tesseral spherical harmonic of polar mode number l and azimuthal mode m , rescaled such that the maxima/minima are at ± 1 . The perturbed piston depends on angular position, shown in Figures 15 and 16 with perturbations of modes $l=4$, $m=2$ and size $\epsilon_0=0.2$.

FIG. 14. Amplitude of the perturbation for a reflected shock front initialised with an azimuthally symmetric spatial perturbation, described in Eq. (6) with $\epsilon_0=0.2$ and $l=4$, benchmarking *ZeusMP* in 3D with 2D.

FIG. 15. Density slice through the spatially perturbed piston at initialisation. The slice runs through the centre of the sphere, normal to the Y direction. The piston is mapped out in a complete sphere around the perturbed shock front shown in Figure 16.

1. Analytical verification

We confirm the behaviour of the model matches the linear theory for small perturbation growth of K. Evans,¹⁸ as in 2D.⁴

Figure 17 shows the perturbation size compared to the linear theory. The computed oscillatory behaviour and growth initially match the analytic solution well, eventually diverging, due to two effects: first, the analytic theory is valid only for “small” perturbations and, second, the perturbation initialized via Eqs. (2) and (7) does not precisely match a single radial eigenmode, consistent with the azimuthally symmetric results.⁴ The solutions only match before the first peak of oscillation, due to lower mode perturbations having a lower rate of oscillation.

K. Evans finds a linear solution for perturbation amplitude $\delta r'$ as a function of radial position r' . These are self-

FIG. 16. The perturbed shock front at initialisation. Driving this strong shock front spherically inwards is a piston of energy, density, and velocity, described by Eqs. (2) and (7). A slice through the density piston is shown in Figure 15.

FIG. 17. Amplitude of the perturbation for a converging shock front initialised with a spatial perturbation, described in Eq. (7), compared to the linear theory of K. Evans,¹⁸ with $l=8$, $m=4$, and $\epsilon_0=0.1$.

similar, allowing rescaling, such that the theoretical perturbation amplitude $\delta r = A\delta r'$ and radial position $r = Br'$, where A and B are fitted constants.

$\delta r'$ is not a function of the azimuthal mode number m , the linear perturbation size being defined by the polar mode number l , consistent with these results, Figure 17. Additionally, as the shock front converges and reflects, the additional modes seeded are predominantly of different l modes and not of different m modes. The azimuthal mode numbers m are isolated for each l mode, perhaps related to the perturbation size $\delta r'$ not being a function of this azimuthal mode. Analytical exploration of this is beyond the scope of this paper, but would involve extending the linear theory to non-linear terms and including mode-mode coupling.

2. Stable perturbation size

We find the existence of the stable perturbation size $\delta r/r|_{l,m}$ for $m \neq 0$ and compare this to $m=0$. We limit this exploration to the single $l=12$, $m=4$ mode as the computational costs are prohibitive. Assuming the 3D stable limit broadly follows those in 2D, $\delta r/r|_l \propto 1/l^2$ (Ref. 4) is a sufficiently rapid decrease in size with respect to l that finding a stable limit requires a large 3D resolution. We exploit the octant-symmetry of modes $l=2n$ and $m=4o$, with n and o as positive integers, only simulating an octant of the 3D grid and exploit that higher modes oscillate more rapidly, allowing observation before perturbation size is no longer properly resolved. We show one and one-half oscillations, after which the behaviour is no longer resolved.

Figure 18 shows the stable limit for $l=12$ and $m=4$, the larger perturbations shrinking and smaller perturbations growing. The stable limit is found as $\delta r/r|_{l=12,m=4} = 0.062 \pm 0.003$, the azimuthally symmetric case $\delta r/r|_{l=12,m=0} = 0.059 \pm 0.004$. These results are comparable, and can only be further studied at great computational cost. We do not pursue this here.

C. Skewed shock front collapse

By initialising the momentum with perturbations pointing away from the centre of collapse, we cause the shock front to collapse to a dispersed 3D region. This cannot be captured in our 2D $R-z$ calculations.

FIG. 18. The stable limit for relative perturbation size for mode $l=12$, $m=4$, varying perturbation size ϵ , Eq. (7). The stable limit $\delta r/r|_{l=12,m=4} = 0.062 \pm 0.003$.

The piston is perturbed with both spatial and momentum perturbations. Spatial perturbations are applied as with Eq. (7), such that $r \rightarrow r_{l,m}(\theta, \phi)$. This leads to a form of the fluid variables that depends on the angular position

$$f(r, \theta, \phi) = \sum_{n=0}^N a_{f,n} r_{l,m}(\theta, \phi)^n. \quad (8)$$

The momentum perturbations are similarly applied by modifying the velocity profile such that

$$a_{v_j} \rightarrow (1 + \Delta_{v_j,l,m}(\theta, \phi)) a_{v_j,0}, \quad (9)$$

$a_{v_j,0}$ is the unperturbed velocity fit parameter, j represents the r , θ and ϕ components of the velocity vector. $\Delta_{l,m}(\theta, \phi)$ has a less obvious form than the spatial perturbations, as we seek to have the shock front collide entirely asymmetrically. We force this at initialisation by having the velocity vectors at the crests of the spatial perturbation point away from the centre of collapse.

In order not to change the zero-order solution we use a perturbation that conserves both total momentum and total angular momentum is implemented. We also conserve the magnitude of the velocity vector at every point with regards to the original velocity vector, $|\vec{v}| = |\vec{v}_0|$. It is also required that the momentum profile has no discontinuities.

These considerations lead to a fairly involved form of $\Delta_{v_j,l,m}(\theta, \phi)$. The form given satisfies the above conditions, but is not unique and is specific for $l=4$ and $m=2$

$$\Delta_{v_{\phi},4,2}(\theta, \phi) = \Delta_{v_{\phi},0} \frac{Y_{4,2}(\theta, \phi)}{N} \Delta'_{v_{\phi}}(\theta, \phi), \quad (10)$$

$$\Delta'_{v_{\phi}}(\theta, \phi) = \begin{cases} 1, & \phi_0 \leq \phi \leq \phi_1, & \theta_1 \leq \theta \\ \frac{1}{\sqrt{2}}, & \phi_1 \leq \phi \leq \phi_2, & \theta_0 \leq \theta \leq \theta_1 \\ -1, & \phi_2 \leq \phi \leq \phi_3, & \theta_1 \leq \theta \\ -\frac{1}{\sqrt{2}}, & \phi_3 \leq \phi \leq \phi_0, & \theta_0 \leq \theta \leq \theta_1 \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

$$\Delta_{v_{\theta},4,2}(\theta, \phi) = \Delta_{v_{\theta},0}(\theta, \phi) \frac{Y_{4,2}(\theta, \phi)}{N} \Delta'_{v_{\theta}}(\theta, \phi), \quad (12)$$

$$\Delta'_{v_{\theta}}(\theta, \phi) = \begin{cases} 1, & \phi_0 \leq \phi \leq \phi_1, & \theta \leq \theta_0 \\ \frac{1}{\sqrt{2}}, & \phi_1 \leq \phi \leq \phi_2, & \theta_0 \leq \theta \leq \theta_1 \\ -1, & \phi_2 \leq \phi \leq \phi_3, & \theta_1 \leq \theta \\ -\frac{1}{\sqrt{2}}, & \phi_3 \leq \phi \leq \phi_0, & \theta_0 \leq \theta \leq \theta_1 \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

$$\Delta_{v_r,l,m}(\theta, \phi) = \sqrt{1 - \{\Delta_{v_{\theta},l,m}(\theta, \phi)^2 + \Delta_{v_{\phi},l,m}(\theta, \phi)^2\}}. \quad (14)$$

θ_i and ϕ_j are the meridians and parallels of latitude, respectively on which the tesseral harmonics vanish, index $i, j=0$ represents those closest to $\theta, \phi = 0$. In the case of θ_i , the bald areas near the poles where $Y_{l,m \neq 0}(\theta, \phi)$ vanish are not included.

In this case, we set $\Delta_{v_{\phi},0} = \Delta_{v_{\theta},0} = \sqrt{\Delta_0^2/(1 + \Delta_0^2)}$, $\Delta_0 = 0.3$ and $\epsilon_0 = 0.2$. The shock front with spatial perturbation $\epsilon_0 = 0.2$ is shown in Figure 16 and a slice through of the corresponding perturbed density piston is shown in Figure 15. The momentum perturbation with $\Delta_0 = 0.3$ and Eqs. (9)–(14) are shown in 3D in Figure 19, and θ, ϕ projections of v_{ϕ} and v_{θ} are shown in Figures 20 and 21, respectively.

When this configuration is run the shock does not collapse to a central point but collapses in a more sheet-like configuration, shown in Figure 22, a density cut-through, achieving the diffuse area of collapse.

Figure 23 shows the shock reflected despite the non-axially symmetric collapse, but is more distorted than without momentum perturbations—the minimum and maximum position of the shock front at each time is greater during collapse and after reflection, as shown in Figure 24. This shows, in the presence of severe momentum perturbations leading to an entirely distorted collapse, the reflected shock recovers

FIG. 19. Vector field plot of the momentum perturbations on a half-sphere of the piston at initialisation, described by Eqs. (9)–(14), $\Delta_0 = 0.3$ and $\epsilon_0 = 0.2$. $|Dv|/|vr| = \sqrt{v_{\theta}^2 + v_{\phi}^2}/v_r$ is the absolute size of the momentum perturbations away from the radial direction.

FIG. 20. Function perturbing the ϕ velocity v_ϕ , $\Delta_{v_\phi,4,2}(\theta, \phi)$, Eq. (10). FIG. 21. Function perturbing the θ velocity v_θ , $\Delta_{v_\theta,4,2}(\theta, \phi)$, Eq. (12).

FIG. 22. A density cut through at the time of collapse of the shock. The high density region at the centre is the region of collapse.

FIG. 23. Surface plot of the shock front with momentum perturbations after reflection.

FIG. 24. The difference between minimum and maximum shock front radius during collapse and expansion, in the presence and absence of momentum perturbations. The vertical line shows the time of collapse. The difficulty of finding the shock front after reflection is increased with momentum perturbations.

intact, the deleterious effects simply give a lower density, corresponding to a lower compression.

VI. CONCLUSIONS

We have shown that the tolerance of shock ignition to moderate asymmetries in drive pressure, shock position, and background density are all underpinned by the remarkable stability of fluid shock waves. Converging shock waves tend to exhibit a maximum value of $\delta r/r$ depending on mode number and even if initialised with larger amplitude they will relax back to the lower value. After reflection at the centre of convergence, asymmetric shock waves tend to recover the same shape as the ingoing shock at the same position. Shock velocity and strength before and after reflection broadly follows the symmetric Guderley solution.

Planar shock waves are tolerant of quite extreme non-uniformity of background density and show an asymptotically steady state deformation which decays as soon as the

shock returns to a uniform medium. Shock waves propagating towards higher densities naturally conform to the shape of the dense core, reducing the temporal dispersion of the collision of ingoing and outgoing shock waves.

Many of these asymmetries share a stable limit in size of perturbation; when travelling through density modulations shock perturbations grow to a stable limit. When the energy driving the shock is modulated in convergent geometry perturbations grow to a stable limit, if the initial piston geometry is perturbed, a similar stable limit appears.

These stable limits occur in all cases that we have investigated, perturbations either of position or drive pressure, single and multi-mode perturbations, and even a damping of perturbations which are initialised with amplitudes larger than the stable limit. The damping at sufficiently large amplitude seems to be associated with many features of shock propagation in ideal gases and may be of a geometric origin and thus strongly non-linear.

All results are robust to 2D and 3D asymmetries and physics which is not included in our simple hydrodynamic model will largely be dissipative and thus tend towards greater symmetry.¹¹

These results show the robustness of shock propagation in shock ignition against modest departures from symmetry is based in the underlying hydrodynamics, and that this robustness is fully 3D.

ACKNOWLEDGMENTS

We thank AWE for supporting C.J.D.'s doctoral research and EPSRC for their financial support for I.A.B. and R.G.E. We also thank the Imperial College High Performance Computing service for their provision of the parallel computing facilities.

¹¹R. Betti, C. D. Zhou, K. S. Anderson, L. J. Perkins, W. Theobald, and A. A. Solodov, "Shock ignition of thermonuclear fuel with high areal density," *Phys. Rev. Lett.* **98**(15), 155001 (2007).

- ²M. Tabak, J. Hammer, M. E. Glinsky, W. L. Kruer, S. C. Wilks, J. Woodworth, E. M. Campbell, and M. D. Perry, "Ignition and high gain with ultrapowerful lasers," *Phys. Plasmas* **1**(5), 1626–1634 (1994).
- ³S. Atzeni, A. Schiavi, and A. Marocchino, "Studies on the robustness of shock-ignited laser fusion targets," *Plasma Phys. Controlled Fusion* **53**(3), 035010 (2011).
- ⁴C. J. Davie and R. G. Evans, "Symmetry of spherically converging shock waves through reflection, relating to the shock ignition fusion energy scheme," *Phys. Rev. Lett.* **110**(18), 185002 (2013).
- ⁵J. M. Stone and M. L. Norman, "Zeus-2d: A radiation magnetohydrodynamics code for astrophysical flows in two space dimensions. I—The hydrodynamic algorithms and tests," *Astrophys. J. Suppl. Ser.* **80**(2), 753–790 (1992).
- ⁶S. Atzeni and J. Meyer-Ter-Vehn, *The Physics of Inertial Fusion* (Oxford Science Publications, 2004).
- ⁷F. V. Shugaev and L. S. Shtemenko, *Propagation and Reflection of Shock Waves* (World Scientific, 1998).
- ⁸R. A. Chevalier and J. Gardner, "The evolution of supernova remnants. 11. Models of an explosion in a plane-stratified medium," *Astrophys. J.* **192**, 457–464 (1974).
- ⁹R. A. Chevalier, "The stability of an accelerating shock wave in an exponential atmosphere," *Astrophys. J.* **359**, 463–468 (1990).
- ¹⁰M. Brouillette, "The Richtmyer-Meshkov instability," *Annu. Rev. Fluid Mech.* **34**, 445–468 (2002).
- ¹¹A. L. Velikovich, J. P. Dahlburg, A. J. Schmitt, J. H. Gardner, L. Phillips, F. L. Cochran, Y. K. Chong, G. Dimonte, and N. Metzler, "Richtmyer–Meshkov-like instabilities and early-time perturbation growth in laser targets and z-pinch loads," *Phys. Plasmas* **7**(5), 1662–1671 (2000).
- ¹²G. Guderley, *Luftfahrtforschung* **19**, 302 (1942).
- ¹³X. Ribeyre, V. T. Tikhonchuk, J. Breil, M. Lafon, and E. Le Bel, "Analytic criteria for shock ignition of fusion reactions in a central hot spot," *Phys. Plasmas* **18**(10), 102702 (2011).
- ¹⁴J. P. Christiansen, D. E. T. F. Ashby, and K. V. Roberts, "Medusa a one-dimensional laser fusion code," *Comput. Phys. Commun.* **7**(5), 271–287 (1974).
- ¹⁵H. Sakagami and K. Nishihara, "Three-dimensional Rayleigh-Taylor instability of spherical systems," *Phys. Rev. Lett.* **65**(4), 432–435 (1990).
- ¹⁶S. Taylor and J. P. Chittenden, "Effects of perturbations and radial profiles on ignition of inertial confinement fusion hotspots," *Phys. Plasmas* **21**(6), 062701 (2014).
- ¹⁷G. A. Kyrala, S. Dixit, S. Glenzer, D. Kalantar, D. Bradley, N. Izumi, N. Meezan, O. L. Landen, D. Callahan, S. V. Weber, J. P. Holder, S. Glenn, M. J. Edwards, P. Bell, J. Kimbrough, J. Koch, R. Prasad, L. Suter, J. L. Kline, and J. Kilkenny, "Measuring symmetry of implosions in cryogenic hohlraums at the NIF using gated x-ray detectors (invited)," *Rev. Sci. Instrum.* **81**(10), 10E316 (2010).
- ¹⁸A. K. Evans, "Instability of converging shock waves and sonoluminescence," *Phys. Rev. E* **54**(5), 5004–5011 (1996).