A study of buckling in steel silos under eccentric discharge flows of stored solids

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Abstract

The most serious loading condition for slender thin-walled metal silos has long been recognized to be the condition of discharge, with eccentric discharge causing more catastrophic failures than any other. Two key reasons for this high failure rate are the difficulties in characterizing the pressure distribution caused by eccentric solids flow, and in understanding the associated unsymmetrical stresses in the silo wall. Few studies have addressed either the linear elastic behavior of such a silo or its buckling failure under eccentric discharge.

In this study, the eccentric discharge pressures are characterized using the new rules of the European Standard EN 1991-4 on Silos and Tanks. This novel description of unsymmetrical pressures permits a study of the structural behavior leading to buckling during eccentric discharge, including the critical effects of change of geometry and imperfection sensitivity, to be undertaken using geometrically and materially nonlinear computational analyses. The mechanics of the behavior are found to be quite complicated. A silo which is safe under axisymmetric loading is found to be susceptible to catastrophic stability failure under eccentric discharge.

ASCE Keywords: Thin shell structures; Solids flow; Structural stability; Buckling; Nonlinear analysis; Computer analysis;

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Introduction

Design criteria for metallic silos are dominated by their susceptibility to buckling failures. The unsymmetrical pressure regime occurring during eccentric discharge of the stored granular material frequently leads to catastrophic buckling failures, and slender silos are particularly susceptible to this failure mode. The commonest failure mode in a slender thin-walled steel silo under eccentric discharge is by serious damage on the side adjacent to the flowing solid, but the cause of such failures has not been widely appreciated in the past. In this paper, it is shown that the failure mode is by buckling under local axial compressive membrane stresses induced by unsymmetrical normal pressures exerted between the solid on the wall. This explanation follows that of Rotter (1986, 2001a, 2001b) but runs counter to those previously offered by Jenike (1967), Wood (1983), Roberts and Ooms (1983) and others. The buckling failure in the example structure occurs well before the material yields and is thus elastic.

Fig. 1 – Elevation and design dimensions of an example cylindrical silo used in this study with eccentric discharge of stored solids (after EN 1991-4, 2007).

Discharge in axisymmetric mass flow results in a significant increase in both the normal pressures and frictional tractions on the silo wall (Rotter, 2001a; 2007), which vary rather erratically and are difficult to characterize with simple equations (Ooi et al, 1990; 2005; Nielsen, 1998). These varying stresses are normally accounted for in design by simple multiplication factors which are based on concepts from quite simple theories (Jenike et al., 1973; Drescher, 1991; Nedderman, 1992) which do not capture the experimentally observed phenomena well (Rotter, 2007). Highly eccentric discharge in dense solids leads to a much

more clearly differentiated pressure pattern associated with the formation of an unsymmetrical pipe flow channel (Fig. 1). The low pressures where the flow channel is in contact with the silo wall were shown by Rotter (1986) to lead to dangerous stress patterns in the silo wall, which is easily catastrophic for structural stability. Despite this risk, eccentric discharge may be necessary for the effective operation of the silo, or it may occur accidentally due to segregation or agglomeration of the contents or partial blockage of an outlet.

In older design standards (AS3774, 1996; DIN 1055-6, 1987; ISO11697, 1995), the effect of eccentricity of solids flow was treated as an unsymmetrical supplement to the axisymmetric solid pressures. Its influence on the stresses was expressed through the application of a rectangular outward 'patch' load of normal pressures at an arbitrary height where its presence was thought to be most damaging. This approach rightfully identified unsymmetrical normal pressures, rather than frictional tractions, as the main catalyst for failure. However, the common misconception that failure in silos is governed by material yielding due to circumferential bending and tension (Jenike, 1967; Wood, 1983; Roberts & Ooms 1983) caused silo designers to treat the shell as a simple ring. Although this model may be appropriate for thick-walled reinforced concrete silos, thin-walled metal silos behave entirely differently. Bursting failure in metal silos is uncommon except where inadequate bolted joints are used (Rotter, 2006).

Some research has since been done to investigate the effect of horizontal patch pressures on silo stability using linear elastic analyses. Gillie and Rotter (2002) and Song (2004) found that the size, magnitude and location of a patch may each have a deleterious effect on both the membrane and bending stresses in the silo wall. Codified representations of patch loads differ considerably from one standard to another. Song and Teng (2003) and Song (2004) showed that although the patch load is indeed very detrimental in linear elastic and linear bifurcation analyses, in geometrically nonlinear analyses the bifurcation loads were little different to those with no patch load. This treatment therefore cannot be relied upon to offer a safe design for eccentric discharge.

In this study, the unsymmetrical pressures caused by eccentric discharge are investigated using the new rules of the European Standard EN 1991-4 (2007), based on the theory of Rotter (1986, 2001a, 2001b). This theory proposes a distribution for the pressures resulting from a parallel-sided circular flow channel forming against the wall, shown in Fig. 2. The solid exerts Janssen pressures outside the channel, elevated pressures at the edges and

decreased pressures within the flow channel, thus simplifying what has been observed in experiments. The relationship between the pressure drop and increase is such that horizontal equilibrium is satisfied, though it does lead to a global overturning moment on the silo. EN 1991-4 requires this distribution to be used on silos whose capacity exceeds 1,000 tonnes with eccentric discharge.

Fig. 2 – Circumferential cross-section of eccentric flow channel horizontal pressures (after EN 1991-4, 2007).

The horizontal static pressures, p_{hse} , at any depth far from the flow channel are taken as equal to the Janssen silo pressures:

$$p_{hse} = \gamma K z_o (1 - e^{-z/z_o}) \text{ where } z_o = \frac{A}{KU\mu} \quad (1)$$

The horizontal pressures within the flowing zone are treated as constant and are given by:

$$p_{hce} = \gamma K z_{oc} (1 - e^{-z/z_{oc}}) \text{ where } z_{oc} = \frac{1}{K} \left(\frac{A_c}{U_{wc}\mu + U_{sc} \tan \phi_i} \right) \quad (2)$$

The pressures at the edges of the channel are simply treated as increased to the same extent as the channel pressure drop over an identical circumferential range:

$$p_{hae} = 2p_{hse} - p_{hce} \quad (3)$$

The associated frictional tractions, p_{wse} , p_{wce} and p_{wae} , are all:

$$p_w = \mu p_h \quad (4)$$

Full definitions can be found in EN 1991-4 (2007) and derivations in Rotter (1986, 2001b). The magnitudes of the flow channel pressures depend on the size of the channel, defined by

its radius and eccentricity, r_c and e_c , respectively. EN 1991-4 recommends at least three different values of r_c to be tested to find the most destructive flow channel geometry. These values are suggested as equal to $k_c r$ where k_c is 0.25, 0.4 or 0.6. The focus in this paper is on the k_c value of 0.6, as it is expected to be the most deleterious.

Silo Design

A simple cylindrical steel silo was designed for symmetrical loads arising from 370 tonnes of wheat, as shown in Fig. 1. Structural design was done according to EN 1993-1-6 (2007). The properties for wheat were taken from EN 1991-4 (2007) using the maximum friction case since buckling resistance dominates in such a design.

The silo of height 18 m and radius 3 m had an aspect ratio of 3 (classed as Slender). The requirement for an unsymmetrical patch load was omitted for simplicity. Action Assessment Class 2 was assumed based on the storage capacity. The beneficial effect of internal pressure was included when designing against buckling. The discharge factors for normal pressures and frictional tractions, C_h and C_w , were taken as 1.15 and 1.1 respectively. The partial safety factor for unfavorable structural actions and the resistance partial safety factor for stability, γ_F and γ_{M3} , were taken as 1.5 and 1.1 respectively, thus separating the characteristic values by a factor of $1.5 \times 1.1 = 1.65$. This value is important in the context of the outcome of later nonlinear calculations against which it may be assessed.

A Fabrication Tolerance Quality Class of C (i.e. 'normal') was adopted making the shell more imperfect and thus requiring a thicker wall. The material of the shell was assumed to be isotropic steel with an elastic modulus of 200 GPa, a Poisson's ratio of 0.3 and a yield stress of 250 MPa.

Two silo designs were produced: one with a uniform wall thickness (6 mm) for clarity of understanding and one with a stepwise varying wall thickness (changing from 3 mm at the top to 6 mm at the base) to follow engineering practice. This made the wall just thick enough at the base of each strake and at the silo base. The design axial membrane stress resultants are shown in Fig. 3, while Fig. 4 shows the corresponding design thicknesses as well as those required to withstand simple bursting failure. Selected wall properties are summarized in Table 1.

Fig. 3 – Axial distribution of the characteristic and design axial membrane stress resultants.

Fig. 4 – Axial distribution of required design thicknesses to resist bursting and buckling.

Numerical Model

The two silos were analyzed using the commercial finite-element package ABAQUS (ABAQUS, 2006). A pinned based was assumed, as well as a thin conical roof of inclination 15° to the horizontal to realistically restricted out of round displacements at the upper boundary. These strongly affect silos under unsymmetrical loads (Rotter, 1987; Calladine 1983).

Using symmetry boundary conditions only half of the silo was modeled with eight-node reduced-integration S8R5 elements. The conical roof was modeled with sufficient four-node S4R5 elements. After careful verification, the mesh resolution was increased near changes of wall thickness, weld depressions, flow channel and edge pressures and at locations of expected buckles. Model mesh details are shown in Fig. 5 for the perfect uniform thickness silo and the imperfect varying thickness silo. The geometrically nonlinear load-deflection path was followed using the modified Riks procedure (Riks, 1979). Plasticity was assumed to be ideal with no hardening. This was sufficient in this study because only limited plasticity develops.

Fig. 5 –ABAQUS model mesh details – left) base of perfect uniform thickness silo (336,666 total dofs); right) – base of imperfect varying thickness silo (627,906 total dofs)

The full suite of computational shell buckling calculations were performed according to EN 1993-1-6 (2007):

LA – Linear Elastic Analysis to find the reference stresses

LBA – Linear Bifurcation Analysis to find the lowest linear buckling eigenvalue and eigenmode

MNA – Materially Nonlinear Analysis to find the reference plastic collapse load

GNA – Geometrically Nonlinear Analysis to find the lowest bifurcation load and associated buckling mode

GNIA – Geometrically Nonlinear Analysis with Imperfections to find the same

GMNA – Geometrically and Materially Nonlinear Analysis to find the same

GMNIA – Geometrically and Materially Nonlinear Analysis with Imperfections to find the same

Characteristic values of loads were adopted in all calculations so that the outcome of the calculation could be used to explore the remaining safety margin between the required and characteristic resistances. The stored solid was assumed to have no stiffness in both the static or flowing zones.

Axisymmetric imperfections representing modified Type A weld depressions as defined by Rotter and Teng (1989), given in Eq. 6 below, were introduced at evenly-spaced intervals up the silo so that the wall had many local imperfections. This imperfection form was developed as a realistic simulation of the joints created during welding of metal sheets, and has been widely used in numerical studies of imperfection sensitivity in cylinders because it is one of the most damaging credible imperfection forms for a uniformly compressed cylinder (Teng & Rotter, 1992; Song et al., 2004).

The purpose of these GNIA and GMNIA analyses is to obtain a realistic estimate of the effects of this commonly found imperfection form (Ding et al., 1996; Teng et al., 2005). This contrasts with the EN 1993-1-6 (2007) requirement that the analyst should seek out the most damaging imperfection form.

The adopted imperfection form (Eq. 5) includes an additional tiny non-symmetric perturbation to help identify the correct first bifurcation point in the load-deflection path (visible change of slope and/or first negative eigenvalue in the tangent stiffness matrix) by introducing a small asymmetry. The perturbation factor, k , was taken as 10^{-2} and n , the integer circumferential mode number, was given a value between 10 and 30, but the outcome of all calculations was insensitive to this choice.

$$\delta = \delta_0 e^{\frac{-\pi z}{\lambda}} \left(\cos \frac{\pi z}{\lambda} + \sin \frac{\pi z}{\lambda} \right) (1 + k \cos n\theta) \quad (5)$$

where $\lambda = \frac{\pi}{[3(1-\nu^2)]^{0.25}} \sqrt{rt}$ is the linear meridional bending half-wavelength (6)

The depression amplitude, δ_0 , was chosen to be the value from the hand design process according to EN 1993-1-6 (2007) Annex D for Fabrication Tolerance Quality Class C. This was done in preference to the special requirement for GMNIA design as specified in Section 8.7.2 of the same standard. Both are shown in Table 1. This choice was made to produce a designed silo that corresponds to typical commercial practice and to retain scientific consistency between the design and the calculations.

Table 1 – Summary of selected properties and dimensionless imperfection amplitude requirements according to EN 1993-1-6 Annex D and Section 8.7.2

Thickness (mm)	Radius to thickness ratio	Membrane stress resultant at yield (N/mm)	Linear meridional bending half- wavelength (mm)	Dimensionless imperfection amplitude (δ/t)	
				Annex D, Eq. D.15	Section 8.7.2, Eq. 8.29
3	1000	750	174.11	1.976	3.162
4	750	1000	201.05	1.712	2.739
5	600	1250	224.78	1.531	2.450
6	500	1500	246.23	1.398	2.236

Results and Discussion

Behavior of the silo under axisymmetric discharge pressures

The silo was first analyzed under axisymmetric loading at characteristic discharge values obtained from the hand design calculations. The hand design process includes many conservative assumptions so it is natural to expect that the design safety margin (= 1.65) will be exceeded when the silo is analyzed using a GMNIA analysis. A summary of the load factors achieved for the case of concentric discharge is given in Table 2. The buckling modes for these load factors are shown in Figs 6 and 7.

For the uniform thickness silo under axisymmetric loading (Fig. 6), the critical buckling location is naturally always at the base of the silo. The similarity of the LBA and MNA load

factors suggest the stability and plasticity will interact, which is confirmed by the large difference between the GNA and GMNA load factors. The load factors at failure relate to the plastic elephant foot's mode (Rotter, 1990; 2006).

Table 2 – Summary of load proportionality factors at failure for concentric discharge

	LBA	MNA	GNA	GMNA	GNIA	GMNIA
Uniform thickness	8.92	6.77	7.44	4.42	5.61	4.13
Varying thickness	7.87	6.78	7.78	4.75	3.31	3.30

The critical locations in the stepped wall thickness silo (Fig. 7) are at the base of each strake, with the location most susceptible to buckling being at the base of the top 3 mm strake at a height of 9.8 m (almost midheight), also in the elephant's foot mode. The MNA, GNA and GMNA load factors are very similar for both silos and this is reflected in the similarity of the buckling modes. However, the GNIA and GMNIA factors are identical for the stepped thickness silo, meaning that the buckling is entirely in the elastic region, reflected in the global diamond buckling mode (Fig. 7). Ultimately, the silo with stepped wall thickness is slightly weaker than the uniform thickness silo because the wall thickness is minimized. The more sophisticated the analysis, the lower the load factor: hence both geometric and material nonlinearity must be included in silo design.

The final GMNIA load factor of 3.30 exceeds the hand calculation value of 1.65 by almost a factor of 2, indicating that the assumptions in the hand calculation design process are very conservative indeed. This is primarily because the EN 1993-1-6 elastic imperfection reduction factor on the buckling strength is obtained empirically as a lower bound to a very wide scatter of experimental data on uniform thickness cylinders (Rotter, 2004), thus making a highly conservative estimate of the buckling strength which in turn leads to a very conservative design. Secondly, this lower bound is so low that no FEA calculation appears to be able to reproduce it for any imperfection mode having the amplitude assumed in the standard (Rotter, 1997). Thirdly, the critical location in any stepped-wall silo is in the thinner plate at a change of plate thickness. The thicker plate at this change provides considerable restraint against buckling and thus raises the buckling resistance (Rotter and Teng, 1989). These three factors

combine to make the computed strength of the structure much greater than the hand design assessed value.

Fig. 6 – Incremental buckling modes for the uniform thickness silo under concentric discharge. From left to right: LBA (8.92), GNA (7.44), GMNA (4.42), GNIA (5.61) and GMNIA (4.13). The geometric scale factors are 1880, 200, 1000, 200 and 1000 respectively.

Fig. 7 – Incremental buckling modes for the stepped thickness silo under concentric discharge. From left to right: LBA (7.87), GNA (7.78), GMNA (4.75), GNIA (3.31) and GMNIA (3.30). The geometric scale factors are 1880, 1000, 1500, 100 and 100 respectively.

Behavior of the silo under eccentric discharge

The goal of this paper is to explore the behavior of the silo under eccentric discharge. The pattern of pressures arising in this condition is quite realistically characterized in EN 1991-4 (2007) and the following calculations, believed to be the first of their kind to use this new pressure model, give a good insight into many silo disasters as well as other phenomena.

Behavior of the uniform thickness silo

Under the set of unsymmetrical pressures associated with a flowing channel of stored solid (Fig. 2), very high axial compressive membrane stresses develop at midheight at the centre of the flow channel. By contrast, the stresses are tensile at the edge of the channel for most of the silo height, becoming compressive at the base of the silo. At these key locations, the axial compressive membrane stresses are enormously greater than those for which the silo was designed (about 2300 N/mm as opposed to 280 N/mm, see Fig. 3). This stress distribution, originally described by Rotter (1986, 2001b), is shown in Fig. 8 for the GMNA analysis at the instant before bifurcation.

Fig. 8 – Axial membrane stress resultant distribution (at the instant before bifurcation, load factor of 1.01) down the vertical axis at various circumferential locations for the uniform thickness silo under eccentric discharge analyzed with GMNA. The LA distribution is at the same load factor.

A summary of the load factors achieved for the uniform thickness silo under the unsymmetrical pressures caused by eccentric discharge with $k_c = 0.6$ is presented in Table 3. Where a cylindrical shell is subjected to non-symmetric pre-buckling displacements, the post-bifurcation deformations naturally contain components of both the pre-buckling and post-buckling modes. The mode that characterizes the bifurcation process is clearly only incremental, and is obtained by subtracting the pre-buckling deformation at bifurcation from the complete deformation just after bifurcation. This is termed the incremental buckling mode, and these modes are shown in Fig. 9, which indicates the change of deformed shape and the nonlinear load-axial deflection curves are shown in Fig. 10.

Table 3 – Summary of load factors for the uniform thickness silo under eccentric discharge

LBA	MNA	GNA	GMNA	GNIA	GMNIA
0.73	1.81	1.99	1.01	1.54	0.98

Fig. 9 – Incremental buckling modes for the uniform thickness silo under eccentric discharge. From left to right: LBA (0.73), GNA (1.99), GMNA (1.01), GNIA (1.54) and GMNIA (0.98).

The geometric scale factors are 500, 500, 500, 2500 and 1000 respectively.

Fig. 10 – Plot of nonlinear load-axial deflection curves for the uniform thickness silo analyses. The node being followed is at the top of the cylindrical shell in the middle of the flow channel.

The compressive stresses at the bottom of the silo at the edge of the flow channel are by far the largest and, since the wall thickness is uniform everywhere, they are responsible for the localized modes seen throughout Fig. 9. The silo wall is also vulnerable at midheight at the centre of the flow channel and this mode could dominate if the wall were thinner at this location. Opposite the channel, the axial membrane stress resultant is unaffected and corresponds to the axisymmetric loading case, not exceeding 280 N/mm.

These load proportionality factors are all much lower than those obtained for concentric discharge and the design which was so conservative under symmetric loads is no longer safe since the GMNIA factor approaches unity. The GNA factor is, astonishingly, almost triple the LBA factor, and the load-deflection curve shows the structure stiffening as it deforms under the unsymmetrical loads. Since the applied pressures induce high circumferential bending, geometric nonlinearity results in a considerable change in geometry and increased stiffness against buckling, a phenomenon not captured by the linear assumption. Indeed, Fig. 8 shows that the LA stresses are still significantly bigger (almost 90% larger at midheight when scaled

to the GMNA factor) than the GMNA stresses. As a result, the LBA predicts a much lower bifurcation load.

Material plasticity (GNA $\rightarrow$ GMNA) reduces the silo strength and changes the buckle shape towards a local elephant's foot mode. The effect of plasticity seems to affect the load-deflection paths very suddenly (Fig. 10), but since the axial deformations are being followed at a node at the very top of the silo they are somewhat insensitive to local buckling deformations at the silo base. The sharp bifurcation and load path reversal shown for all uniform wall thickness silos demonstrates unstable post-buckling behavior. The plastic collapse MNA calculation relates to a circumferential bending mechanism, which was the misguided focus of many previous studies of eccentric discharge (Jenike, 1967; Wood 1983). The corresponding load factor is very high and by itself does not reflect the true structural behavior in this condition.

Fig. 11 – Combined membrane stress resultant distribution (at the instant before bifurcation, load factor of 1.01) down the vertical axis at various circumferential locations for the uniform thickness silo under eccentric discharge analyzed with GMNA.

The GMNIA result is very similar to the GMA result because the critical mode is near the base and is distant from any of the local imperfections. The circumferential distributions of the axial and circumferential membrane stress resultants are shown in Fig 11 through the buckle near the silo base and at midheight.

Behavior of the stepped thickness silo

The more realistic stepped wall design was next explored in the same manner. A summary of the load factors achieved under eccentric discharge with $k_c = 0.6$ is given in Table 4. Those marked with an asterisk represent values at the first inflexion point on the load-displacement curve, see Fig. 12. The incremental buckling modes are shown in Fig. 13. The axial membrane stress resultant distribution for the perfect shell just before bifurcation is shown in Fig. 14.

Fig. 12 – Plot of nonlinear load-axial deflection curves for the stepped thickness silo analyses.

The node being followed is at the top of the cylindrical shell in the middle of the flow channel.

Table 4 – Summary of load factors for the stepped thickness silo under eccentric discharge

LBA	MNA	GNA	GMNA	GNIA	GMNIA
0.43	1.14	0.62	0.62	0.28*	0.28*

These load factors are all significantly below unity, with the exception of MNA which does not contribute to the behavior at all. This silo, designed to EN 1991-4 (2007) to comfortably withstand axisymmetric loading, is now wholly inadequate under eccentric discharge. Many of the observations for the uniform wall thickness silo are also valid here.

Fig. 13 – Incremental buckling modes for the stepped thickness silo under eccentric discharge.

From left to right: LBA (0.43), GNA & GMNA (0.62) and GNIA & GMNIA (0.28). The geometric scale factors are 100, 6000 and 200 respectively.

The base wall thickness is same for both silos, so any difference in behavior between the uniform and stepped wall silos relates to failures not occurring at the base. From Fig. 13 it is evident that buckling occurs at midheight in all stepped wall analyses, as this region experiences high axial compressive membrane stresses and is always a part of the silo that has a thinner wall. The thinner wall reduces the linear bifurcation stress and also causes a greater strength reduction due to imperfections. Consequently, these buckling stresses are lower than they were for the uniform thickness silo (Figs 14 and 15). Thus, in changing from a uniform to a stepped wall, the critical buckling location moves from the edge of the flow channel at the base to the centre of the channel at midheight. This elastic midheight buckle relates well to known failures in service. Eccentric discharge leads to very serious elastic stability failure at very low stresses near the midheight of a realistic stepped wall silo.

Fig. 14 – Axial membrane stress resultant distribution (at the instant before bifurcation, load factor of 0.62) down the vertical axis at various circumferential locations for the stepped thickness silo under eccentric discharge analyzed with GNA and GMNA. The LA distribution is at the same load factor.

Fig. 15 – Membrane stress resultant distribution around the circumference at the location of the buckle at the instant before bifurcation for the stepped thickness silo under eccentric discharge analyzed with GNA or GMNA.

In the stepped wall silo, the GNA factor is 44% higher than the LBA factor. This surprising outcome is again caused by geometric nonlinearity reducing the stresses in the structure, leading to a lower elastic bifurcation load. The GNA and GMNA load-deflection path, shown in Fig. 12, is typical of unstable post-buckling behavior. With imperfections, however, the GNIA and GMNIA path shows a clear point of inflexion followed by indefinite geometric hardening with a progressive growth of the imperfection mode. With no negative eigenvalues reported at the change of slope, it is evident that the bifurcation point has been lost. This phenomenon often occurs when imperfection amplitudes are large, leading to a smooth transition from pre- to post-buckling behavior.

Conclusions

The following conclusions may be drawn based on the results of this study:

1. A silo designed according to the new rules of the European Standard for the condition of concentric filling, storage and discharge of contents is found by non-linear finite element analysis to have a significant reserve of strength under the design loads (GMNIA load factor of 3.3) beyond what is required in the structural assessment (safety factor of 1.65). This is due to the conservatism of the assumptions upon which the hand design procedure is founded.
2. The European Standard EN 1991-4 has divided silos into 3 categories according to size and complexity. The largest capacity silos must be designed explicitly for eccentric discharge. This example silo, still a considerable structure (6 x 18 meters), falls far short of this requirement for an explicit eccentric discharge calculation. The low load factors obtained in this study indicate that a silo that functions perfectly under normal symmetrical conditions may experience a catastrophic stability failure if an eccentric pipe flow channel develops. This has often occurred in service.
3. Under the eccentric discharge pressures of EN 1991-4, the regions of highest compressive axial membrane stress are at the centre of the channel at midheight and at the edge of the channel at the base of the silo. These are both critical regions where this pressure model indicates that the silo may buckle.

4. At midheight, very high axial membrane stresses develop in the thin wall and the buckling mode is elastic. The low internal pressure in the flow channel also reduces the imperfect pressurized buckling strength. This midheight buckling mode has often been observed in practice and is responsible for many failures.
5. At the base of the silo, the internal pressures are much higher so the buckling mode becomes plastic. There are two main reasons why this mode is not observed in practice: the narrower or absent flow channel in real structures and the elastic restraint provided by the stationary solid.
6. The behavior of the cylindrical shell under eccentric discharge pressures has been found to be very complicated with several counter-intuitive phenomena. A clear explanation for the enhanced strength caused by geometric nonlinearity when the shell curvature is reduced at the centre of the flow channel is still needed.
7. Geometric nonlinearity, which reduces buckling loads under axisymmetric conditions and is commonly thought to be highly detrimental, surprisingly gives additional strength under this form of unsymmetrical load. Geometric imperfections in the form of axisymmetric weld depressions were explored and were found to reduce the strength only slightly. Calculations involving deeper imperfections require very careful interpretation.

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