Flexural buckling of elliptical hollow section columns

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Abstract

Complementary to recent studies on the cross-section behavior of elliptical hollow sections, investigations into the member response have also been performed and described in this paper. The first stage of these investigations involved a series of precise column buckling tests on hot-finished steel elliptical hollow sections. In total, 24 flexural buckling tests about the minor and major axes were carried out. Specimen sizes and lengths were carefully chosen in order to cover a wide range of both cross-section and member slendernesses. The non-dimensional column slenderness of the test specimens varied between 0.19 and 1.58. Measurements were taken of specimen geometry, global initial geometric imperfections and tensile material properties. Key results from the tests including the full load-deformation histories have been presented and discussed. The test results have also been supplemented by 158 numerically generated results, allowing a wider range of geometries to be investigated. Design rules for the member buckling resistance of elliptical hollow section columns have been proposed and verified by means of reliability analysis. The presented results are the first member buckling tests on elliptical hollow sections, and represent part of the development of comprehensive structural design rules for these sections.

CE Database subject headings: Buckling; Columns; Compression; Design; Hollow sections; Laboratory tests; Numerical analysis; Slenderness ratio; Steel structures.

Introduction

Tubular members are widely used for exposed structural elements owing to their efficiency and aesthetic appearance. The familiar range of tubular members currently comprises square, rectangular and circular hollow sections. However, elliptical hollow sections have been recently introduced to the construction market, and structural columns are becoming a popular application. Elliptical hollow sections can offer greater efficiency than circular hollow sections, particularly when subjected to eccentric loading (generating a bending moment about the stronger axis) or when differing end restraints or bracings exist about the two principal axes (altering the effective column buckling lengths). Recent examples of the use of elliptical hollow sections as columns may be found at the airports at Heathrow in London, UK (Corus 2006a) and Barajas in Madrid, Spain (Viñuela-Rueda and Martinez-Salcedo 2006). Previous studies of elliptical hollow sections conducted by the authors include design proposals for cross-section classification (Gardner and Chan 2007), cross-section design strength in compression (Chan and Gardner 2008a) and bending about both principal axes (Chan and Gardner 2008b) and combined shear and bending (Gardner et al. 2008). Recent studies on the elastic buckling of elliptical hollow sections (Zhu and Wilkinson 2006; Ruiz-Teran and Gardner 2008), the response of filled elliptical tubes (Roufegarinejad and Bradford 2007; Zhao et al. 2007; Yang et al. 2008) and the behaviour of connections to EHS (Bortolotti et al. 2003; Choo et al. 2003; Pietrapertosa and Jaspart 2003; Willibald et al. 2006) have also been performed. This paper describes an investigation of the buckling strength of elliptical hollow section columns. Detailed experimental and numerical studies are described and structural design rules are proposed.

Experimental studies

A comprehensive full-scale laboratory testing programme on EHS (grade S355), manufactured by Corus Tubes (Corus 2006b), has been conducted at Imperial College London. To date, material tensile coupon tests and cross-section capacity tests in compression (Chan and Gardner 2008a), bending (Chan and Gardner 2008b) and shear (Gardner et al. 2008) have been performed. This paper focuses on the member behavior of pin-ended, elliptical hollow section columns. The tested EHS had an aspect ratio of two, overall cross-section dimensions of 150×75 mm and thicknesses of 4 mm, 5 mm and 6.3 mm. A total of six material tensile coupon tests and twenty four column buckling tests were carried out.

Tensile coupon tests

Material tensile coupon tests were performed to determine the basic engineering stress-strain response of the material for each of the tested section sizes. The obtained material data were used to facilitate the analysis of the column test results and were incorporated into the numerical simulations to replicate the response of the tested specimens. Full details of the tensile tests have been reported by the authors (Chan and Gardner 2008a), whilst a summary of the test results is given in Table 1.

Flexural buckling tests

The primary aim of the column tests was to investigate the flexural buckling response of EHS pin-ended compression members under axial loading. The pin-ended conditions were provided by hardened steel knife-edges fixed to the ends of the specimens. The specimen lengths were carefully chosen to provide a spectrum of member slendernesses. The nominal pin-ended column lengths were 0.7 m, 1.5 m, 2.3 m and 3.1 m (Fig. 1), with the corresponding L_{cr}/i ranging from 15 to 117

(and non-dimensional column slenderness ranging from 0.19 to 1.58), where L_{cr} is the buckling (effective) length in the buckling plane considered and i is the radius of gyration about the relevant buckling axis (determined using the properties of the gross cross-section). The general testing configuration is depicted in Fig. 2. For column specimens where the measured global imperfection was less than $L/1000$, an eccentricity of loading was applied such that the combined imperfection plus eccentricity was equal to $L/1000$. For three of the columns – 150×75×6.3-C1, 150×75×5-C3 and 150×75×5-C5 – the measured initial imperfections were greater than $L/1000$; for these specimens, the load was applied concentrically. The loading was recorded by a 1000 kN load cell located at the top end of the columns. Vertical displacement was measured at the loaded end of the columns by two LVDTs, whilst two additional LVDTs were positioned at each end of the columns to measure end rotation. Two LVDTs were also located at the mid-height of the columns to measure the lateral deflection in the major and minor axes directions. Four linear electrical resistance strain gauges were affixed to the extreme fibers of the section at a distance of 20 mm from the mid-height to avoid contact with the LVDTs. Load, strain, displacement and input voltage were all recorded using the data acquisition equipment DATASCAN and logged using the DSLOG computer package.

The geometry of an elliptical hollow section is depicted in Fig. 3 and the mean measured dimensions and maximum global geometric imperfections ω_g (determined by employing feeler gauges to measure the clearance between the columns and a flat datum) of the test specimens (based on specimen lengths ignoring the knife edges) are summarized in Table 2. Local geometric imperfections were not measured in this study since the proportions of the cross-sections examined were relatively stocky and insensitive to local buckling. For thin-walled hollow sections, such measurements would typically be performed (Young and Chen 2008). Geometric properties for the EHS specimens are defined using the exact formulae adopted by the authors in previous studies

(Chan and Gardner 2008a, b; Gardner and Chan 2007). The key results from the column tests have been reported in Table 2. Full load-deformation relationships for the column tests are reported in Chan (2007) and typical structural responses are described in the following sections.

Load versus vertical displacement response

The normalized load-vertical displacement response for the EHS 150×75×6.3 pin-ended columns of 0.7 m length are shown in Fig. 4. In the figure, C1 represents the column buckling response about the z-z axis (minor axis) whilst C2 illustrates the response about the y-y axis (major axis). According to the cross-section classification measure defined by Gardner and Chan (2007) and by Ruiz-Teran and Gardner (2008), this cross-section is deemed to be fully effective in pure compression. Both columns have demonstrated a similar peak load because the columns are relatively stocky, though a significant difference in the unloading behavior may be observed. This can be explained by considering the effects of inelastic local buckling and plastic hinge formation at the most heavily loaded cross-section (at the column mid-height). For the column buckling about the z-z axis (column C1), the maximum compressive stress coincides with the flattest portion of the cross-section which is most susceptible to local buckling. Local buckling heralds a loss of second moment of area resulting in a marked drop in load. Conversely, for column C2 where buckling is about the y-y axis, the maximum compression coincides with the stiffest region of the cross-section, which is resistant to local buckling, resulting in a more gradual loss of load carrying capacity.

Load versus lateral deflection response

Figs 5 and 6 show the load versus lateral deflection response of the two 3.1 m long EHS 150×75×6.3 pin-ended columns, buckling about the minor and major axes. In the figures, the elastic buckling load N_{cr} (given by Eq. (1)) and the plastic yield load N_y (given by Eq. (2)) have also been plotted.

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2} \quad (1)$$

$$N_y = Af_y \quad (2)$$

where E is the Young's modulus, I is the second moment of area about the relevant buckling axis, L_{cr} is the buckling (effective) length in the buckling plane considered, A is the gross cross-section area and f_y is the material yield stress. Eqs (1) and (2) represent the two theoretical upper bounds to column resistance. For stockier members, the ultimate load is dominated by the plastic yield load whilst for slender members, the ultimate load approaches the Euler buckling load. For the column under consideration, when buckling about the minor axis (Fig. 5), the elastic buckling load N_{cr} is below the yield load N_y , whilst when buckling about the major axis (Fig. 6), the elastic buckling load is above the yield load. The relative influence of the two bounds may be seen in Figs 5 and 6.

In addition to the inclusion of N_{cr} and N_y in Figs 5 and 6, the results of a second order elastic analysis and a rigid plastic analysis have also been shown. The second order elastic analysis was performed on the assumption that the unloaded column has an initial sinusoidal bow of maximum amplitude ω_i . The maximum additional lateral deflection ω arising under increased loading N may be determined from Eq. (3) provided the column remains elastic.

$$\omega = \frac{\omega_i}{1 - (N/N_{cr})} - \omega_i \quad (3)$$

The amplitude of the initial bow was selected to achieve the best representation of the response of the tested columns – for column 150×75×6.3-C7 (Fig. 5), the required initial bow was found to be approximately $L/500$, whilst for column 150×75×6.3-C8 (Fig. 6), a value of approximately $L/1000$ was found to be suitable. In the latter case, the required initial bow was similar to the total test imperfection (initial bow plus applied load eccentricity), whilst for the 150×75×6.3-C7, a larger

imperfection than was measured in the test was required – interestingly $L/500$ was also the imperfection amplitude that resulted in best agreement between the FE model and the test (see Table 3), suggesting that a slightly larger imperfection (geometric or residual stresses) may have actually existed for this test specimen.

For the second order rigid plastic analysis, reference may be made to the concentrically loaded pin-ended column shown in Fig. 7. By assuming that all deformation is concentrated in a plastic hinge at the mid-height of the column, the axial load N that can be sustained under increased lateral deflection may be determined by examining the stress distribution of Fig. 8. As illustrated in Fig. 8, the central compressive region (zone 2) is responsible for resisting the axial load N , whilst the two outer regions (zones 1 and 3) form a couple to resist the second order moment $M_2 = N\omega$ arising from the lateral deflection ω (Allen and Bulson 1980). Based on this stress model, the second order rigid plastic boundary has been derived and plotted in Figs 5 and 6. The test results demonstrate sound agreement with the theoretical models, with the general response being characterized by the envelope of the two second order boundaries. The initial elastic response concurs with the second order elastic analysis while the unloading behavior merges into the second order rigid plastic line. These test results have been replicated numerically and used for the validation of column buckling curves in the following sections.

Numerical simulations

A numerical modeling investigation, utilizing the finite element (FE) package ABAQUS (2006), was implemented alongside the experimental programme. The objectives of the investigation were to (1) replicate the experimental results, (2) validate the numerical models and (3) perform parametric studies. The elements chosen for the FE models were 4-noded, reduced integration shell

elements with six degrees of freedom per node, designated as S4R in the ABAQUS (2006) element library, and suitable for thin or thick shell applications. A uniform mesh density was carefully chosen by carrying out a mesh convergence study based on elastic eigenvalue predictions with the aim of achieving accurate results whilst minimizing computational effort. A suitable mesh size was found to be $2a/10(a/b) \times 2a/10(a/b)$ mm, where a and b are defined in Fig. 3. In total, 158 numerical simulations were carried out, including validation studies, sensitivity studies and parametric studies.

The column tests were modeled using the measured dimensions of the test specimens and measured material stress-strain data. Initial global geometric imperfections were incorporated in the numerical model by means of a global half-sine wave given by $\omega_g \sin(\pi x / L)$, where x is the distance along the specimen, L is the specimen length and ω_g is the global imperfection amplitude. A range of global imperfection amplitudes ω_g was studied – in addition to the measured imperfection values (excluding applied load eccentricity), four fixed fractions of the specimen length L ($L/250$, $L/500$, $L/1000$ (representing the total test imperfection including applied load eccentricity) and $L/2000$) were considered. A similar spectrum of global imperfection amplitude was studied by Batterman and Johnston (1967) and Gardner and Nethercot (2004). The amplitudes of $L/250$ and $L/500$ reflect the current allowable out-of-straightness tolerance for elliptical hollow section tubes in Europe (EN 10210-2 2006), whilst $L/500$ is the tolerance limit in Europe (EN 10210-2 2006), North America (ASTM A501 2005) and Australia (AS 1163 1991) for other tubular members.

The presence of longitudinal residual stresses in structural members can have a significant influence on column buckling strengths, by causing premature yielding of the cross-section resulting in a reduction in stiffness and loss of load carrying capacity. Residual stresses are induced primarily during the production process. For cold-formed sections, residual stresses are principally induced

through plastic deformation, whilst for hot-finished and welded sections, uneven cooling is the main source of residual stresses. The elliptical hollow sections considered in the present study are hot-finished structural sections; the residual stresses are therefore primarily induced through uneven cooling. In tubes, uneven cooling arises as a result of the differing thermal conditions present at the outer and inner surfaces of the sections, with the outer surface cooling more rapidly. This generally leaves the outer surface of the tube in longitudinal and radial compression, with equilibrating tension on the inner surface. In the development of the European column buckling curve for hot-finished tubular sections (Beer and Schulz 1970), a representative residual stress pattern based on measurements from circular tubes was employed. The adopted distribution was symmetrical through the thicknesses with compressive residual stresses on the outer surface and tensile residual stresses on the inner surface, though the experimental findings of Stamenkovic and Gardner (1983) exhibited a contradictory trend. The magnitude of both tensile and compressive residual stresses was in the region of 15% of the material yield stress. It may be inferred that residual stresses in elliptical tubes would be of similar magnitude and distribution to those in circular tubes, though since no residual stresses measurements were performed in this study, further investigation would be required to verify this. Two observations indicated that the level of residual stresses in the studied hot-finished EHS was low: (1) negligible deformations occurred when the material tensile coupons were machined from the cross-sections and (2) a distinct yield point was seen in both the tensile coupon results and the stub column results (Chan and Gardner 2008a), which high residual stresses would erode. For these reasons, residual stresses were not explicitly incorporated into the numerical models.

The true material stress-strain relationships were generated from the engineering stress-strain curves obtained from the tensile coupon tests. The material non-linearity was incorporated into the numerical models by means of a piecewise linear stress-strain model to replicate, in particular, the strain-hardening region. Boundary conditions were applied to model pin end conditions at the ends

of the columns. The modified Riks method (ABAQUS 2006) was employed to solve the geometrically and materially non-linear column models, which enabled the post-ultimate behavior to be traced. The numerical failure mode of $150 \times 75 \times 6.3$ -C4 is shown in Fig. 9 and compared with the corresponding deformed test specimen. Results of the numerical simulations are tabulated in Table 3, in which, the ratios between the ultimate FE and experimental axial loads are shown and compared for different imperfection levels.

Replication of test results was found to be satisfactory with the numerical models able to successfully capture the initial stiffness, ultimate capacity, general load-deformation response and failure patterns observed in the tests. Comparison between test and FE results are shown for columns $150 \times 75 \times 4$ -C5 (minor axis buckling) and $150 \times 75 \times 6.3$ -C8 (major axis buckling) in Figs 10 and 11, respectively. The anticipated sensitivity to imperfections has been reflected in the numerical results in particular among the specimens of intermediate slenderness which showed the greatest variation in response. For example, in the case of the $150 \times 75 \times 6.3$ -C8 models ($L_{cr}/i = 67$), the ultimate load reduces by approximately 30% with an increase of imperfection amplitude from $L/2000$ to $L/250$.

Having verified the general ability of the FE models to replicate the column test behavior for EHS with an aspect ratio of two, a series of parametric studies was conducted. The primary aim of the parametric studies was to investigate the influence of cross-section slenderness, aspect ratio and member slenderness on the column load carrying capacity. The obtained results were also used to assess column buckling design curves. A piecewise linear material stress-strain model was developed from the tensile coupon tests conducted on the $150 \times 75 \times 6.3$ sections and adopted throughout the parametric studies (see Fig. 12). Initial geometric imperfections in the non-linear parametric analyses were of the form of a half-sine wave with an amplitude ω_g of $L/1000$, which corresponded to the total test imperfection (geometric bow plus applied load eccentricity) and

provided the best agreement between FE and test results (Table 3). This magnitude is the same as that employed in the formulation of the European column buckling curves (Beer and Schulz 1970; Galambos 1998) and the Australian column curves (Rotter 1982; Beedle 1991). It is worth noting that the current AISC Standard employed $L/1500$ (Beedle 1991; Galambos 1998) as the governing out-of-straightness in developing the column design curves. The section sizes considered in the parametric studies were 150×150 , 150×75 and 150×50 with varying thicknesses and a range of column lengths to extend the investigation to higher member slenderness. The results have been utilized for the validation of proposed column buckling curves for elliptical hollow sections, as discussed in the following section.

Buckling resistance of members

In this section, the results of the EHS column buckling tests are examined and compared with the current column design specifications adopted in Europe, North America and Australia; on the basis of the comparisons, design recommendations are presented.

Member slenderness for flexural buckling

European Standard (EC 3)

According to EN 1993-1-1 (2005), the non-dimensional member slenderness λ_{EC} is given by

$$\lambda_{EC} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i\pi} \sqrt{\frac{f_y}{E}} \quad \text{for Class 1 to 3 (fully effective) sections} \quad (4)$$

$$\lambda_{EC} = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} = \frac{L_{cr}}{i\pi} \sqrt{\frac{(A_{eff}/A)f_y}{E}} \quad \text{for Class 4 (slender) sections} \quad (5)$$

where L_{cr} is the buckling (effective) length in the buckling plane considered, i is the radius of

gyration about the relevant buckling axis (determined using the properties of the gross cross-section) and A_{eff} is the effective cross-sectional area, a proposal for which has been made for EHS by Chan and Gardner (2008a).

North American Standard (AISC 360)

In accordance with AISC 360 (2005) and Tide (1985, 2001), the corresponding non-dimensional member slenderness λ_{AISC} is given by

$$\lambda_{\text{AISC}} = \frac{L_{\text{cr}}}{i\pi} \sqrt{\frac{f_y}{E}} \quad \text{for fully effective sections} \quad (6)$$

$$\lambda_{\text{AISC}} = \frac{L_{\text{cr}}}{i\pi} \sqrt{\frac{Qf_y}{E}} \quad \text{for slender sections} \quad (7)$$

where Q is the slenderness reduction factor which defines the ratio of the stress at local buckling to the yield stress (AISC 360 2005).

Australian Standard (AS 4100)

According to AS 4100 (1998), the equivalent non-dimensional member slenderness λ_{AS} is given by

$$\lambda_{\text{AS}} = \frac{L_{\text{cr}}}{i} \sqrt{\frac{f_y}{250}} = \frac{L_{\text{cr}}}{i\pi} \sqrt{\frac{f_y}{E}} \left(\pi \sqrt{\frac{E}{250}} \right) \quad \text{for fully effective sections} \quad (8)$$

$$\lambda_{\text{AS}} = \frac{L_{\text{cr}}}{i} \sqrt{\frac{k_f f_y}{250}} = \frac{L_{\text{cr}}}{i\pi} \sqrt{\frac{k_f f_y}{E}} \left(\pi \sqrt{\frac{E}{250}} \right) \quad \text{for slender sections} \quad (9)$$

where k_f is defined as the ratio of the effective area to the gross area of the cross-section.

Table 4 summarizes the key results of the column tests where the ultimate load N_u has been normalized by the yield load $N_y (=Af_y)$ or the effective yield load $A_{\text{eff}}f_y$. According to the slenderness parameters and limits proposed by Gardner and Chan (2007) and by Ruiz-Teran and

Gardner (2008), the tested EHS with wall thicknesses of 4 mm are Class 4 (slender), and the ultimate loads have therefore been normalized by $A_{eff}f_y$. The 5 mm and 6.3 mm thick sections are classified as Class 1-3 (fully effective) and thus, the ultimate loads have been normalized by Af_y . The normalized test results have also been plotted in Figs 13 and 14. For comparison, existing column test data for circular hollow sections (CHS) have also been added to Figs 13 and 14 (Janns et al. 1989). From these figures, it may be seen that the test results for buckling about the minor (z-z) and major (y-y) axes follow a similar trend alongside their circular counterparts.

Column buckling curves

European Standard (EC 3)

The concept of multiple column curves adopted in Europe (Beer and Schultz 1970; Jacquet 1970; Sfintesco 1970) forms the basis of the column design criteria in Eurocode 3 (EN 1993-1-1 2005). A set of five buckling curves described by the Ayrton-Perry (Ayrton and Perry 1886; Robertson 1925; Maquoi and Rondal 1978) formula is adopted in this European standard. For hollow sections, the choice of buckling curve depends on the forming route (hot-finished or cold-formed) and the material yield strength. For hot-finished hollow sections of grade S355, buckling curve ‘a’ is recommended. In Eurocode 3, buckling curves are presented in the form given by Eq. (10). The resulting buckling reduction factor χ = buckling resistance/cross-section resistance (N_b/N_c) has been plotted in Fig. 13.

$$\chi = \frac{N_b}{N_c} = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda_{EC}^2}} \quad \text{but } \chi = N_b/N_c \leq 1 \quad (10)$$

where $\Phi = 0.5[1 + \alpha(\lambda_{EC} - 0.2) + \lambda_{EC}^2]$ and α is an imperfection factor (equal to 0.21 for buckling curve ‘a’).

North American Standard (AISC 360)

A single column curve is currently adopted in North America (AISC 360 2005). This column curve is derived from the three column curves proposed by the Structural Stability Research Council (Bjorhovde and Tall 1971; Bjorhovde 1972, 1978; Galambos 1998) and can be described by basic column equations which have been derived empirically based on test data (Tide 1985, 2001; Beedle 1991). The AISC column curve is defined by Eqs (11) and (12) and has been plotted in Fig. 13.

$$\frac{N_b}{N_c} = 0.658^{\lambda_{AISC}^2} \quad \text{for } \lambda_{AISC} \leq 1.5 \quad (11)$$

$$\frac{N_b}{N_c} = \frac{0.877}{\lambda_{AISC}^2} \quad \text{for } \lambda_{AISC} > 1.5 \quad (12)$$

Australian Standard (AS 4100)

Column design curves, based on the multiple column curve concept are also adopted in the Australian Standard (Rotter 1982; Trahair and Bradford 1998; AS 4100 1998). For hollow sections, the choice of buckling curve depends on the forming route (hot-formed, cold-formed (stress relieved) or cold-formed (not stress relieved)) and the cross-section slenderness. For hot-finished hollow sections of Class 1-3 (fully effective), the reduction factor $\alpha_c = N_b/N_c$ can be described by Eq. (13) which has been plotted in Fig. 13. The slenderness λ_{AS} has been divided by $\pi\sqrt{E/250}$ for consistency and direct comparison with the other Standards considered.

$$\alpha_c = \frac{N_b}{N_c} = \xi \left\{ 1 - \sqrt{1 - \left(\frac{90}{\xi(\lambda_{AS} + \alpha_a \alpha_b)} \right)^2} \right\} \quad (13)$$

where α_b is equal to -1.0 for Class 1-3 (fully effective) sections and ξ and α_a are defined in AS 4100 (1998).

Proposals, reliability analyses and discussion

As shown in Fig. 13, the buckling curves for hot-finished hollow sections from the three Standards considered generally follow each other closely, with the AS 4100 curve being slightly higher over the full range of member slenderness. The AISC curve is marginally lower than the EC 3 curve at low and intermediate slenderness. At higher slenderness, all curves converge towards the Euler elastic buckling curve.

Partial (resistance) factors are applied to the nominal column equations given by Eqs (10) to (13) to ensure that the required level of reliability is achieved. In EC 3, this partial factor γ_{M1} appears in the denominator and is set equal to unity, whereas in AISC 360 and AS 4100, the resistance factors (denoted ϕ_c and ϕ , respectively) appear in the numerator and have a value of 0.9. The ‘design’ column curves are plotted in Fig. 14. The buckling curves recommended for hot-finished hollow sections in the three Standards considered generally provide a lower bound to the EHS test data and to the numerical results from the described parametric studies on elliptical hollow sections with aspect ratios a/b of 1.0 (CHS), 2.0 and 3.0 (shown by lines in Fig. 14). It is therefore proposed that these buckling curves may also be applied to elliptical hollow sections.

In order to verify the reliability of the buckling resistance functions, standard statistical analyses in accordance with EN 1990 (2002) and AISC 360 (2005) were performed. In the Eurocode analysis, the ratio of mean to nominal yield strengths (i.e. the material over-strength) was taken as 1.16 and the coefficients of variation of yield strength and geometric properties were taken as 0.05 and 0.02, respectively (Byfield and Nethercot 1997). These values originate from industrial data obtained from European steel producers. For the AISC analysis, the ratio of mean to nominal yield strengths was taken as 1.028 and the coefficients of variation of yield strength and geometric properties were taken as 0.058 and 0.05, respectively (Bartlette et al. 2003).

The results of the analyses and a summary of the key statistical parameters are presented in Tables 5 (European analysis) and 6 (AISC analysis). The following symbols are used in the Eurocode analysis: $k_{d,n}$ is the design (ultimate limit states) fractile factor for n tests, where n is the population of test data under consideration; b is the average ratio of experimental to model resistance based on a least squares fit to the test data; V_δ is the coefficient of variation of the tests relative to the resistance model; V_r is the combined coefficient of variation incorporating both model and basic variable uncertainties. The following symbols are used in the AISC analysis: V_Q is the coefficient of variation of load effects; V_R is the combined coefficient of variation incorporating both model and basic variable uncertainties (equivalent to V_r in Eurocode terminology); β is the reliability index. For an acceptable level of reliability, a value of γ_{M1} less than unity (unity being the adopted partial factor for member in EN 1993-1-1 (2005)) and a value of β greater than 2.6 is sought. The results (Tables 5 and 6) indicate that when the 24 tests are considered alone, whilst the AISC requirements are satisfied, the Eurocode outcome is marginal. However, the required level of reliability is achieved for both codes when the test and parametric FE results are considered together. The AS 4100 (1998) buckling curve lies below the Eurocode curve and it may be assumed that this also yield acceptable reliability. Overall, it is recommended that the buckling curves from EN 1993-1-1 (2005), AISC 360 (2005) and AS 4100 (1998) that are currently applied to SHS, RHS and CHS columns can also be adopted for hot-finished EHS columns, buckling about either the major or minor axis.

It is worth noting that the initial out-of-straightness implied by modern column curves is generally less than the corresponding specified maximum fabrication tolerances (Beedle 1991; Galambos 1998). This disparity raises concern over the structural adequacy of real columns in relation to the design strengths provided by the Standards (Bild and Trahair 1989). However, in essence, the formulation of modern column curves is based on fitting a design model to available laboratory test

results for real columns, and performing statistical analyses to determine suitable resistance factors to achieve lower bound predictions or the required level of reliability. A similar process is also undertaken in the development of other structural design rules. Inherently embedded within the test results are a range of geometric imperfections, residual stresses, load eccentricities and material characteristics.

The fabrication limit on out-of-straightness for circular hollow sections is $L/500$, where L is the length of the member (EN 10210-2 2006; ASTM A501 2005; AS 1163 1991). However, the basic initial out-of-straightness implied in the development of the current American column curve is $L/1500$ (Beedle 1991; Galambos 1998), and in the derivation of the current European curve for tubular sections is $L/1000$ (Beer and Schultz 1970). Clearly, the initial geometric imperfections assumed in the development of both design codes is smaller than the fabrication tolerance, but still significantly greater than the mean value of out-of-straightness observed in tubular columns of approximately $L/6300$, as reported by Bjorhovde (1977). In the current study, the mean measured initial out-of-straightness for the tested elliptical tubes was approximately $L/2000$, whilst the fabrication tolerance is $L/250$ for EHS with major outer diameter (2a) less than 250 mm and $L/500$ for larger sections (EN 10210-2 2006). Since it is proposed to adopt the current CHS buckling curves for EHS, the implied initial imperfections in the buckling curves are again lower than the fabrication tolerances. The results obtained herein therefore support the wider research findings that fabrication tolerances may be unduly lax as evidenced by both the observed structural performance and measured imperfections of real columns. Thus, it is recommended that fabrication tolerances for structural steelwork be re-assessed in preference to adjusting buckling curves to accord with current tolerances. It should also be added that fabrication tolerances are based on mill products of substantial length, and may not reflect the initial out-of-straightness of columns of typical structural slenderness (Davison and Birkemoe 1982), and furthermore, for columns in real structures, the presence of end restraint and end moments may significantly reduce the influence of the out-of-

straightness. In the context of this research, the tested elliptical hollow sections, buckling about either the major or minor axis, have demonstrated similar structural behavior to their circular counterparts, and it is therefore deemed appropriate to adopt the same column curves.

Conclusions

The compressive response of hot-finished elliptical hollow section (EHS) columns has been examined in this study. A total of twenty four flexural buckling experiments was performed – twelve about the minor axis and twelve about the major axis. All tested specimens had a cross-sectional aspect ratio of two. Various column lengths were considered to give a range of non-dimensional member slenderness from 0.19 to 1.58. Measurements of initial global geometric imperfections in the EHS columns revealed average values of out-of-straightness of approximately $L/2000$, and all were within specified tolerances. Residual stresses were not explicitly measured, but minimal deformation of coupons upon machining from the complete sections and a sharply defined yield point observed in stub column tests are both indicative of low residual stress magnitudes in EHS, as is generally the case for other hot-finished tubular sections. The key material properties, geometric measurements and test results from the column buckling tests have been reported. Comparisons of sample load-deflection histories have been made against second order elastic and rigid plastic analytical approaches, revealing good agreement between experimental results and theoretical models.

To extend the pool of structural performance data on EHS columns, a detailed numerical modelling programme was conducted. Verification of the models was carefully performed against all test results using measured material properties and geometries – satisfactory replication of the full load-deflection histories and failure modes was achieved. Parametric studies were subsequently

performed to assess the structural response of EHS over a wider range of aspect ratios (from 1.0 to 3.0) and member slendernesses. A total of 158 results were generated numerically.

With the aid of both the experimentally and numerically derived results, the column buckling curves utilized in Europe, North America and Australia were examined. The elliptical hollow section columns were found to exhibit similar structural performance to their circular counterparts and it is therefore proposed that the buckling curves from EN 1993-1-1 (2005), AISC 360 (2005) and AS 4100 (1998) that are currently applied to SHS, RHS and CHS columns can also be adopted for hot-finished EHS columns, buckling about either the major or minor axis. The suitability of these design proposals was verified by means of reliability analyses in accordance with the European and AISC requirements.

Acknowledgements

The authors are grateful to the Dorothy Hodgkin Postgraduate Award Scheme for the project funding, and would like to thank Corus Tubes for the supply of test specimens and funding contributions, Andrew Orton (Corus Tubes) and Ada Law (Imperial College London) for their technical input and Ron Millward (Imperial College London) for his assistance in the laboratory works.

Notation

The following symbols are used in this paper:

A	=	gross cross-sectional area;
A_{eff}	=	effective cross-sectional area;
E	=	Young's modulus;
I	=	second moment of area;
L	=	specimen length;
L_{cr}	=	buckling (effective) length;
M_2	=	second order moment ($N\omega$);
N	=	applied axial load;
N_b	=	buckling resistance;
N_c	=	cross-section resistance;
N_{cr}	=	elastic buckling load;
N_u	=	ultimate axial load;
N_y	=	plastic yield load;
Q	=	slenderness reduction factor which defines the ratio of the stress at local buckling to the yield stress (AISC 360 2005);
V_δ	=	coefficient of variation of tests relative to resistance model (EN 1990 2002);
V_Q	=	coefficient of variation of load effects (AISC 360 2005);
V_T	=	combined coefficient of variation incorporating both model and basic variable uncertainties (EN 1993-1-1 2005);
V_R	=	combined coefficient of variation incorporating both model and basic variable uncertainties (AISC 360 2005);
a	=	half of the larger outer diameter;

'a'	=	buckling curve 'a' in EN 1993-1-1 2005;
b	=	correction factor (EN 1990 2002);
b	=	half of the smaller outer diameter;
b _{tc}	=	tensile coupon width;
f _u	=	ultimate tensile stress;
f _y	=	material yield stress;
k _{d,n}	=	design fractile factor for n tests (EN 1990 2002);
k _f	=	form factor, defined as the ratio of the effective area to the gross area of the section (AS 4100 1998);
t	=	thickness;
i	=	radius of gyration;
x	=	displacement along the specimen;
y-y	=	cross-section major axis;
z-z	=	cross-section minor axis;
Φ	=	$0.5[1 + \alpha(\lambda_{EC} - 0.2) + \lambda_{EC}^2]$ (EN 1993-1-1 2005);
α	=	imperfection factor (EN 1993-1-1 2005);
β	=	reliability index (AISC 360 2005);
α _a	=	$\frac{2100(\lambda_{AS} - 13.5)}{\lambda_{AS}^2 - 15.3\lambda_{AS} + 2050}$ (AS 4100 1998);
α _b	=	section constant (AS 4100 1998);
α _c	=	member slenderness reduction factor (AS 4100 1998);
χ	=	buckling reduction factor (EN 1993-1-1 2005);
δ	=	vertical displacement;

ϕ	=	capacity factor (AS 4100 1998);
ϕ_c	=	resistance factor (AISC 360 2005);
γ_{M1}	=	partial factor (EN 1993-1-1 2005);
η	=	$0.00326(\lambda_{AS} + \alpha_a \alpha_b - 13.5) \geq 0$ (AS 4100 1998);
λ	=	non-dimensional member slenderness;
λ_{AISC}	=	non-dimensional member slenderness for AISC 360 2005;
λ_{AS}	=	non-dimensional member slenderness for AS 4100 1998;
λ_{EC}	=	non-dimensional member slenderness for EN 1993-1-1 2005;
ω	=	lateral displacement at mid-height of column;
ω_g	=	global imperfection amplitude;
ω_i	=	initial global imperfection amplitude and
ξ	=	$\frac{\left(\frac{\lambda_{AS} + \alpha_a \alpha_b}{90}\right)^2 + 1 + \eta}{2\left(\frac{\lambda_{AS} + \alpha_a \alpha_b}{90}\right)^2} \text{ (AS 4100 1998).}$

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Fig. 1. Range of tested column lengths

(a) Schematic setup

(b) Experimental setup

Fig. 2. Column test arrangement

Fig. 3. Geometry of an elliptical hollow section

Fig. 4. 150×75×6.3 column normalized load-vertical displacement curves

Fig. 5. $150 \times 75 \times 6.3$ – C7 column load-lateral deflection curve

Fig. 6. $150 \times 75 \times 6.3$ – C8 column load-lateral deflection curve

Fig. 7. Rigid plastic model

Fig. 8. Plastic stress distribution

Fig. 9. Typical column failure mode ($150 \times 75 \times 6.3$ – C4 column)

Fig. 10. 150×75×4 – C5 column load-lateral deflection curves
(FE imperfection = $L/1000$)

Fig. 11. 150×75×6.3 – C8 column load-lateral deflection curves
(FE imperfection = $L/1000$)

Fig. 12. Piecewise linear stress-strain model

Fig. 13. Normalized test results and column buckling curves (nominal)

Fig. 14. Normalized test results and column buckling curves (design)

Table 1. Mean measured dimensions and key results from the tensile coupons tests

Tensile coupons	Width b_{tc} (mm)	Thickness t (mm)	Young's modulus E (N/mm ²)	Yield stress f_y (N/mm ²)	Ultimate tensile stress f_u (N/mm ²)
150×75×4.0-TC1	19.99	4.15	217400	380	512
150×75×4.0-TC2	20.03	4.16	217700	373	514
150×75×5.0-TC1	20.06	5.10	216900	374	506
150×75×5.0-TC2	20.15	5.08	217200	364	503
150×75×6.3-TC1	19.90	6.43	217700	381	509
150×75×6.3-TC2	19.93	6.36	215200	400	515

Table 2. Mean measured dimensions and key results from the column tests

Columns	Axis of buckling	Larger outer diameter 2a (mm)	Smaller outer diameter 2b (mm)	Thickness t (mm)	Buckling Length L _{cr} (mm)	Measured maximum global imperfection in the direction of buckling ω_g (mm)	Ultimate load N _u (kN)
150×75×4.0-C1	Minor	150.56	75.48	4.14	700	0.29	495
150×75×5.0-C1	Minor	150.08	76.00	5.13	700	0.10	614
150×75×6.3-C1	Minor	150.37	75.25	6.27	700	1.23	820
150×75×4.0-C2	Major	150.54	75.40	4.24	700	0.13	573
150×75×5.0-C2	Major	150.21	75.64	5.11	700	0.15	677
150×75×6.3-C2	Major	150.28	75.53	6.35	700	0.49	866
150×75×4.0-C3	Minor	150.44	75.53	4.20	1500	0.15	507
150×75×5.0-C3	Minor	150.31	75.48	5.19	1500	1.80	647
150×75×6.3-C3	Minor	148.36	75.62	6.30	1500	0.20	789
150×75×4.0-C4	Major	150.05	75.51	4.26	1500	0.43	538
150×75×5.0-C4	Major	150.10	76.08	5.10	1500	0.38	680
150×75×6.3-C4	Major	148.47	75.90	6.33	1500	0.25	836
150×75×4.0-C5	Minor	150.26	75.40	4.22	2300	1.09	365
150×75×5.0-C5	Minor	150.11	75.40	5.12	2300	3.38	393
150×75×6.3-C5	Minor	148.82	75.92	6.31	2300	0.89	452

150×75×4.0-C6	Major	150.34	75.46	4.17	2300	0.46	489
150×75×5.0-C6	Major	150.05	75.54	5.09	2300	1.02	611
150×75×6.3-C6	Major	148.77	75.78	6.21	2300	0.25	814
150×75×4.0-C7	Minor	150.50	75.45	4.22	3100	1.93	234
150×75×5.0-C7	Minor	149.93	75.79	5.09	3100	1.55	242
150×75×6.3-C7	Minor	148.77	75.85	6.28	3100	0.74	292
150×75×4.0-C8	Major	150.46	75.43	4.18	3100	0.25	429
150×75×5.0-C8	Major	150.03	75.67	5.13	3100	1.32	509
150×75×6.3-C8	Major	148.60	75.91	6.21	3100	0.42	648

Table 3 Comparison of the column test results with FE results for varying imperfection amplitude ω_g

Columns	FE N_u /Test N_u				Measured ω_g
	$\omega_g=L/250$	$\omega_g=L/500$	$\omega_g=L/1000$	$\omega_g=L/2000$	
150×75×4.0-C1	1.00	1.05	1.08	1.09	1.10
150×75×5.0-C1	0.96	1.02	1.04	1.07	1.07
150×75×6.3-C1	0.92	0.97	1.00	1.02	0.98
150×75×4.0-C2	0.94	0.96	0.97	0.97	0.98
150×75×5.0-C2	0.93	0.95	0.96	0.96	0.97
150×75×6.3-C2	0.94	0.96	0.97	0.98	0.98
150×75×4.0-C3	0.78	0.90	0.98	1.03	1.08
150×75×5.0-C3	0.73	0.84	0.92	0.97	0.90
150×75×6.3-C3	0.74	0.85	0.94	0.99	1.04
150×75×4.0-C4	0.92	0.98	1.01	1.03	1.03
150×75×5.0-C4	0.84	0.90	0.93	0.95	0.95
150×75×6.3-C4	0.88	0.94	0.97	0.99	1.00
150×75×4.0-C5	0.74	0.87	0.98	1.06	1.06
150×75×5.0-C5	0.81	0.95	1.07	1.15	1.00
150×75×6.3-C5	0.87	1.01	1.13	1.21	1.24
150×75×4.0-C6	0.86	0.96	1.04	1.08	1.11
150×75×5.0-C6	0.81	0.91	0.98	1.02	1.03
150×75×6.3-C6	0.76	0.86	0.93	0.97	1.00
150×75×4.0-C7	0.76	0.87	0.94	0.98	0.97
150×75×5.0-C7	0.86	0.98	1.07	1.12	1.12
150×75×6.3-C7	0.87	0.98	1.05	1.10	1.12
150×75×4.0-C8	0.80	0.94	1.05	1.14	1.25
150×75×5.0-C8	0.80	0.94	1.06	1.14	1.16
150×75×6.3-C8	0.77	0.90	1.02	1.11	1.21
MEAN	0.85	0.94	1.00	1.05	1.06
COV	0.09	0.06	0.06	0.07	0.09

Table 4 Summary of normalized results from the column tests

Columns	Cross-section classification (compression)	N_u/N_y or $N_u/A_{eff}f_y$	λ_{EC}
150×75×4.0-C1*	Slender	0.95	0.33
150×75×5.0-C1	Fully effective	0.93	0.34
150×75×6.3-C1	Fully effective	0.97	0.36
150×75×4.0-C2*	Slender	1.07	0.19
150×75×5.0-C2	Fully effective	1.03	0.20
150×75×6.3-C2	Fully effective	1.01	0.20
150×75×4.0-C3*	Slender	0.96	0.71
150×75×5.0-C3	Fully effective	0.97	0.73
150×75×6.3-C3	Fully effective	0.94	0.77
150×75×4.0-C4*	Slender	0.99	0.41
150×75×5.0-C4	Fully effective	1.04	0.42
150×75×6.3-C4	Fully effective	0.99	0.44
150×75×4.0-C5*	Slender	0.68	1.10
150×75×5.0-C5	Fully effective	0.60	1.13
150×75×6.3-C5	Fully effective	0.53	1.17
150×75×4.0-C6*	Slender	0.93	0.63
150×75×5.0-C6	Fully effective	0.93	0.64
150×75×6.3-C6	Fully effective	0.98	0.68
150×75×4.0-C7*	Slender	0.44	1.48
150×75×5.0-C7	Fully effective	0.37	1.51
150×75×6.3-C7	Fully effective	0.35	1.58
150×75×4.0-C8*	Slender	0.82	0.85
150×75×5.0-C8	Fully effective	0.77	0.87
150×75×6.3-C8	Fully effective	0.78	0.91

* Results normalized by $A_{eff}f_y$

Table 5. Summary of statistical analysis parameters for EN 1990 (2002)

Data set	No. of tests	$k_{d,n}$	b	V_{δ}	V_r	γ_{M1}
Test results	24	3.56	1.083	0.0567	0.0735	1.02
Test + FE results	48	3.31	1.076	0.0463	0.0656	0.99

Table 6. Summary of statistical analysis parameters for AISC 360 (2005)

Data set	No. of tests	V_Q	V_R	ϕ_c	β
Test results	24	0.20	0.11	0.9	3.11
Test + FE results	48	0.20	0.10	0.9	3.17