The influence of previous stress history and stress path direction on the surface settlement trough induced by tunnelling

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The pre-failure behaviour of overconsolidated soils is well known to be non-linear and inelastic. Over the last two decades experimental work has shown that pre-failure behaviour is also dependent on the previous stress history of the soil. However, some research has also suggested that, if enough time is allowed for creep to take place, then there is no effect of recent stress history; moreover, what affects soil stiffness is the direction of the stress path relative to the gross yield surface. This paper investigates the effect of two scenarios on the surface settlement trough associated with tunnelling. The first assumes that the previous stress history of the soil has an effect on soil stiffness, whereas the second assumes that creep/ageing periods have erased any memory of the previous stress history, and it is only the stress path direction that influences soil stiffness. A three-surface kinematic hardening model, which can simulate both scenarios, is employed in finite element analyses of tunnel construction. The predictions of the kinematic hardening model for the simulation of each scenario are compared with predictions using a non-linear elastic model and with field measurements.

KEYWORDS: ground movements; numerical modelling and analysis; tunnels

Le comportement préalable à la rupture de sols surconsolidés est réputé comme étant non linéaire et inélastique. Depuis une vingtaine d'années, des travaux expérimentaux ont démontré que le comportement préalable à la rupture est également fonction des contraintes historiques auxquelles le sol a été soumis. Toutefois, d'après Clayton et Heymann, si on laisse un temps suffisant pour qu'un cheminement se produise, les contraintes historiques récentes n'ont aucun effet; de plus, le facteur affectant la rigidité du sol est la direction du chemin des contraintes relativement à la surface élastique brute. La présente communication se penche sur l'effet de deux scénarios sur la dépression du tassement de la surface dans le cadre du percement de tunnels. Le premier scénario présuppose que l'historique des contraintes précédentes du sol influe sur sa rigidité, tandis que le deuxième scénario présuppose que les périodes de cheminement / vieillissement ont effacé tout souvenir de l'historique des contraintes précédentes, et que seule la direction du cheminement de la contrainte influe sur la rigidité du sol. Un modèle de durcissement cinématique de trois surfaces, pouvant simuler ces deux scénarios, est utilisé dans les analyses aux éléments finis de la construction des tunnels. On effectue une comparaison des prédictions du modèle de durcissement cinématique pour la simulation de chacun des scénarios avec les prédictions faisant usage d'un modèle élastique non linéaire, avec des mesures sur le terrain.

INTRODUCTION

It is now well established that the pre-failure behaviour of overconsolidated soils is highly non-linear and inelastic (e.g. Burland & Symes, 1982; Jardine *et al.*, 1984). Constitutive models that incorporate non-linear behaviour have been used in the numerical analysis of geotechnical structures, and have been shown to improve considerably the predictions of ground movements for overconsolidated clays (e.g. Simpson *et al.*, 1979). More specifically, in the case of tunnels, Addenbrooke *et al.* (1997) demonstrated that the introduction of non-linear pre-failure stiffness improved the predicted surface settlement profile, as compared with linear elastic models.

Subsequently, a number of experimental investigations on reconstituted overconsolidated clays have shown that soil stiffness is not only non-linear but also depends on its previous or recent stress history (Atkinson *et al.*, 1990). Atkinson *et al.* (1990) studied the effect of recent stress history by performing a number of tests on reconstituted

overconsolidated London Clay. The tests involved bringing the samples to a common stress state by following different approaching paths and then shearing along a common loading path: see Fig. 1(a). This resulted in an angle θ between the approaching path and the loading path OA equal to 0° , $+90^\circ$, -90° and 180° . A rest period of 3 h was allowed between the approaching and loading paths. The stiffness along the common loading path (i.e. path OA) was found to depend on the direction of the previous approaching path (Fig. 1(b)). Atkinson *et al.* (1990) noted that the highest stiffness at small strains is observed for the case of $\theta = 180^\circ$ (see Fig. 1(b)), which constitutes a complete reversal in the stress path direction, whereas the lowest stiffness is observed for $\theta = 0^\circ$, which is a continuation of the previous stress path direction. The effect of recent stress history decreased as the soil was loaded, and became negligible after strains of about 0.5%. On the basis of these tests and further testing Stallebrass & Taylor (1997) developed a three-surface kinematic hardening model (3-SKH) that can simulate non-linearity and the effect of recent stress history. When this model was applied in the numerical analysis of tunnel excavation, Stallebrass *et al.* (1994a) demonstrated that the assumption of different previous stress histories before the excavation of a tunnel significantly influenced the predicted undrained settlement trough above the tunnel.

However, Clayton & Heymann (2001), on the basis of similar experiments on natural Bothkennar Clay, have recently suggested that if, at the end of the approaching path,

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Fig. 1. (a) Stress paths followed in constant- p' tests; (b) stiffness of reconstituted London Clay against strain measured in constant- p' tests (from Atkinson *et al.*, 1990)

enough time is allowed for creep to take place, then recent stress history does not have an effect on soil stiffness, at least for natural soils undergoing modest excursions. Fig. 2(a) shows the stress paths followed in a single sample of Bothkennar Clay (Clayton & Heymann, 2001). Three incoming stress paths BA, CA and DA, preceding a common undrained triaxial compression path initiating from A, were investigated. A rest period, of the order of 1 to 3 days, was applied at point A before the beginning of the common loading stage. Plots of undrained stiffness against strain for the three loading paths, as presented by Clayton & Heymann (2001) (see Fig. 2(b)), show no perceptible difference in either the very small strain or the small strain region. These results indicate that sufficient creep/ageing time can erase the memory of the recent stress history. Clayton & Heymann (2001) suggested that in this case what becomes important and really influences stiffness is the direction of the current loading path, 'in as much as it dictates the rate at which the gross yield surface is reached'. Fig. 3(a) presents the compression and extension stress paths applied to a natural

sample of London Clay. The sample was initially reconsolidated to its estimated in situ stresses, point B (through path OAB), and was then sheared in compression (path BC). The sample was then taken to point B (through path CDB) and sheared in extension (path BE). Before each shearing stage, rest periods were maintained. Plots of normalised stiffness, as presented by Clayton & Heymann (2001) (see Fig. 3(b)), show that the stiffness at very small strains is the same for both paths; however, on further loading, the extension path shows lower stiffness than the compression path. It was suggested that the rest period resulted in the stiffness at very small strains reverting back to the maximum value; this was attributed to healing processes. The difference in the rate of decay of stiffness on further loading was attributed to the direction of the stress path relative to the gross yield surface: that is, softer behaviour was observed for the path that takes the loading closer to the gross yield surface (i.e. the extension path).

Clayton & Heymann (2001) noted that the above conclusion is applicable to natural soils undergoing modest

Fig. 2. Bothkennar Clay: (a) stress paths followed in single sample; (b) stiffness against strain for single specimen subjected to three identical loading paths (from Clayton & Heymann, 2001)

excursions. Hence, from the above investigations, it is not clear whether the geological stress history of a deposit, which constitutes more than a modest excursion for the soil, can be considered to have an effect on soil stiffness, when engineering construction takes place (i.e. as Stallebrass *et al.*, 1994a, have assumed), or whether the long creep/ageing period that has intervened has erased the geological stress history, leaving only the effect of stress path direction. This paper investigates these two scenarios, and their effect on the surface settlement trough induced by tunnel construction, at a site with a known previous stress history. The problem is studied through a number of finite element analyses, using the finite element program ICFEP (Potts & Zdravkovic, 1999). In order to model the two scenarios, the modified three-surface kinematic hardening model M3-SKH (Grammatikopoulou

Fig. 3. London Clay: (a) stress paths; (b) normalised stiffness against strain from undrained triaxial compression and extension tests (from Clayton & Heymann, 2001)

(2004), Grammatikopoulou *et al.* (2006)), which is a modification of the 3-SKH model (Stallebrass and Taylor (1997)), is employed. Similar to the 3-SKH model, the M3-SKH model is capable of simulating non-linearity and plasticity from the early stages of loading, as well as the effect of recent stress history. This paper also shows how this model can simulate the effect of stress path direction, as reported by Clayton & Heymann (2001).

The predictions of the M3-SKH model are also compared with the predictions of an elasto-plastic model with a non-linear pre-yield behaviour, which has already been used in the analyses of Addenbrooke *et al.* (1997). Further comparisons are made with field measurements.

The paper is structured as follows: first the boundary value problem analysed is introduced; then the constitutive model and the parameters used in the finite element analyses are briefly described. This is followed by a presentation of the ability of the M3-SKH model to predict the effect of stress path direction in simulations of undrained triaxial compression and extension tests. The details of the finite element analyses of the boundary value problem and the presentation of the results then follow. Finally the conclusions of this investigation are drawn.

BOUNDARY VALUE PROBLEM ANALYSED

For this study the geometry of the westbound tunnel of the Jubilee Line Extension (JLE) of the London Underground network beneath St James's Park (London, UK) was adopted. The soil profile and tunnel geometry are presented in Fig. 4. The analyses presented in this paper are concerned with the construction of the westbound tunnel, which took place first. No previous construction activity is known to have taken place before the construction of the tunnels.

CONSTITUTIVE MODELS

London Clay

The London Clay deposit was modelled with the M3-SKH model (Fig. 5). The M3-SKH model is an extension of the 3-SKH model (Stallebrass & Taylor, 1997), which itself is an extension of the two-surface 'bubble' model (Al-Tabbaa & Wood, 1989). Following the 3-SKH model, the M3-SKH model employs two kinematic surfaces—the yield and history surfaces—within the modified Cam Clay bounding surface (Roscoe & Burland, 1968). The main modified features of the M3-SKH model, as compared with the 3-SKH model, are as follows.

- The M3-SKH model incorporates a variety of shapes of the yield and plastic potential surfaces in the deviatoric plane, which can simulate the failure conditions of most soils better than the circular deviatoric shape adopted in the 3-SKH model (that is, if the circular shape is chosen to match the strength in triaxial compression, then this overestimates the strength in triaxial extension and under plane-strain conditions; Potts & Gens, 1984). The deviatoric shapes incorporated in the M3-SKH model include the Mohr–Coulomb hexagon and the general shape proposed by Van Eekelen (1980). The latter, depending on the choice of the input parameters, can simulate different deviatoric shapes, including the one proposed by Lade (Lade & Duncan, 1975): see Fig. 5(b).
- The M3-SKH model adopts a different hardening modulus from the 3-SKH model. This is because the

Fig. 4. Soil profile and tunnel geometry at St James's Park

Fig. 5. Modified three-surface model: (a) representation of yield, history and bounding surfaces in triaxial stress space; (b) deviatoric section through yield, history and bounding surfaces (with Van Eekelen equation taking the Lade shape)

hardening modulus of the 3-SKH model results in a non-smooth elasto-plastic transition and a sudden drop in stiffness once yielding is initiated. The new hardening modulus of the M3-SKH model ensures a smooth elasto-plastic transition (Grammatikopoulou *et al.*, 2006). Fig. 6 shows the stiffness–strain curves predicted by the two models for an undrained triaxial compression test.

Appendix 1 gives some basic equations of the model. For more details on the generalised formulation and implementation of the new model into ICFEP the reader is referred to Grammatikopoulou (2004) and Grammatikopoulou *et al.* (2006).

In the analyses presented in this paper a Mohr–Coulomb hexagon was adopted as the deviatoric shape of the yield

Fig. 6. Predictions of three-surface (3-SKH) and modified three-surface (M3-SKH) models for undrained triaxial compression test: normalised stiffness–strain curve (p'_i is mean effective stress at the start of shearing)

surface. The plastic potential was assumed to have a circular deviatoric shape, resulting in a non-associated flow rule (following Potts & Gens, 1984, who have demonstrated that, for problems involving plane strain deformation, if a Mohr–Coulomb hexagon is adopted for the deviatoric shape of the yield surface—and hence the failure surface—in a critical-state model, then a different deviatoric shape must be adopted for the plastic potential surface; otherwise plane strain failure occurs at either $\theta_f = 30^\circ$ or $\theta_f = -30^\circ$, which is clearly unrealistic). The adoption of these deviatoric surfaces introduces two additional parameters to the eight parameters of the original 3-SKH model. Moreover, one additional parameter is introduced in the new hardening modulus of the M3-SKH model. Finally, for the calculation of the elastic shear modulus G_e within the kinematic yield surface, an equation of the form proposed by Viggiani & Atkinson (1995) was adopted, hence introducing two extra parameters. In total, this form of the M3-SKH model requires 13 parameters. These are: λ^* , the slope of the isotropic normal compression line in $\ln v - \ln p'$ space; κ^* , the slope of the elastic part of the swelling line in $\ln v - \ln p'$ space; ϕ' , the angle of shearing resistance at the critical state; Y_p and Z_p , constants that define the shape of the plastic potential in the deviatoric plane; T , the ratio of the size of the history surface to that of the bounding surface; S , the ratio of the size of the yield surface to that of the history surface; α_1 and α_2 , parameters in the hardening modulus; and A_G , n and m , parameters for the calculation of the elastic shear modulus, after Viggiani & Atkinson (1995) (note that here the notation A_G is adopted for the parameter A). Finally an additional parameter is required to fix the model in $\ln v - \ln p'$ space, and this is usually taken as N , the specific volume on the isotropic normal compression line at $p' = 1$ kPa.

In addition to the M3-SKH model, a non-linear elastic pre-yield model combined with a Mohr–Coulomb yield surface (and a non-associated flow rule) was also used in the analyses. This model has already been used by Addenbrooke *et al.* (1997) for the analysis of the twin tunnels beneath St James's Park, and was called model J4. The expressions that describe the variation of the tangent shear and bulk moduli, in the non-linear range, as implemented into ICFEP, are given in Appendix 2.

Terrace Gravel

The Terrace Gravel deposit was modelled with model J4.

Made Ground/Alluvium

The Made Ground and Alluvium were modelled as a single layer using an isotropic linear elastic model.

MODEL PARAMETERS

London Clay

In the absence of test data from the St James's Park site, a set of parameters was derived for the M3-SKH model (Grammatikopoulou, 2004), on the basis of a series of laboratory tests on reconstituted overconsolidated London Clay from the Canons Park site, London (Jardine, 1985; Hight *et al.*, 2003). The parameters are given in Table 1. It should be noted that the available experimental investigation did not include any stress paths from which the size of the history surface, and hence the parameter T , could be determined. Hence this parameter was derived on the basis of a parametric study. The value of T adopted was compared with values reported in the literature. As an example, the predictions of the M3-SKH model for the simulation of an un-

Table 1. M3-SKH material parameters for London Clay

N	λ^*	κ^*	A	n	m	ϕ'	Y_p	Z_p	T	S	α_1	α_2
3.43	0.097	0.003	590.0	0.87	0.28	22.5°	0.0	1.0	0.2	0.08	6.0	35.0

drained triaxial compression test initiating from an OCR of 7 can be seen in Fig. 7. The simulation of the test involved modelling of the stress history of the sample (i.e. one-dimensional compression followed by swelling) before modelling of the shearing stage. A better match to the experimental results can be obtained if a slightly smaller size of the history surface is adopted ($T = 0.1$ rather than $T = 0.2$), keeping the size of the yield surface the same ($TS = 0.016$ in both cases); the predictions for this case can also be seen in Fig. 7. However, at the time when the tunnel analyses were carried out a value of $T = 0.2$ was adopted, so that comparisons with predictions using other three-surface and two-surface models could be made (refer to Grammatikopoulou, 2004). This value of T is in accordance with values reported in the literature: for example, Stallebrass & Taylor (1997) report a value of $T = 0.25$ for speswhite kaolin, while Stallebrass *et al.* (1994b) and Ingram (2000) adopted a value of $T = 0.2$ for London Clay in their tunnel analyses).

For the model J4 the use of the parameters adopted by Addenbrooke *et al.* (1997), for the non-linear elastic part of the model, resulted in a softer response when compared with the available test data from the Canons Park site. For this reason a new set of parameters was derived; the predicted response can be seen in Fig. 7, and the parameters are given in Tables 2 and 4 of Appendix 2.

Terrace Gravel

The parameters for the Terrace Gravel, which was modelled with model J4, were taken from Addenbrooke *et al.* (1997) and are given in Tables 3 and 4 of Appendix 2.

Fig. 7. Predictions of M3-SKH model and J4 model for undrained triaxial compression test (OCR = 7): (a) normalised stiffness–strain curve (p'_i is mean effective stress at start of shearing); (b) stress–strain curve

Made Ground/Alluvium

A Young's modulus $E = 5000$ kPa and a Poisson's ratio $\mu = 0.3$ were assumed for these layers.

MODELLING THE EFFECT OF STRESS PATH DIRECTION

The previous stress history, before the excavation of the twin tunnels, at St James's Park can be assumed to be the typical geological history of the London Clay deposit (since there is no previous activity known to have taken place). Fig. 8 shows a typical stress path followed by a London Clay element in the vicinity of the tunnels. This involved erosion of the overlying deposits and the upper part of the London Clay (path AB), followed by deposition of the gravel layers and Made Ground/Alluvium (path BC).

If it is assumed that the subsequent excavation of the westbound tunnel is influenced by the previous stress history, then the stiffness of the soil in compression is expected to be lower, at least initially, than the stiffness in extension,

since the former path is a continuation of the previous stress path, whereas the extension path is a reversal to the previous stress path direction. In the kinematic hardening models the configuration of the kinematic surfaces in stress space provides some memory of the previous stress history. The configuration of the kinematic surfaces at the end of the geological history is shown in Fig. 8(a). It can be seen that loading in extension would re-invoke the high elastic stiffness within the yield surface, whereas loading in compression would result in lower initial stiffness, since the model would continue behaving plastically.

If it is assumed that creep/ageing has erased the effect of the geological stress history and that the subsequent tunnel excavation is influenced only by the stress path direction, as reported by Clayton & Heymann (2001), then compression and extension loading is expected to show initially similar very small-strain stiffness: see Fig. 3. On further loading the stiffness in the small-strain range would be lower for the stress path that takes the loading closer to the gross yield surface: that is, in this case, the extension path. One way of simulating the scenario where creep/ageing has erased the effect of the geological stress history and there is only the effect of the stress path direction in these models is to centre the kinematic surfaces around the current stress state at the end of the previous stress history: see Fig. 8(b). In this way the kinematic surfaces do not retain any memory of the previous geological history. Moreover, loading either in compression or in extension would start from inside the elastic region, and the models would predict the high elastic stiffness initially. On further loading, the stress state would reach the kinematic yield surface, and since in these models the hardening modulus depends on the relative position of the surfaces, the stress path that takes the loading closer to the bounding surface would show a faster rate of decay of stiffness than the one that takes it away.

In order to demonstrate this, two sets of single element analyses simulating undrained triaxial compression (TXC) and extension (TXE) were carried out using the M3-SKH model. The first set of analyses simulated the stress history shown in Fig. 8(a), before the simulation of the shearing stages. The second set of analyses followed the same stress history, but at the end of it and before the simulation of the shearing stages the surfaces were centred around the current stress state (as shown in Fig. 8(b)), simulating in this way the effect of stress path direction. The former set of analyses is denoted by the letters 'sh' and the latter set is denoted by the letters 'sh-c'. The parameters used in these analyses are those given in Table 1. Fig. 9(a) shows the predictions of the model. Comparison of the compression and extension paths predicted by the analyses 'sh-c' shows that the same stiffness is predicted initially in both compression and extension. Moreover, when yielding is initiated, the stiffness predicted in extension is lower than the stiffness predicted in compression, which is in agreement with the fact that the extension path takes the loading closer to the bounding surface. Hence, by centring the kinematic surfaces, the model can predict a behaviour similar to that shown in Fig. 3(b). Comparison of the predictions of analyses 'sh' reveals that, as expected, the stiffness in extension is higher initially than the stiffness in compression. Finally, comparison of the two sets of analyses ('sh' and 'sh-c') shows that generally a different relative stiffness response between compression and extension is predicted by each set of analyses.

If the previous stress history is different from the one assumed for this site—for example, it involves only erosion—then a different stiffness response to the one shown in Fig. 9(a) would be expected, at least for the scenario in which the previous stress history influences soil stiffness. The stress history followed by Clayton & Heymann (2001)

Fig. 8. (a) Configuration of kinematic surfaces at end of geological history for element of London Clay in vicinity of tunnels; (b) centring of kinematic surfaces at end of geological history

Fig. 9. Comparison of predictions of modified three-surface model (M3-SKH) for analyses 'sh-c' and 'sh'. Normalised stiffness–strain curves for: (a) stress history of Fig. 8; (b) stress history of Fig. 3(a)

in Fig. 3(a) differs from the stress history in Fig. 8 in that it involves only unloading and no reloading. A set of single element analyses simulating the stress history and the stress paths shown in Fig. 3(a) were carried out. These analyses (denoted by the letters 'sh') were again compared with similar analyses in which the same stress history was followed, but the surfaces were centred around the current stress state, before the simulation of the undrained TXC and TXE shearing stages. The latter set of analyses is again denoted by the letters 'sh-c'. In all these analyses an assumption was made for the size of the bounding surface at point O. Fig. 9(b) plots the stiffness–strain curves predicted by these analyses. It has to be noted that although these analyses follow the stress paths of Clayton & Heymann (2001), the parameters used are those summarised in Table 1. Hence the predicted stiffness–strain curves can only be compared qualitatively with the curves plotted in Fig. 3(b) (moreover, Fig. 9 plots tangent stiffness whereas Fig. 3(b) plots secant stiffness). Comparison of the stiffness–strain curves predicted by analyses 'sh' in Fig. 9(b) shows that, for this stress history, the stiffness in compression (which is a reversal in the stress path direction) is higher than the stiffness in extension (which is a continuation of the stress path). This is in contrast to what the analyses 'sh' show in Fig. 9(a). However, the stiffness–strain curves predicted by analyses 'sh-c' show a behaviour similar to Fig. 9(a) and Fig. 3(b), although in Fig. 9(b) the difference between compression and extension, once yielding is initiated, is slightly more significant than in Fig. 9(a). This is due to the shearing stages of Fig. 9(b) initiating from a higher K_0 value than the ones in Fig. 9(a) (initial K_0 in Fig. 9(b) equal to 2.1 as compared with 1.1 in Fig. 9(a)). This means that the extension path in Fig. 9(b) starts closer to the bounding surface, and hence the stiffness along this path degrades quicker than the extension path in Fig. 9(a). Still, Figs 9(a) and 9(b) can only be compared qualitatively. This is because in the stress paths of Fig. 9(b) an assumption has to be made for the size

of the bounding surface at the beginning of the approach path, since the complete stress history of the sample is not known, whereas in Fig. 9(a) the size of the bounding surface at the beginning of the erosion stage is determined by the amount of overburden dictated by the assumed stress history.

FINITE ELEMENT ANALYSES

Analysis details

The finite element mesh used in the analyses is presented in Fig. 10. All the analyses performed were plane strain, and used eight-noded isoparametric quadrilateral elements with 2×2 integration. A modified Newton–Raphson scheme, with an error-controlled sub-stepping stress point algorithm, was used as the non-linear solver (see Potts & Zdravkovic, 1999).

Analyses with the M3-SKH model

Two analyses were performed with the M3-SKH modelling the London Clay stratum. The first analysis, analysis 1, assumed that there is an effect of the previous geological stress history, whereas the second analysis, analysis 2, assumed that creep/ageing erased the effect of the geological stress history and there is only the effect of stress path direction.

In analysis 1, the geological stress history of the London Clay deposit was modelled, followed by the subsequent excavation of the westbound tunnel. The initial conditions at the start of the geological history represented the London Clay stratum before erosion took place (point A in Fig. 8). The elements of Made Ground/Alluvium and the Terrace Gravel in the finite element mesh were absent, and an overburden stress of 1835 kPa existed at the surface of the mesh, representing approximately 180 m of overburden material. (An indication of the possible overburden depth is provided by Chandler, 2000, who noted that for Ashford Common geological evidence suggests that approximately 200 m of the upper part of the London Clay and the overlying deposits have been eroded.) The assumed overburden stress results in an overconsolidation ratio OCR of approximately 6 at the westbound tunnel axis. An initial effective stress ratio $K_0 = 0.617$ ($K_0 = 1 - \sin \phi'$) was prescribed. The modelling of the geological history involved the gradual removal of the overburden stress, representing erosion (point A to point B in Fig. 8). This was followed by the construction of the Terrace Gravel and Made Ground/Alluvium layers, modelling the reloading of the London Clay due to subsequent deposition (point B to point C in Fig. 8). During the reloading stage the water level was raised from the top of today's London Clay to the top of the Terrace Gravel, maintaining a pore water pressure distribution hydrostatic with depth. An effective stress ratio $K_0 = 0.5$ was prescribed for the Made Ground/Alluvium and Terrace Gravel. This first stage of the analysis was carried out drained. The schematic configuration of the model's kinematic surfaces at the end of the geological history, in the vicinity of the westbound tunnel, and before the tunnel excavation, can be seen in Fig. 8(a) (configuration A). The

Fig. 10. Finite element mesh

K_0 profile obtained at the end of the modelling of the geological history can be seen in Fig. 11 together with the profile calculated by the equation of Mayne & Kulhawy (1982). (The profiles show reasonable agreement, although the model underpredicts K_0 at depths greater than 15 m: at westbound tunnel depth the model predicts $K_0 = 0.95$ as compared with $K_0 = 1.1$ calculated by the equation.) The subsequent excavation of the westbound tunnel was then carried out undrained.

In analysis 2 the geological stress history was simulated as described in analysis 1. However, at the end of this stage the kinematic surfaces were centred around the current stress state, erasing in this way any memory of the previous geological stress history. The configuration of the surfaces before the excavation of the tunnel can be seen in Fig. 8(b)) (configuration B). The excavation of the westbound tunnel was carried out undrained.

Analyses with model J4

In this third analysis the J4 model was adopted. The prescribed initial conditions involved a pore water pressure distribution hydrostatic with depth, with the water level located at the top of the Terrace Gravel. The K_0 profile predicted by the M3-SKH model and shown in Fig. 11 was adopted. The excavation of the westbound tunnel was carried out undrained.

In order to model the three-dimensional tunnel excavation using plane strain analyses the 'volume loss control' method was employed (Potts & Zdravkovic, 2001). According to this method the tunnel excavation is simulated by reducing incrementally the nodal forces (equivalent to the excavated soil) around the tunnel perimeter. The volume loss developing during this process is monitored. Once the desired volume loss is achieved, the tunnel lining can be constructed. The excavation of the tunnel is then completed, allowing the forces in the tunnel lining to build up. Preliminary analyses using tunnel lining properties adopted by Addenbrooke *et al.* (1997) showed that the undrained settlement trough did not change once the tunnel lining was installed. Hence in this case the properties of the lining are unimportant.

Analyses results

Figure 12 compares the surface settlement troughs predicted by the M3-SKH model for analyses 1 and 2 at a volume loss of 1%. Also shown in the same figure are the

Fig. 11. K_0 profiles predicted at end of geological history

Fig. 12. Ground surface settlement profiles predicted by M3-SKH model (analyses 1 and 2), model J4, and Gaussian distribution

results for the non-linear model J4 and the settlement trough corresponding to the Gaussian distribution at the same volume loss.

Comparison of analyses 1 and 2 using the M3-SKH model

Figure 12 shows that analysis 1 predicts a deeper and narrower settlement profile than analysis 2.

As will be shown, the above observation can be attributed to the stiffness response experienced by the soil elements during each analysis. The tunnel excavation subjects some soil elements around the tunnel to compression paths and other elements to extension paths. As will be further discussed, the relative stiffness response between the elements subjected to compression and the elements subjected to extension influences the predicted surface settlement profile.

Owing to the different configurations of the kinematic surfaces at the start of the tunnel excavation in analyses 1 and 2, different relative stiffness responses in compression and extension would be expected for the two analyses: see Fig. 9(a). An example of an element subjected to a compression path during the tunnel excavation is the element at the springline of the tunnel, whereas an example of an element subjected to an extension path is the element at the crown of the tunnel. Fig. 13 plots the shear stiffness-strain curves experienced by these soil elements for analyses 1 and 2. (The tangent stiffness G_{oct} is defined as $G_{\text{oct}} = dq/(3d\epsilon_s) = dJ/dE_d$, where $q = J\sqrt{3} = (3/2(\mathbf{s}:\mathbf{s}))^{0.5}$ is the generalised deviatoric stress and $\epsilon_s = E_d/\sqrt{3} = (2/3(\mathbf{e}_s:\mathbf{e}_s))^{0.5}$ is the generalised deviatoric strain; $\mathbf{s}$ is the deviatoric stress tensor and $\mathbf{e}_s$ is the deviatoric component of the strain tensor. The equation for E_d is also given in Appendix 2.) Fig. 13 shows that indeed different stiffness responses are obtained for the two analyses. For analysis 1 the element at the springline (subjected to compression) shows initially much lower stiffness than the element at the crown (subjected to extension),

Fig. 13. Comparison of stiffness-strain curves at springline and crown of tunnel predicted by M3-SKH model for analyses 1 and 2

which is consistent with kinematic surface configuration A. For analysis 2 (configuration B) the stiffness response at the springline is similar to that at the crown. In the latter case this occurs because the K_0 value is close to unity at tunnel depth. The different stiffness responses predicted for the two configurations A and B are the reason why a different settlement trough is predicted by analyses 1 and 2, since this is the only difference in the analyses.

It should be also noted that analysis 1 predicts a very small heave at some distance from the tunnel axis. Presumably this is a consequence of the elastic unloading due to the removal of the weight of the soil from within the tunnel.

Comparison with non-linear model J4

In Fig. 12 the predictions of model J4 plot closer to the prediction of analysis 2 (kinematic surface configuration B) than to analysis 1. This is because the J4 model predicts similar stiffness in compression and extension, which is similar to the stiffness that the M3-SKH model predicts for configuration B. This is demonstrated by comparison of Fig. 14, which plots the stiffness-strain curves at the springline and crown of the tunnel for this analysis, with the equivalent stiffness-strain curves for analysis 2 using the M3-SKH model in Fig. 13.

Comparison with the Gaussian distribution

The empirical approach to the prediction of the transverse ground surface settlement profile caused by tunnel construction is to assume that it follows a Gaussian error curve (Peck, 1969), given by

$$S = S_{\max} e^{-\frac{x^2}{2i^2}} \quad (1)$$

where S is the vertical ground surface settlement at a distance x from the centreline of the tunnel, $S_{\max}$ is the maximum vertical settlement at the centreline of the tunnel, and i is the distance of the point of inflexion of the curve from the centreline of the tunnel. On the basis of field observations (O'Reilly & New, 1982; Rankin, 1988), a value of i equal to half the depth of the tunnel axis is usually adopted for clays, and is used here. The maximum vertical settlement at the centreline of the tunnel can be calculated from (O'Reilly & New, 1982)

$$S_{\max} = \frac{V_s}{i\sqrt{2\pi}} \quad (2)$$

where V_s is the settlement volume per unit advance.

The settlement trough corresponding to the Gaussian distribution for a volume loss of 1% is plotted in Fig. 12,

Fig. 14. Comparison of stiffness-strain curves at springline and crown of tunnel for model J4

alongside the settlement troughs predicted by the M3-SKH model (analyses 1 and 2) and model J4. Moreover, Fig. 15 compares the Gaussian distribution with the normalised surface settlement troughs predicted by the models. In both figures the predictions of analysis 1 can be seen to plot closer to the Gaussian distribution than the predictions of analysis 2 or the predictions of model J4.

Comparison with field measurements

The ground movements due to tunnelling at St James's Park were measured extensively as part of the JLE project (Standing *et al.*, 1996). A volume loss of 3.3% was measured for the westbound tunnel. This volume loss was unexpectedly high, compared with the previously measured values for tunnels driven in the London Clay, which usually varied between 1.0 and 2.0%, and is believed to be the result of a number of factors, including lower than usual undrained strengths, varying permeabilities and excavation methods associated with a rapid rate of advance (Nyren *et al.*, 2001). It should also be noted that the measured settlement trough does not follow a Gaussian distribution.

Figure 16 compares the predictions of analysis 1 with the field measurements at a volume loss of 3.3%. It can be seen that although analysis 1 predicts a deeper settlement trough than analysis 2 (in Fig. 12), and compares better with the Gaussian distribution, it still does not match the observed settlement profile.

Discussion

It is interesting to note that analysis 1 reached the volume loss of 3.3% at a high percentage of unloading (i.e. 95%). This can be attributed to two causes. First, as aforementioned, the measured volume loss was unusually high for

Fig. 15. Comparison of normalised ground surface settlement profiles predicted by M3-SKH model (analyses 1 and 2) and model J4 with Gaussian distribution

Fig. 16. Comparison of ground surface settlement profiles predicted by M3-SKH model (analysis 1) and field measurements for westbound tunnel

tunnels constructed in London Clay. Second, the parameters adopted in the analyses were derived from reconstituted tests from the Canon's Park site, and they could be too stiff for modelling the London Clay at St James's Park (for example, the parameters adopted in the analyses of Addenbrooke *et al.*, 1997 for model J4 resulted in softer behaviour than the test data from Canon's Park). For this reason an analysis with softer parameters was also undertaken (by changing the parameters α_1 and α_2 in Table 1 to $\alpha_1 = 2.0$ and $\alpha_2 = 10.0$ and repeating only the last stage of the analysis, i.e. tunnel excavation), and the results are also plotted in Fig. 16. This analysis predicted a deeper settlement trough than the analysis with the original parameters. However, the predictions are still shallower than the field measurements.

Analysis 2 was also repeated with softer parameters (again by changing the parameters α_1 and α_2 in Table 1 to $\alpha_1 = 2.0$ and $\alpha_2 = 10.0$ and repeating only the last stage of the analysis, i.e. tunnel excavation) and the results of this analysis are compared with the results of analyses 1 with softer parameters in Fig. 17. It can be seen that again analysis 1 predicts a deeper and narrower settlement trough than analysis 2. The difference in the settlement trough predicted by analyses 1 and 2 with softer parameters, in Fig. 17, is similar to that shown in Fig. 12 (note that the results are presented for different volume losses, i.e. 1% in Fig. 12 and 3.3% in Fig. 17).

Moreover, as mentioned earlier in the paper, Grammatikopoulou (2004) repeated analyses 1 and 2 with a version of the 3-SKH model implemented into ICFEP. This version of the 3-SKH model incorporates the same variety of yield and plastic potential surfaces as the M3-SKH model, but keeps the hardening modulus of the original 3-SKH model. The results of these analyses showed the same difference between analyses 1 and 2 as presented in this paper, and showed that the conclusions of the paper are independent of the kinematic hardening model adopted, as long as it can simulate the effects of both recent stress history and stress path direction. For further details on the comparison of kinematic hardening models, the reader is referred to Grammatikopoulou (2004).

The numerical analyses of this paper show that a deeper settlement trough can be predicted if the assumption is made that the previous geological stress history influences the subsequent soil stiffness, even though the predicted settlement trough still does not match the field observations. These conclusions are applicable only to the previous stress history of the site, which was simulated in the analyses. As already shown, the assumed stress history together with the predicted K_0 profile controls the relative stiffness response in compression and extension for each scenario, and hence the difference in the predicted settlement troughs. For a different geological stress history (i.e. one involving only unloading)

Fig. 17. Comparison of ground surface settlement profiles predicted by M3-SKH model (analyses 1 and 2) for softer parameters

the results could be very different, even if the K_0 profile before the tunnel excavation was the same as the one shown in Fig. 11. For example, in such a case analysis 1 would predict a stiffer behaviour in compression than in extension, at least initially (refer to Fig. 9(b)). Similarly, the results of this exercise would be different if a tunnel in normally consolidated clay was modelled, in which both the geological stress history and the K_0 profile at the end of the geological history is different. This investigation is applicable to stiff clays, in which case $K_0 \geq 1.0$. In the analyses presented in this paper $K_0 \approx 1.0$ at the level of the tunnel, which renders the stiffness in analysis 2 to be very similar in compression and extension (see Fig. 13). For higher values of K_0 , the stiffness for analysis 2 would be higher in compression than in extension (i.e. as shown in Fig. 3(b)). Hence the relative difference in stiffness between analyses 1 and 2 would be even more exaggerated—and hence, also, the difference in the predicted settlement troughs. Additional analyses of shallower tunnels, with $K_0 = 1.2$ at the tunnel level, yield the same conclusions as the current paper (Grammatikopoulou, 2004). It is important to emphasise that this paper investigated the two scenarios for a site with a known geological stress history, and all the analyses presented assumed the same K_0 profile before the tunnel excavation.

SUMMARY AND CONCLUSIONS

This study investigated the effect of two scenarios on the ground surface settlement trough induced by tunnel construction. The first scenario assumed that there is an effect of the previous geological stress history on soil stiffness, whereas the second scenario assumed that creep/ageing erased any memory of the previous geological stress history, leaving only the effect of stress path direction. These two scenarios were investigated through the use of a kinematic hardening model, the modified three-surface model (M3-SKH). A comparison of the predictions of the two scenarios showed the following results.

- The simulation of the two effects results in different relative stiffness responses between compression and extension paths.
- This difference in the predicted stiffness response influences the undrained settlement trough that is developed as a result of tunnel construction.
- For the same tunnel volume loss, a settlement trough that was deeper (and was closer to the measured trough) was predicted for the analysis that assumed that there is an effect of the previous geological stress history, and which resulted in a softer response in compression than in extension.

The predictions of the M3-SKH model were also compared with the predictions of an elasto-plastic model (model J4) with a non-linear pre-yield response and with field measurements. These comparisons showed the following.

- For the same tunnel volume loss, the predictions of the non-linear model J4 fall close to the predictions of the M3-SKH model when this simulates the effect of stress path direction. This is because model J4 predicts similar stiffness in compression and extension, which is similar to the stiffness that the M3-SKH model predicts for the effect of stress path direction.
- The predictions of the M3-SKH model for the analysis that accounted for the effect of previous stress history are closer to the Gaussian distribution than the predictions of the same model for the analysis that accounted for the effect of stress path direction.

However, the predictions are still shallower and wider than the field observations.

- (c) The adoption of softer parameters improves the predicted settlement trough, but still does not match the field measurements. Parameters that can match the field data require justification by further laboratory testing.

In summary, the numerical investigation presented in this paper shows that a more realistic, and closer to the measured, undrained settlement trough induced by tunnelling can be simulated if the assumption is made that there is an effect of the previous stress history on soil stiffness. However, it has to be emphasised that this conclusion is applicable only to the previous geological stress history of the site analysed in this paper, which involves erosion followed by redeposition. For a different geological history, different conclusions might be obtained.

The above conclusions are independent of the kinematic hardening model used in the analyses, as long as it can simulate the effects of both recent stress history and stress path direction.

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APPENDIX 1. MODIFIED THREE-SURFACE KINEMATIC HARDENING MODEL M3-SKH

The equation of the bounding surface is

$$F_b = \left(p' - \frac{p'_0}{2}\right)^2 + \frac{1}{2} \frac{\mathbf{s}:\mathbf{s}}{g^2(\theta_b)} - \frac{p_0'^2}{4} = 0 \quad (3)$$

The equation of the history surface is

$$F_h = (p' - p'_a)^2 + \frac{1}{2} \frac{(\mathbf{s} - \mathbf{s}_a):(\mathbf{s} - \mathbf{s}_a)}{g^2(\theta_h)} - T^2 \frac{p_0'^2}{4} = 0 \quad (4)$$

The equation of the yield surface is

$$F_y = (p' - p'_b)^2 + \frac{1}{2} \frac{(\mathbf{s} - \mathbf{s}_b):(\mathbf{s} - \mathbf{s}_b)}{g^2(\theta_y)} - T^2 S^2 \frac{p_0'^2}{4} = 0 \quad (5)$$

In the above equations p'_0 represents the value of the mean effective stress at the intersection of the current swelling line with the isotropic compression line, p'_a and $\mathbf{s}_a$ are the mean effective stress and the deviatoric stress tensor at the centre of the history surface, p'_b and $\mathbf{s}_b$ are the mean effective stress and the deviatoric stress tensor at the centre of the yield surface, T is the ratio of the size of the history to that of the bounding surface, and S is the ratio of the size of the yield to that of the history surface. The functions $g(\theta_b)$, $g(\theta_h)$ and $g(\theta_y)$ define the shape of the surfaces in the deviatoric plane, where θ_b , θ_h and θ_y are the values of the Lode's angle for the bounding, history and yield surfaces respectively. The Lode's angle, θ_b , is

$$\theta_b = -\frac{1}{3} \sin^{-1} \left[\frac{3\sqrt{3}}{2} \frac{\det(\mathbf{s})}{\left\{ \left[\frac{1}{2} (\mathbf{s}:\mathbf{s}) \right]^{\frac{1}{2}} \right\}^3} \right] \quad (6)$$

The Lode's angles θ_h and θ_y , for the history and yield surfaces, are calculated by using as origin the centre of the history and yield surfaces respectively (and hence replacing $\mathbf{s}$ in equation (6) with $(\mathbf{s} - \mathbf{s}_a)$ in the former case or $(\mathbf{s} - \mathbf{s}_b)$ in the latter case).

Two options are adopted for the function $g(\theta)$. The first one gives the shape of a Mohr–Coulomb hexagon in the deviatoric plane (see Fig. 5(b)),

$$g(\theta) = \frac{\sin \phi'}{\cos \theta + \frac{\sin \theta \sin \phi'}{\sqrt{3}}} \quad (7)$$

where ϕ' is the angle of shearing resistance.

The second option gives the family of continuous surfaces in the deviatoric plane proposed by van Eekelen (1980),

$$g(\theta) = \frac{X}{(1 + Y \sin 3\theta)^Z} \quad (8)$$

where X , Y and Z are material constants. In particular, $X = \sin(\phi'_{\theta=0^\circ})$, where $\phi'_{\theta=0^\circ}$ is the angle of shearing resistance associated with a Lode's angle $\theta = 0^\circ$.

The plastic potential surfaces are given by equations of the form of equations (3) to (5) but with the functions $g(\theta)$ replaced by functions $g_p(\theta)$, which define the shape of the plastic potential surfaces in the deviatoric plane. In the case of the plastic potential the continuous surfaces proposed by van Eekelen (1980) are adopted; Y and Z in equation (8) are replaced by the material constants Y_p and Z_p , whereas X is replaced by X_p , which varies such that when the soil is yielding, the current plastic potential surface always passes through the current stress state.

For the case where the yield and history surfaces are moving within the bounding surface, the hardening modulus is expressed by an equation of the form

$$A = A_0 + A_1 + A_2 \quad (9)$$

where

$$A_0 = \frac{4}{\lambda^* - \kappa^*} \left\{ (p' - p'_b) \left[p'_b(p' - p'_b) + \frac{1}{2} \left(\frac{\partial F_y}{\partial \mathbf{s}} : \mathbf{s}_b \right) + \frac{T^2 S^2 p_0'^2}{4} \right] \right\} \quad (10)$$

$$A_1 = \frac{4}{\lambda^* - \kappa^*} \alpha_1 \left(\frac{b_1^*}{b_{1\max}^* - b_1^*} \right) \left(\frac{p'_0}{2} \right)^3 T^2 S^2 \quad (11)$$

$$A_2 = \frac{4}{\lambda^* - \kappa^*} \alpha_2 \left(\frac{b_2^*}{b_{2\max}^* - b_2^*} \right) \left(\frac{p'_0}{2} \right)^3 T^2 S^2 \quad (12)$$

In the above equations, α_1 and α_2 are material parameters, b_1^* is a measure of proximity of the yield and bounding surfaces, b_2^* is a measure of proximity of the yield and history surfaces and $b_{1\max}^*$ and $b_{2\max}^*$ are their maximum values.

APPENDIX 2. MODEL J4

The equations describing the variation of the tangent shear modulus G and tangent bulk modulus K in the non-linear range are

$$\frac{3G}{p'} = A + B \cos(\alpha X^\gamma) - \frac{B \alpha \gamma X^{\gamma-1}}{2.303} \sin(\alpha X^\gamma) \quad (13)$$

$$\frac{K}{p'} = R + S \cos(\delta Y^\eta) - \frac{S \delta \eta Y^{\eta-1}}{2.303} \sin(\delta Y^\eta) \quad (14)$$

where

$$X = \log_{10} \left(\frac{E_d}{\sqrt{3}C} \right) \quad (15)$$

$$Y = \log_{10} \left(\frac{\varepsilon_v}{T} \right) \quad (16)$$

where A , B , C , R , S , T , α , γ , δ and η are material constants; ε_v is the volumetric strain; and E_d is the deviatoric strain, defined as

$$E_d = \frac{2}{\sqrt{6}} \left[(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_2)^2 \right]^{\frac{1}{2}} \quad (17)$$

Owing to the trigonometric nature of equations (13) and (14), minimum ($E_{d\min}$, $\varepsilon_{v\min}$) and maximum ($E_{d\max}$, $\varepsilon_{v\max}$) strain limits are set, below and above which the tangent shear and bulk moduli vary only with mean effective stress. Furthermore, the magnitude of the stiffness is prevented from falling below specified minimum values, $G_{\min}$ and $K_{\min}$.

Table 2. Material parameters for the non-linear elastic part of model J4 for London Clay

A	B	C : %	α	γ	E_{dmin} : %	E_{dmax} : %	G_{min}
1600.0	1550.0	1.0×10^{-4}	1.2	0.617	3.464×10^{-3}	0.692 820	2333.3
R	S	T : %	δ	η	ε_{vmin} : %	ε_{vmax} : %	K_{min}
600.0	580.0	1.0×10^{-3}	1.9	0.420	5.0×10^{-3}	0.15	3000.0

Table 3. Material parameters for the non-linear elastic part of model J4 for Terrace Gravel

A	B	C : %	α	γ	E_{dmin} : %	E_{dmax} : %	G_{min}
1380.0	1248.0	5.0×10^{-4}	0.974	0.940	8.833×10^{-4}	0.34641	2000.0
R	S	T : %	δ	η	ε_{vmin} : %	ε_{vmax} : %	K_{min}
275.0	225.0	2.0×10^{-3}	0.998	1.044	2.1×10^{-3}	0.20	5000.0

Table 4. Mohr–Coulomb yield and plastic potential parameters for model J4

Parameters	Terrace Gravel	London Clay
c'	0	0
ϕ'	35.0°	22.5°
v	17.5°	11.25°

NOTATION

A_G	parameter in calculation of elastic shear modulus
E_d	deviatoric strain invariant in general stress space
$E_{u,tan}$	undrained tangent Young's modulus
$G_{oct,tan}$	tangent octahedral shear stiffness
J	deviatoric stress invariant in general stress space
K_0	earth pressure coefficient at rest
m	parameter in calculation of elastic shear modulus
N	specific volume on isotropic compression line at $p' = 1$ kPa
n	parameter in the calculation of elastic shear modulus
p'	mean effective stress
p'_i	mean effective stress at beginning of shearing
q	deviatoric stress invariant in triaxial plane
S	ratio of size of yield surface to that of history surface
T	ratio of size of history surface to that of bounding surface
v	specific volume
α_1	parameter in hardening modulus
α_2	parameter in hardening modulus
ε_s	deviatoric strain in triaxial plane
φ'	angle of shearing resistance
κ^*	slope of elastic part of swelling line in $\ln v$ – $\ln p'$ space
λ^*	slope of isotropic normal compression line in $\ln v$ – $\ln p'$ space

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