A progressive failure model for mesh-size-independent FE analysis of composite laminates subject to low-velocity impact damage

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Abstract

An original, ply-level, computationally efficient, three-dimensional (3D) composite damage model is presented in this paper, which is applicable to predicting the low velocity impact response of unidirectional (UD) PMC laminates. The proposed model is implemented into the Finite Element (FE) code ABAQUS/Explicit for one-integration point solid elements and validated against low velocity impact experimental results.

Mesh-regularisation of the impact results is one of the greatest challenges in impact damage modelling of composite targets. Currently published mesh-regularisation strategies can produce mesh-size-independent impact results for a very limited range of mesh-sizes. The originality of the model presented in this paper is based on a novel mesh-regularisation strategy, which is applicable to predicting mesh-size-independent impact results for any target discretisation. Accurate modelling of composites' impact damage using a relatively coarse mesh, and thus at a much reduced computational cost when compared to existing techniques, can be achieved with the proposed method.

Key-words: A. Laminate; B. Polymer-matrix composites (PMCs); B. Impact behaviour; C. Damage mechanics; C. Finite element analysis (FEA)

1. Introduction

When progressive failure is simulated with FEA by means of a Damage Mechanics (DM) approach based on a smeared crack formulation, mesh refinement results in a smaller localisation band width and thus reduces the global energy dissipated by the numerical fracture process. A negative softening slope, adjusted as a function of a characteristic element length, and energy release rates [1], provides a simple and physically sound solution to the problem of objective energy dissipation with respect to the FE mesh size. This approach, which is commonly referred to as the “cohesive crack model”, was first proposed for the numerical modelling of damage in concrete by Bazant and Oh [2]. As emphasised by Cervera and

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Chiumenti [3], it represented a milestone in the road to crack modelling, as it was the first successful attempt to link fracture mechanics and continuum mechanics theories. The majority of non-linear commercial FE codes implemented the cohesive crack model. Furthermore, many researchers have applied this approach to the modelling of impact damage in advanced composite materials, e.g. [4], [5], [6], [7], [8], [9], [10], [11].

Advanced composite material models are increasingly being used in the design of safety-critical components and structures for aerospace or defence applications and it is crucial that predicting impact damage is as accurate as possible. However, a general lack of detailed validation of the cohesive crack model for impact analysis, or discussions of the effects of varying the mesh-size on the simulated impact results, is evident. Alternative strategies for the modelling of composite impact damage have also been proposed in the open literature, e.g. [12], [13], [14]. These works do not describe in detail the mesh size-objective damage formulations, but emphasise that one of the largest obstacles, which must be overcome, is the mesh sensitivity of the impact analyses [12], [13].

In the current paper, a composite damage model for 3D progressive failure analysis of composite laminates, subject to low-velocity impact damage is presented. The damage model features an original DM approach, which, contrary to the classic cohesive crack model, can be applied to predicting mesh-size independent impact solutions for any target discretisation.

2. Formulation

2.1 3D transversely isotropic elastic behaviour and non-linear elastic-plastic shear behaviour

During the explicit analysis, the total strains and stresses are computed at the generic simulation time t in incremental form, for a Δt time step, as follows:

$$\begin{cases} \boldsymbol{\varepsilon}_i^{t+\Delta t} = \boldsymbol{\varepsilon}_i^t + \dot{\boldsymbol{\varepsilon}}_i \Delta t = \boldsymbol{\varepsilon}_i^t + \Delta \boldsymbol{\varepsilon}_i^{t+\Delta t} \\ \boldsymbol{\sigma}_i^{t+\Delta t} = \boldsymbol{\sigma}_i^t + \Delta \boldsymbol{\sigma}_i^{t+\Delta t} \end{cases} \quad \text{Equation (1)}$$

In the above Equation (1) a vector representation is used for strains and stresses with the following shorthand convention: $i = 1(\equiv x), 2(\equiv y), 3(\equiv z), 4(\equiv xy), 5(\equiv yz), 6(\equiv zx)$. The stress increment vector $\Delta \boldsymbol{\sigma}_i^{t+\Delta t}$ is computed at each time step assuming linear elastic direct behaviour and non-linear (plastic) orthotropic shear stress-strain behaviour as:

$$\begin{bmatrix} \Delta \sigma_x^{t+\Delta t} \\ \Delta \sigma_y^{t+\Delta t} \\ \Delta \sigma_z^{t+\Delta t} \\ \Delta \tau_{xy}^{t+\Delta t} \\ \Delta \tau_{yz}^{t+\Delta t} \\ \Delta \tau_{zx}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \frac{1-\nu_{yz}\nu_{zy}}{E_y E_z A} & \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_y E_z A} & \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_y E_z A} & 0 & 0 & 0 \\ \frac{\nu_{xy}+\nu_{xz}\nu_{zy}}{E_x E_z A} & \frac{1-\nu_{zx}\nu_{xz}}{E_x E_z A} & \frac{\nu_{zy}+\nu_{zx}\nu_{xy}}{E_x E_z A} & 0 & 0 & 0 \\ \frac{\nu_{xz}+\nu_{xy}\nu_{yz}}{E_y E_x A} & \frac{\nu_{yz}+\nu_{xz}\nu_{yx}}{E_y E_x A} & \frac{1-\nu_{xy}\nu_{yx}}{E_y E_x A} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{G}_{xy} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{G}_{yz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{G}_{zx} \end{bmatrix} \begin{bmatrix} \Delta \varepsilon_x^{t+\Delta t} \\ \Delta \varepsilon_y^{t+\Delta t} \\ \Delta \varepsilon_z^{t+\Delta t} \\ \Delta \gamma_{xy}^{t+\Delta t} \\ \Delta \gamma_{yz}^{t+\Delta t} \\ \Delta \gamma_{zx}^{t+\Delta t} \end{bmatrix}$$

Equation (2)

With,

$$A = \frac{1-\nu_{xy}\nu_{yx}-\nu_{yz}\nu_{zy}-\nu_{zx}\nu_{xz}-2\nu_{xy}\nu_{yz}\nu_{zx}}{E_x E_y E_z} \quad \text{Equation (3)}$$

And where $\gamma_i = 2\varepsilon_i$ are the engineering shear strains, with $i \geq 4$.

The direct behaviour is assumed transversely isotropic, i.e. $E_y = E_z$, and $\nu_{yz} = \nu_{zy}$. The tangent shear moduli $\bar{G}_i$, are defined as:

$$\bar{G}_i^{t+\Delta t} = \sum_{k=1}^4 G_{i,k} (\gamma_i^{t+\Delta t})^k, i \geq 4 \quad \text{Equation (4)}$$

The four coefficients $G_{i,k}$ are determined from polynomial fit of the experimental data for the $\tau_{xy}(\gamma_{xy})$, $\tau_{yz}(\gamma_{yz})$ and $\tau_{zx}(\gamma_{zx})$ behaviours. A maximum shear strain up to which the polynomial fits are valid, $\gamma_{P,\max}$, is user defined as:

$$\begin{cases} \gamma_{xy}^{t+\Delta t} > \gamma_{P,\max} \rightarrow \bar{G}_{xy}^{t+\Delta t} = hG_{xy,1} \\ \gamma_{yz}^{t+\Delta t} > \gamma_{P,\max} \rightarrow \bar{G}_{yz}^{t+\Delta t} = \bar{G}_{yz}(\gamma_{P,\max}) \\ \gamma_{zx}^{t+\Delta t} > \gamma_{P,\max} \rightarrow \bar{G}_{zx}^{t+\Delta t} = hG_{zx,1} \end{cases} \quad \text{Equation (5)}$$

where for $\gamma_{xy}^{t+\Delta t} > \gamma_{P,\max}$ the tangent in-plane and out-of-plane shear stiffness are user defined through the parameter h to improve the correlation between the curve fit and the test results at large shear strains. The model assumes that the material unloads in shear with initial shear stiffness, i.e. $G_{i,1}, \forall i \geq 4$.

2.2 Failure criteria

The following failure criteria are implemented.

For fibre tensile failure:

$$\sigma_x^{t+\Delta t} > 0 \rightarrow f_{ft} = \left(\frac{\sigma_x^{t+\Delta t}}{X_t} \right)^2 - 1; f_{ft} \geq 0 \quad \text{Equation (6)}$$

Where X_t is the tensile strength of the composite in the fibre direction.

For fibre compressive failure:

$$\sigma_x^{t+\Delta t} < 0 \rightarrow f_{fc} = \left(\frac{\sigma_x^{t+\Delta t}}{X_c} \right)^2 - 1; f_{fc} \geq 0 \quad \text{Equation (7)}$$

where X_c is the compressive strength of the composite in the fibre direction.

The criteria originally proposed in [15] are implemented for matrix tensile and compressive failure:

$$\left\{ \begin{array}{l} (\sigma_i^{t+\Delta t}) \xrightarrow{\perp x(\varphi)} (\sigma_{i'}^{t+\Delta t}) \\ \sigma_{y'}^{t+\Delta t} > 0 \rightarrow f_{mt} = \left(\frac{\sigma_{y'}^{t+\Delta t}}{Y_t} \right)^2 + \left(\frac{\tau_{x'y'}^{t+\Delta t}}{S_{xy}} \right)^2 + \left(\frac{\tau_{y'z'}^{t+\Delta t}}{S_{yz}} \right)^2 - 1; f_{mt} \geq 0 \\ \sigma_{y'}^{t+\Delta t} < 0 \rightarrow f_{mc} = \left(\frac{\tau_{x'y'}^{t+\Delta t}}{S_{xy} - \mu_t \sigma_{y'}^{t+\Delta t}} \right)^2 + \left(\frac{\tau_{y'z'}^{t+\Delta t}}{S_{yz} - \mu_t \sigma_{y'}^{t+\Delta t}} \right)^2 - 1; f_{mc} \geq 0 \end{array} \right. \quad \text{Equation (8)}$$

where Y_t is the matrix tensile strength and S_{xy} is the shear strength. The transverse friction coefficient μ_t is defined from the Mohr-Coulomb theory as:

$$\mu_t = -\frac{1}{\tan(2\varphi_0)} \quad \text{Equation (9)}$$

where the angle φ_0 identifies the fracture surface orientation for pure transverse compressive failure. This angle needs to be measured experimentally and it is typically found that $\varphi_0 > 50^\circ$ for UD polymer matrix composites. The longitudinal friction coefficient μ_l can be derived using a simple orthotropic model [16]:

$$\mu_l = \mu_t \frac{S_{xy}}{S_{yz}} \quad \text{Equation (10)}$$

Where S_{yz} is the transverse shear strength, which is inversely calculated from the transverse compressive strength Y_c as:

$$S_{yz} = \frac{Y_c}{2 \tan(\varphi_0)} \quad \text{Equation (11)}$$

In Equation (8), the symbol $\perp x(\varphi)$ signifies a 3D rotation around the x-axis (fibre-direction axis) of an angle φ . The angle $\bar{\varphi}$, which satisfies the failure criteria for matrix tensile failure, f_{mt} , and matrix compressive failure, f_{mc} , determines the orientation of the fracture surface. The fracture surface can assume any orientation $0 \leq \bar{\varphi} \leq \pi$ depending on the loading conditions, see Figure 1. The orientation $\bar{\varphi} = \pi/2$ is for a fracture plane parallel to the plane of a UD ply. When a stack of plies is modelled using this approach, $\bar{\varphi} = \pi/2$ identifies a fracture surface which has the same orientation of a delamination.

2.3 Modelling damage propagation

The DM approach implemented in this work uses a total of three independent damage variables, namely a fibre tensile damage variable, d_{xt} , a fibre compressive damage variable, d_{xc} and a matrix damage variable d_m .

The damage variables are forced to evolve, irreversibly, from 0 to 1 as a function of strain after failure initiation. For the generic damage mode, the relationship between failure strain ε^f , characteristic element length L , onset failure stress σ^o and fracture energy Γ is the following:

$$\varepsilon^f = \frac{2\Gamma}{\sigma^o L} \quad \text{Equation (12)}$$

The following damage law prescribes the evolution of the damage variable with respect of the driving strain $\varepsilon^{t+\Delta t, d}$ for the generic damage mode as:

$$d^{t+\Delta t} = \max \left\{ 0, \min \left\{ 1, \varepsilon^f \frac{\varepsilon^{t+\Delta t, d} - \varepsilon^o}{\varepsilon^{t+\Delta t, d} (\varepsilon^f - \varepsilon^o)} \right\} \right\} \quad \text{Equation (13)}$$

When one of the interactive criteria in Equation (6)-(8) is met, an equivalent stress and equivalent strains are defined.

For matrix damage modelling:

$$\begin{aligned} \sigma_m^o &= \sqrt{\langle \sigma_{y'}^{t+\Delta t} \rangle^2 + (\tau_{xy'}^{t+\Delta t})^2 + (\tau_{y'z'}^{t+\Delta t})^2} \Big|_{d_m^{t+\Delta t}=0} \\ \varepsilon_m^o &= \sqrt{\langle \varepsilon_{y'}^{t+\Delta t} \rangle^2 + (\gamma_{xy'}^{t+\Delta t})^2 + (\gamma_{y'z'}^{t+\Delta t})^2} \Big|_{d_m^{t+\Delta t}=0} \\ \varepsilon_m^{t+\Delta t, d} &= \sqrt{\langle \varepsilon_{y'}^{t+\Delta t} \rangle^2 + (\gamma_{xy'}^{t+\Delta t})^2 + (\gamma_{y'z'}^{t+\Delta t})^2} \Big|_{d_m^{t+\Delta t}>0} \end{aligned} \quad \text{Equation (14)}$$

Where $\langle x \rangle$ is the McCauley operator, defined as $\langle x \rangle = \max(0, x)$. Figure 2 illustrates the matrix failure and damage coupling strategy, where two-dimensional (2D) stress interaction is assumed for reasons of simplicity of representation.

The damage law described in Equations (12) and (13) is fracture mechanics (energy) based and its formulation is identical to the formulation originally proposed by Bazant and Oh [2].

However, experimental evidence indicates that several matrix cracks develop in a composite ply prior to the complete loss of its load bearing capability. On the other hand, experimental evidence indicates that interlaminar cracks develop in a composite laminate and coalesce into one interface crack, i.e. a delamination. Also, for composite laminates loaded in longitudinal parallel-to-the-fibre direction, there is no evidence of distributed fibre damage, but rather one kink band or one fibre tensile failure surface develop in the laminate under compressive or tensile loading, respectively.

Thus, the cohesive crack model is modified to allow multiple intralaminar matrix cracks per element. There is only one out-of-plane matrix crack per element and one fibre fracture surface per element.

A value for $\mathcal{E}_m^f$ is obtained from application of Equation (10) with a simple quadratic interpolation function for the fracture energy in the mix-mode case. The matrix fracture energy is made dependent on the orientation of the fracture surface, which is predicted by the matrix failure criteria, Equation (8).

When a tensile force act on the fracture surface, i.e. $\sigma_{y'} \geq 0$ and $\bar{\varphi} = 90$, the following expression is used to compute the matrix fracture energy:

$$\Gamma_m = \Gamma_I^m \left(\frac{\sigma_{y'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{x'y'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{y'z'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 \quad \text{Equation (15)}$$

Where Γ_I^m is the mode I matrix fracture energy and Γ_{II}^m is the mode II matrix fracture energy.

On the other hand, when a compressive force acts on the fracture surface, i.e. $\sigma_{y'} < 0$, or when a tensile force acts on the fracture surface with $\bar{\varphi} \neq 90$, the matrix fracture energy is computed as:

$$\Gamma_m = c_{num} \left[\Gamma_I^m \left(\frac{\sigma_{y'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{x'y'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{y'z'}^{t+\Delta t} |d_m^{t+\Delta t}=0|}{\sigma_m^o} \right)^2 \right] \quad \text{Equation (16)}$$

The parameter c_{num} quantifies the number of intralaminar matrix cracks at saturation. This parameter is defined as a function of the intralaminar matrix crack density at saturation parameter, c_{dens} , and element dimensions as follows:

$$c_{num} = c_{dens} L_y = c_{dens} l_y \quad \text{Equation (17)}$$

In which L_y is the characteristic element length in the y-direction and l_y is defined next.

ABAQUS only provides the VUMAT with a smeared element length that is strictly valid only for perfectly cubic elements, as the ABAQUS calculation does not take into account element's aspect ratio, and nodal coordinates are not accessible to the VUMAT. Thus, the three input parameters l_x , l_y and l_z , which correspond to the FE lengths measured in the global coordinate system along the x, y and z direction, are used in the present model.

The characteristic element lengths for matrix damage modelling are defined as:

$$\begin{aligned} \bar{\varphi} = 90^\circ &\Rightarrow L_{yz} = l_z \\ \bar{\varphi} \neq 90^\circ &\Rightarrow L_{yz} = \frac{l_y}{\cos(\bar{\varphi})} \end{aligned} \quad \text{Equation (18)}$$

For fibre damage modelling, Equation (12) is applied as follows:

$$\begin{cases} f_{ft} \geq 0 \rightarrow \varepsilon_x^f = \frac{2\Gamma_{Xt}}{X_t l_x} \\ f_{fc} \geq 0 \rightarrow \varepsilon_x^f = \frac{2\Gamma_{Xc}}{X_c l_x} \end{cases} \quad \text{Equation (19)}$$

Where Γ_{Xt} and Γ_{Xc} are the intralaminar fracture toughness values for tensile and compressive modes, respectively.

By combining Equations (16), (17) and (18), the following equation is obtained:

$$\varepsilon_m^f = \frac{2c_{dens} \left[\Gamma_I^m \left(\frac{\sigma_{y'}^{t+\Delta t} | d_m^{t+\Delta t} = 0}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{x'y'}^{t+\Delta t} | d_m^{t+\Delta t} = 0}{\sigma_m^o} \right)^2 + \Gamma_{II}^m \left(\frac{\tau_{y'z'}^{t+\Delta t} | d_m^{t+\Delta t} = 0}{\sigma_m^o} \right)^2 \right]}{\sigma_m^o \cos(\bar{\varphi})} \quad \text{Equation (20)}$$

i.e., when intralaminar matrix fracture energy is scaled with a number of cracks per element, the characteristic element length disappears from the computation of the ultimate failure strain: the potential damage energy increases with increasing FE volume. It could be speculated that the topological information, which is provided by the characteristic length in the cohesive crack model, and which is required for mesh-regularisation, is not absent in the modified cohesive crack formulation here proposed, Equation (20). The crack density parameter, which has

replaced the characteristic length that appeared at the denominator of Equation (12), has also units of mm^{-1} .

The stresses are finally updated using the following strategy:

$$\left\{ \begin{array}{l} \left(\sigma_i^{t+\Delta t} \right) \xrightarrow{\perp x(\bar{\varphi})} \left(\tilde{\sigma}_i^{t+\Delta t} \right) = \begin{bmatrix} \sigma_x^{t+\Delta t} \\ \sigma_{y'}^{t+\Delta t} \left(1 - d_m^{t+\Delta t} \right) \sigma_{y'}^{t+\Delta t} > 0 \\ \sigma_{z'}^{t+\Delta t} \\ \tau_{xy'}^{t+\Delta t} \left(1 - d_m^{t+\Delta t} \right) \\ \tau_{y'z'}^{t+\Delta t} \left(1 - d_m^{t+\Delta t} \right) \\ \tau_{z'x}^{t+\Delta t} \left(1 - d_m^{t+\Delta t} \right) \end{bmatrix} \\ \left(\tilde{\sigma}_i^{t+\Delta t} \right) \xrightarrow{\perp x(-\bar{\varphi})} \left(\tilde{\tilde{\sigma}}_i^{t+\Delta t} \right) = \left(\tilde{\sigma}_i^{t+\Delta t} \right) \left(1 - d_{X_t}^{t+\Delta t} \right) \left(1 - d_{X_c}^{t+\Delta t} \right) \end{array} \right. \quad \text{Equation (22)}$$

Where $\left(\sigma_i^{t+\Delta t} \right)$ is the stress vector in the rotated reference frame; $\left(\tilde{\sigma}_i^{t+\Delta t} \right)$ is the stress vector for a volume of material with matrix damage; and $\left(\tilde{\tilde{\sigma}}_i^{t+\Delta t} \right)$ is the stress vector for a volume of material in which both matrix and fibres are damaged.

3. Low velocity impact tests

The low velocity impact response of three composite lay-ups was investigated using an instrumented drop tower. The three lay-ups were: 1) Quasi-Isotropic (QI), $[+45/90/-45/0]_{2s}$; 2) AIRBUS (AIR) lay-up, $[+45/-45/0/90/-45/0/0/+45/0]_s$; and 3) Cross-ply lay-up (090), $[0/90]_{4s}$. All three lay-ups were made with UD HTS carbon fibre-reinforced MVR-444 epoxy matrix composite laminates with 0.34mm thick UD plies. The QI and 090 lay-ups were 5.4mm thick and the AIR lay-up was 6.4mm thick.

All targets had in-plane dimension of $150 \times 100 \text{mm}^2$. They were impacted at their centre using a varying impactor height and a mass of 5Kg. The impacts were introduced onto the targets using a 15.875mm diameter hemispherical tup made of hardened steel, and each target only received one impact. Upon impact, the force-time history and incident velocity-time history were recorded by a digital oscilloscope data acquisition unit. The acceleration, velocity and the displacement of the impactor were deduced by subsequent numerical integrations of the force-time trace using the trapezoidal rule. The force-time history was output by a Load Cell PCB Piezo sensor. The incident velocity of the impactor was measured by means of a ruled grid trigger attached to the impactor side (zebra), which passed a photo-emitter/photo-diode device. A modified BOEING fixture was used to support the targets. The fixture prescribed side clamped boundary conditions (BC's) by means of two-sided, 3mm diameter, hemispherical supports over a perimeter with sides 127mm and 76mm long. Following impacting, the targets were also C-scanned using a defect gate setting to determine the damage area.

4. Numerical simulations

4.1 Model definition and input properties

For generating the first series of results, which are presented in Section 4.2, the numerical targets were meshed with one element through-thickness per each ply in the laminate with a relatively coarse structured mesh; interface elements were not used. In these simulations, the FE's in the central target window had all dimensions of 1.5mm x 1.5mm x 0.34mm. The region of the target outside the clamped window was discretized using larger FE elements. The tests BC's were simulated by constraining the z-direction of the nodes at the clamped region. These nodes are highlighted in Figure 3, which also shows the target/impactor FE model. Displacements in the x and y-direction were also constrained for all nodes at two corners of this region, which avoided unrealistic in-plane rotations of the target during the impact simulations. Only the lower half of the semi-spherical tip of the impactor was modelled, and it was discretized as a rigid surface with a lumped mass of 5Kg. Initial velocity BC's, which matched the experimental impact velocities, were assigned to this part.

For one lay-up results were also obtained for varying mesh densities using elements with 2mm², 1.5mm² and 1mm² in-plane areas, respectively. Simulations took about 12 (3), 28 (7) and 52 (13) CPU (clock) hours for the three mesh densities, respectively.

The input properties used for the simulation were the following: $E_x = 114GPa$;

$$E_y = E_z = 8.6GPa; \nu_{xy} = \nu_{zx} = 0.3; \nu_{yz} = 0.46; G_{xy,1} = G_{zx,1} = 4.45GPa;$$

$$G_{xy,2} = -110.65GPa; G_{xy,3} = G_{zx,3} = 1223.6GPa; G_{xy,4} = G_{zx,4} = -3230GPa;$$

$$G_{yz,1} = 4000MPa; G_{yz,2} = G_{yz,3} = G_{yz,4} = G_{zx,2} = G_{zx,3} = G_{zx,4} = 0; \gamma_{p,max} = 0.025, h = 0.05;$$

$$X_t = 1.85GPa; X_c = 1.2GPa; Y_t = 35MPa; S_{xy} = 75MPa; S_{zx} = 40MPa; \phi_0 = 53^\circ;$$

$$Y_c = 170MPa; \Gamma_{Xt} = 40kJ/m^2; \Gamma_{Xc} = 40kJ/m^2; \Gamma_I^m = 170J/m^2; \Gamma_{II}^m = 1.2kJ/m^2;$$

$$c_{dens} = 5.25mm^{-1};$$

All the above elastic and failure properties were measured using standard tests. Fibre tensile strength and matrix tensile strength were characterised in [17]. Γ_{Xt} , Γ_{Xc} and c_{dens} were assumed. Γ_{Xt} was assumed conservatively equal to 40kJ/m², e.g. [18]; Γ_{Xc} was assumed equal to this value. The parameter c_{dens} was set equal to 5.25mm⁻¹ using an inverse approach: this value was varied to find a best match between the numerical and experimental force-time histories for one impact case, see Section 4.2.

The application of the classic cohesive crack model to an impact problem allows the dissipation of the correct amount of energy only for a mesh in which the FE's have dimensions which are smaller than the distance between two adjacent intralaminar matrix cracks. Thus, the need for defining the value for the parameter c_{dens} is equivalent to the need for using the correct mesh

size when applying the cohesive crack model with constant mode I and II matrix input fracture energies. For the targets investigated in this paper, the maximum FE size for applicability of the cohesive crack model can be inversely determined from the crack density parameter as

$$L_{y,MAX} = 1 / c_{dens} = 0.19mm.$$

4.2 Impact simulations

4.2.1 Validation of the damage formulation

Numerical and experimental force-time histories and force-displacement plots are compared in this Section. Also, the envelope of the matrix damage variable, d_m , is compared to the C-Scan tests results. Figure 4 shows the numerical and experimental results for the QI target impacted at 37J impact energy; Figure 5 for the QI target impacted at 74J; Figure 6 for the 090 target impacted at 37J; Figure 7 for the AIR target impacted at 75J. The damage model appears to over-predict matrix damage areas, especially at lower impact energies. This is especially evident in Figure 6. Also, certain features of the delamination in the C-scans are not captured by the model, e.g. the 45° delamination of the back ply (Figure 7). This is due to the fact that the delamination is simulated at the mid-plane of each ply rather than in between adjacent plies. This kinematic approximation inevitably lowers the accuracy of the model at capturing the correct stress concentrations and thus variation of energy release rate characteristics that depend on the orientation of adjacent plies. However, the comparison appears generally favourable for all lay-ups and impact energies investigated, which validates the failure and damage formulation proposed in this paper as well as the use of a simplified representation of the tests BC's in the models.

4.2.2 Validation of the mesh-regularisation approach

The impact against the QI target, 74J impact energy, was chosen as the test case for the validation of the mesh-regularisation approach proposed in this paper. This target was discretized with 0.34mm thick solid elements with square areas of 1mm², 1.5mm² and 2mm², respectively.

Figure 8 shows a comparison between experimental and numerical force-time histories, force displacement plots, velocity-time histories and displacement-time histories, when using the different discretisation strategies.

Figures 9 shows a comparison of the experimental and numerical results, which were obtained for two different target discretisation strategies, i.e. FE's with in-plane dimensions of 1mm² and 2mm² (in both cases 0.34mm thick) respectively, and two different mesh-regularisation approaches, i.e. the classic cohesive crack model and the modified approach, which were both implemented in the current work. The simulations were conducted in two steps. The constant input matrix fracture energies that resulted in the best fit with the experimental results for the

simulation with the 1mm^2 mesh were used as a reference for the simulations with the 2mm^2 mesh. For the latter, matrix fracture energies were scaled using either:

- 1) Equation (15) for all matrix damage modes for the case labelled as “2mm Mesh – Classic approach”; or
- 2) Equation (15) and Equation (16), with $c_{dens} = 5.25\text{mm}^{-1}$ for the case labelled as 2mm Mesh – Proposed model”;

Force-time histories, force displacement plots, and the damage areas are plotted for these three numerical targets in this Figure 9. The results presented in Figure 8 and Figure 9 show that mesh-size independent impact damage analysis can be successfully achieved with the proposed method. Both quantitative (force, velocity and displacement-time histories and force-displacement plots) and qualitative (extent and shape of matrix damage areas) results were independent of target discretisation. Small differences between the results obtained for the different mesh densities can be attributed to the deterioration of the kinematic representation of the problem with increasingly coarser FE sizes.

The results in Figure 9 A), B) and F) compared to A), show that the cohesive crack model, with constant mode I and II input matrix fracture energies, is inapplicable for mesh-regularisation of the impact results, when a relatively coarse mesh is used for discretisation of the target.

Relatively large FE's are generally used when the target is discretized with conventional shell finite elements, e.g. [19] and [11], in which $1\text{mm} \times 1\text{mm}$ FE's were used at the impact point, or [12], in which FE's had a size up to $3.6\text{mm} \times 3.6\text{mm}$ at the impact point. In industrial applications, which require results to be produced quickly, the use of 3D solid elements with larger aspect ratio is often necessary.

5. Discussion

An approach for accurately modelling interlaminar and intralaminar fracture mechanisms is required when modelling impact in laminated composites. This typically comprises the coupling of an in-plane ply-level damage model with interface cohesive elements, e.g. [9], [12].

However, the use of interface cohesive elements in between each ply of a typical standard composite impact coupon with 16 plies, when using 3D solid elements for the plies discretisation, results in increasing the CPU time up to 30-40 folds (depending on the refinement of the discretisation), when compared to a model with no interface elements. The formulation proposed here uses 3D phenomenological-based failure criteria for tensile and compressive matrix failure, and 3D DM, which can predict the 3D orientation of matrix cracks, including an orientation that can mimic delamination failure. Thus, approximated modelling of delamination failure can be achieved at a much reduced computational cost when using the proposed method. This modelling approach results in extremely rough kinematic approximations, as the delamination is simulated at the mid-plane of each ply rather than at the plies' interface. However, the results presented in this paper show that its predictive capabilities

are qualitatively and quantitatively comparable to those obtained with more expensive methods, e.g. [9].

A parameter, which quantifies saturation levels for in-plane matrix cracks, was introduced to dissipate the correct amount of fracture energy associated with distributed damage in the form of multiple intralaminar matrix cracks per element. It is emphasised that an enhancement of damage energy based on crack density must be used when targets are discretised with solid elements that have in-plane dimensions larger than the minimum physical distance between two adjacent intralaminar matrix cracks. Because this distance is system dependent and may be difficult to characterise for impact loading conditions, a crack density parameter should always be used for impact damage modelling with a smeared crack (energy based) formulation, regardless of the mesh-size.

6. Conclusions

A computationally efficient, phenomenological-based, 3D damage model was formulated and implemented into the FE code ABAQUS/Explicit for one-integration point solid elements. The numerical results were validated against experimental results, and the proposed formulation was shown to predict the low velocity impact behaviour of carbon PMC laminates with different lay-ups with reasonable accuracy.

The cohesive crack model produces mesh-size independent solutions only for applications where a single crack is simulated, or when the loading conditions and the mesh-size both allow to simulate the opening of multiple cracks at some regular intervals. An example is given by the meso-scale simulations carried out by Maimi' et al. [20], where several FE's were used through-thickness of a 90° ply in a cross-ply laminate subjected to uniaxial tensile loading conditions. Thus, application of the cohesive crack model for impact damage analysis dissipates the correct amount of damage energy only for a mesh in which the FE's have dimensions which are smaller than the distance between two adjacent intralaminar matrix cracks. Because this distance is generally not known, a crack density parameter should always be used for composites impact damage analysis with a smeared formulation. Alternatively, results for different mesh-densities must also be presented when the cohesive crack model is applied with constant mode I and mode II input matrix fracture energies.

A method for mesh-regularisation of the impact results, which is valid for any mesh-size, was proposed in this paper. Mesh-regularisation of the impact results can successfully be achieved for any mesh-size by simply scaling the energy release rates by the number of intralaminar cracks in one element at cracks saturation. The number of intralaminar matrix cracks was related to the characteristic element dimensions and the intralaminar crack saturation density parameter, which could only be reasonably obtained using an inverse method.

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Figure 1. Idealisation of a UD composite Representative Volume Element (RVE). The fibre direction is the x-direction. The potential fracture plane is identified by a rotation ϕ along the x-direction with respect of the RVE principal material symmetry plane (xyz).

Figure 2. Illustration of local model behaviour for matrix failure (2D stress interaction is

assumed for reasons of simplicity of representation).

Figure 3. FE model of the target and impactor, with highlighted nodes used to prescribe the target's BC's.

Figure 4. Numerical and experimental impact results for a QI target for 37J impact energy. A) Force-time history; B) Force-displacement; C) Envelope of matrix damage; D) C-Scan tests results.

Figure 5 Numerical and experimental impact results for a QI target for 74J impact energy. A) Force-time history; B) Force-displacement; C) Envelope of matrix damage; D) C-Scan tests results.

Figure 6. Numerical and experimental impact results for a 090 target for 37J impact energy. A) Force-time history; B) Force-displacement; C) Envelope of matrix damage; D) C-Scan tests results.

Figure 7. Numerical and experimental impact results for an AIR target for 75J impact energy. A) Force-time history; B) Force-displacement; C) Envelope of matrix damage; D) C-Scan tests results.

Figure 8. Comparison between experimental and numerical results for a QI target impacted at 74J impact energy. The numerical results are from simulations in which the target was discretized using FE's, 0.34mm thick, and with in-plane square areas of 1mm², 1.5mm² and 2mm², respectively.

Figure 9. Numerical and experimental impact results for a QI target discretized with two different mesh-densities, and impacted at 75J impact energy. A) Force-time histories; B) Force-displacement plots; Envelope of the matrix damage variable: C) 1mm² FE size; D) 1.5mm² FE size; E) 2mm² FE size using the proposed model; F) 2mm² FE size using the cohesive crack model.