Towards a Miniaturized Needle Steering System with Path Planning for Obstacle Avoidance

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Abstract—Percutaneous intervention is among the preferred diagnostic and treatment options in surgery today. Recently, a biologically inspired needle steering system was proposed, where a novel “programmable bevel” is employed to control the tip angle as a function of the offset between interlocked needle segments. The new device, codenamed STING, can steer along arbitrary curvilinear trajectories within a compliant medium, and be controlled by means of an embedded position sensor. In this work, we provide details of our latest attempt to miniaturize the STING, with the design and manufacture of a 4 mm outer diameter (OD) two-part prototype that includes unique features, such as a bespoke trocar and insertion mechanism, which ensure that the segments do not come apart or buckle during the insertion process. It is shown that this prototype can steer around tight bends (down to a radius of curvature of \( \sim 70 \) mm), a performance which is comparable to the best systems in this class. With the need to comply with the specific mechanical constraints of STING, this paper also introduces a novel path planner with obstacle avoidance which can produce a differentiable trajectory that satisfies constraints on both the maximum curvature of the final trajectory and its derivative. In vitro results in gelatin for the integrated prototype and path planner demonstrate accurate 2D trajectory following (0.1 mm tracking error, with 0.64 mm standard deviation), with significant scope for future improvements.

Index Terms—Biologically inspired robots, Closed-loop control, Medical robots and systems, Needle steering, Nonholonomic path planning

I. INTRODUCTION

MINIMALLY Invasive Surgery (MIS) for diagnosis and treatment has become the preferred choice in many clinical applications [1]. In particular, percutaneous intervention has attracted significant interest, due to the minimal access requirements for this type of procedure [2]. To date, it is generally performed with straight needles because surgeons can extrapolate their behavior in the tissue intuitively during the insertion process. Recently, however, there have been efforts to use flexible needles to reach a target while avoiding ‘no-go’ areas. Apart from ours [3-5], these researches can be broadly classified into three main categories: needle steering methods based on the lateral motion of an external base and deformation of the soft tissue [6, 7]; steering control of flexible needles with a fixed-shape bevel tip [8, 9] and steering control using concentric tubes [10, 11]. These methods have substantially advanced the state of the art, but there remain a number of outstanding research challenges which stand in the way of a widespread adoption of these technologies. Needles which employ a lateral motion of the base provide a compact solution which can be used with off-the-shelf flexible needles, but the insertion process can cause large stresses on the tissue, especially near the surface. Needles with a fixed-shape bevel tip change direction by means of an axial rotation of the whole body (i.e. “duty-cycling”), which may result in large stresses at the tool tissue interface, especially if the needle already lies on a curvilinear path. Concentric tubes offer a highly configurable and dexterous solution, but for needle steering applications the number of curves in the path is irreversibly tied to the number of segments (e.g. 3 concentric tubes can only produce a curved path with up to 3 constant radius curves), which limits the range of applications for which such a system would be suitable (for a more detailed description of each of these classes see [4, 5]). A few additional needle steering approaches have been proposed in recent years which exploit the concept of an active tip to steer within a medium. A few notable examples are the magnetically driven needle navigation system proposed in [12], where a magnetic force is employed to steer the needle tip around obstacles in the liver, and the tendon-driven system marketed under the name Seeker Steerable Biopsy Needle™ (PneumRx Inc.; Mountain View, CA), for which a robotic assisted application for lung biopsy is described in [13]. Active needle designs, however, still appear to be at a very preliminary stage and, as such, have yet to gain traction within the mainstream needle steering community.

In an attempt to tackle some of the challenges associated with current needle steering systems, such as reducing tissue trauma and improving the range of applicability, our work centers on the development of a biologically inspired steerable needle for soft tissue surgery. The needle employs a “programmable bevel” which can be used to control the steering angle of the needle tip [3, 4] along an arbitrary, smooth, curvilinear path without the need for large contact forces. Our
previous reports have shown that the concept of a programmable bevel tip is viable through experiments with a first prototype which was 12 mm in outer diameter (OD). Two-dimensional (2D) trajectory following was achieved by means of a state feedback controller which employed the chained form representation [4] and a model predictive control approach [5]. To close the control loop, a small electromagnetic (EM) position sensor was incorporated into the prototype for the measurement of the tip position and orientation. A kinematic model of the bio-inspired needle, codenamed Soft Tissue Intervention and Neurosurgical Guide (STING), was based on a set of experimental results which demonstrated an approximately linear relationship between the relative offset between needle segments and the curvature of the generated path during insertion. Encouraged by the success of these early experimental results, this paper describes our latest attempt to miniaturize the needle and guide it along smooth planar curvilinear trajectories. Section II describes the development of a 4 mm, two-part needle which is specifically designed to follow a path defined in 2D space. Details of a bespoke trocar and insertion mechanism are also provided. Section III describes the development of a path planner, which produces a smooth path constrained both the maximum curvature (a function of the needle’s flexural stiffness) and/or the maximum rate of change of curvature (a function of the need to restrict the speed of each segment in the interest of safety), both of which are required for correct operation of the needle. Section IV outlines the integration of the 4 mm OD two-segment flexible needle and path planner, and their evaluation through in vitro experiments in gelatin. This paper concludes with Section V, where the current results are placed in the context of future work.

II. 4 MM OD TWO-SEGMENT FLEXIBLE NEEDLE

A. Needle Design and Manufacture

In order to reduce the outer diameter of our first 12 mm OD prototype [4], the choice of a two-segment design (as opposed to the 4-segment design in [4]) was made in this study. With this temporary simplification, which would prevent the needle from being able to steer in full three dimensions, rapid prototyping technology could then be used to manufacture suitable samples, thus allowing us to demonstrate the application and function of a 4 mm needle.

As in [4], the 4 mm STING was manufactured with a rapid prototyping technique. “Wing-shaped guides”, shown in a lighter color in Fig. 1(a), were incorporated into the design to ensure that, with the reduced cross sectional area, each segment could still be pushed from the back. The wings were manufactured out of a rigid material, VeroWhite (Objet Ltd.; tensile strength of 50MPa; hardness of 83 Shore Scale D; elongation at break of 20%) and are 40 mm in length [14]. The tips of the two segments (also shown in Fig. 1) were manufactured in the same material, VeroWhite, as stiff bevels were shown empirically to improve the needle’s steering capabilities. The body is made of two flexible materials, DM_9885 (Objet Ltd.; tensile strength of 6MPa; hardness of 85 Shore Scale A; elongation at break of 55%) and DM_9895 (Objet Ltd.; tensile strength of 20MPa; hardness of 95 Shore Scale A; elongation at break of 30%), both of which are combinations of VeroWhite and TangoBlackPlus materials from Objet Ltd. [14]. The choice of a variable stiffness design of the male segment, which was possible due to the RP machine’s ability to print any combination of materials in one pass, was driven by the discrepancy in cross sectional area between the male and female segments. Indeed, since the cross-section of the male segment is larger than that of the

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**Fig. 1.** (a) Manufactured and (b) designed 4 mm OD two-segment flexible needle. (c) Cross-sectional view from distal end. (d) Cross-sectional view of needle body. Large arrows describe direction of layer stack.

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**Fig. 2.** A flexible needle and its bespoke trocar with guiding rails
female segment due the presence of the interlocking mechanism, as shown in Fig. 1(d), the female was manufactured out of the harder material, DM_9895, while the male was made in combination with a softer material, DM_9885. This particular multi-material design was arrived to by simple trial and error. A 1 mm diameter through hole was incorporated into the male segment to accommodate an electromagnetic sensor for position tracking, as shown in Fig. 1(c) and Fig. 1 (d). As a point of note, the print direction for the prototype during manufacture is indicated by arrows in Figs. 1(b) and 1(c).

B. Trocar Design

To guide the new STING prototype accurately, a bespoke trocar was also designed which includes a recess for the main needle body, alongside two additional hollow “rails”, which help to constrain the wing-shaped guides of the needle during the insertion process. In this way, a push from the back of each segment, which would otherwise produce a moment about the needle’s long axis, can be used both to control the offset between the parts and the overall insertion process. The rails are designed to have a 0.1~0.2 mm gap for smooth sliding motion, as shown in Fig. 2(a). The trocar was designed so that it can be separated into two parts to ease the manufacture, as shown in Fig. 2(b). It was machined out of a transparent acrylic plastic to allow any separation between the two parts of the needle as a result of the interlock failing within the trocar to be clearly visible during the experiments. Stiff shape memory alloy (Nickel Titanium) transmission cables are used to couple each wing-shaped guide to a dedicated linear actuator, details of which are provided in Section IV.

III. SMOOTH PATH PLANNER

A. STING Constraints

Considering the unique mechanism of motion of the STING, the following three constraints must be considered during development of a suitable path planner for an integrated percutaneous system:

- The maximum curvature of the overall path should be bounded, since the curvature is related to the offset between the two segments of the flexible needle and an excessive offset between steering segments may cause the leading one to buckle [5].
- The maximum rate of change of the curvature should be bounded. Our feedback controller restricts the maximum speed of each segment to prevent damage to the surrounding tissue [5]. This requirement results in a restriction to the maximum rate of change of steering offset, which in turn results in a restriction to the maximum rate of change of the path’s curvature.
- The thickness of the flexible needle and any control inaccuracies should be considered when planning a suitable path for the flexible needle.

The following sections describe our work on a novel path planner which finds a smooth path between any entry and target location pair, which takes into account the aforementioned constraints.

B. Related Work

For needle steering by application of a moment to the needle base, DiMaio and Salcudean proposed potential fields to avoid obstacles [7]. For beveled tip needles, Alterovitz, Goldberg and their research group [15, 16], and Park et al. [17] developed a series of path planning algorithms, which are used to obtain non-holonomic paths consisting of a series of arcs, the curvature of which is either a positive or negative constant value. Although Patil and Alterovitz suggested a modified path planner for variable curved paths [18], the generated paths still consist of a series of arcs and thus the first derivative of their curvature is discontinuous. For concentric tubes, Lyons et al. proposed a planning algorithm to identify the tubes’ configuration which minimizes the amount of contact with the tissue, assuming that the concentric tubes move inside of a tubular environment without interaction with the tissue [19].

These techniques cannot readily be used for the STING because of the mechanical constraints described in Section III.A. On this account, there was a need to develop a smooth non-holonomic path planning method, which satisfies the constraints on both the curvature and/or the rate of change of the curvature.

There have been many path planning algorithms developed for mobile robots [20], for instance potential fields, deterministic sampling-based methods such as A*, deterministic methods using straight lines and arcs [16, 21], level set-based approaches such as Fast Marching [3, 20, 22-24], probabilistic sampling-based searches such as Rapidly-exploring Random Trees (RRTs) [15, 18, 25] and Stochastic Motion, and approaches using curvature polynomials [26-28]. Out of these, Fast Marching-based approaches can produce a continuous path, and Cohen and Kimmel [23] suggested a relationship between the minimum radius of curvature of the extracted path and the input map’s features; however, it proved difficult to control the final path’s maximum curvature value precisely [20, 24]. On this account, Bano et al. proposed a novel path planning algorithm which employs curvature polynomials for a flexible needle [28]. However, it does not restrict the path’s curvature and its derivative explicitly and instead, selects the final path among a large number of candidates obtained by setting various inserting and/or targeting angles.

C. Curvature Polynomials and Gradient-Based Optimization

This section introduces a method to compute a smooth path for which the curvature and its derivative are continuous and also bounded. As explained in other published literature [26-28], here the path is first described as a curvature polynomial to obtain a smooth path, then its coefficients are calculated using a gradient-based optimization method.

Given the start state $X_0 = [x_0 \ y_0 \ \theta_0 \ \rho_0]^T$ and target state $X_T = [x_T \ y_T \ \theta_T \ \rho_T]^T$, where $x_i$ and $y_i$ indicate the
2D location of the needle, $\theta_i$ indicates the approach angle, and $\rho_i$ is the curvature, a smooth path can be expressed in the form of a curvature polynomial, from which the other states can be obtained by integration, as shown in (1) [26] below:

$$\begin{align*}
\rho(s) &= \rho_0 + as + bs^2 + cs^3 \\
\theta(s) &= \theta_0 + \int_0^s \rho(\tau) d\tau \\
x(s) &= x_0 + \int_0^s \cos(\theta(\tau)) d\tau \\
y(s) &= y_0 + \int_0^s \sin(\theta(\tau)) d\tau
\end{align*}$$

(1)

where $a$, $b$, $c$ are the coefficients of a curvature polynomial and $s$ is the arc length. Note that the order of the curvature polynomial depends on the number of constraints.

The path planning problem can be considered as an optimization problem which identifies the parameter vector $P = [a\ b\ c\ s_i]^T$, consisting of the coefficients of a curvature polynomial and the total path length. The set of non-linear equations (1) can be rewritten in vector form as in (2), which can then be linearized as in (3):

$$\begin{align*}
\Delta X = [\frac{\partial}{\partial P} f] \Delta P
\end{align*}$$

(3)

If we choose the order of the polynomial so that the dimensions of $\Delta X$ and $\Delta P$ are the same, the Jacobian has sufficient rank for inversion. Therefore, the parameter vector can be found by iterative optimization as in (5):

$$\begin{align*}
\Delta P &= \left[\frac{\partial}{\partial P} f\right]^{-1} \Delta X \\
\Delta X &= \Delta X
\end{align*}$$

(4)

where $\mu$ is a constant value used to tune the convergence rate.

In order to avoid obstacles, an additional metric, $L$, can be introduced as shown in [27]. In this paper, $L$ is modified for simpler calculation as follows:

$$L = \sum_{i=0}^{N-1} \max(D_c - D_{c,max}, 0)$$

(6)

where $D_{c,max} = \min(\sqrt{(x(s) - x_i)^2 + (y(s) - y_i)^2})$ indicates the minimum distance from the path to the $i$th obstacle, and $N$ and $D_c$ indicate the number of obstacles and the allowed clearance distance respectively.

To restrict the maximum path’s curvature and its derivative within a specific range explicitly, this paper introduces two additional metrics. To allow the curvature and its derivative to vary within the allowed range and be bounded when they exceed these values, the metrics $M$ and $N$ are defined as in (9) and (10) for the curvature and its derivative, respectively:

$$M = \max(\rho_{max} - \rho_{limit}, 0)$$

(9)

$$N = \max(\dot{\rho}_{max} - \dot{\rho}_{limit}, 0)$$

(10)

where $\rho_{max}$ is the maximum curvature, $\rho_{limit}$ is the allowed range limit for the curvature.

One last point to consider is the definition of the state vector $X$. It defines which states are considered during the planning process. Contrary to previous approaches where the states include the desired approach angle and the desired curvature at the target point [26-28], for our flexible needle these do not need to be considered although it is assumed that the target position will be identified manually by the user during the planning stage. Therefore, when the restriction on the curvature is considered, the state vector and the parameter vector become:

$$X(s) = [x(s)\ y(s)\ L\ M]^T$$

(11)

where $a$, $b$ and $c$ are the coefficients of the third-order curvature polynomial:

$$\rho(s) = \rho_0 + as + bs^2 + cs^3$$

(12)

When the restrictions on both the curvature and its derivative are considered, the state vector becomes as in (13) and thus a fourth-order curvature polynomial needs to be used:

$$X(s) = [x(s)\ y(s)\ L\ M\ N]^T$$

(13)

where $a$, $b$, $c$, $d$ and $s_i$ are the coefficients of a curvature polynomial:

$$\rho(s) = \rho_0 + as + bs^2 + cs^3 + ds^4$$

(14)

In the following simulations and experiments, the pseudo-inverse was used to find $\Delta P$ because the determinant of the Jacobian can be zero when $x$ and $y$ are still converging and some constraints such as $L$, $M$ and $N$ are already satisfied:

$$\Delta P = \left[\frac{\partial}{\partial P} f\right]^{+} \Delta X$$

(14)
IV. EXPERIMENTAL VALIDATION

A. Experimental Setup

Fig. 3 shows the experimental setup with the 4 mm OD STING prototype. The linear actuators are controlled via a CompactRIO embedded controller programmed in LabView (National Instruments inc.) [4, 5]. A Labview-based graphical user interface (GUI) was developed and integrated into the setup in order to program desired trajectories, monitor performance and log key control parameters. An EM tracking sensor (Aurora 5 DOF sensor with 0.5 mm diameter and 8 mm length, Northern Digital Inc.), with a root-mean square (RMS) accuracy of 0.7 mm/0.2° [29] was included to measure the needle’s tip position and orientation. The gelatin phantom was prepared with 6 weight %, as used in previous works [4, 5] to mimic in some simplified fashion the stiffness of brain tissue. The approximate Young’s modulus for the gelatin phantom used in this work was experimentally found to be 7kPa [30].

B. Steering Offset vs. Curvature Calibration

The first step in evaluating the performance of the prototype was to find the relationship between steering offsets and corresponding resultant curvatures. For this purpose, ten simple insertion tests with constant steering offset (δ) of -20, -15, -10, -5, 0, 5, 10, 15, 20 and 25 mm were performed (note that offset sign relates to steering direction) and the EM tracking system was used to gather the tip’s trace as in [4]. After analyzing the results and in order to be able to use the state-feedback controller developed in previous work [4], we assumed that the curvature is linearly related to the steering offset, in this case by a constant coefficient κ of 4.824×10^4 (mm^-2), which indicates that an offset of about 21 mm is required to produce a 100 mm radius of curvature (Fig. 4(b)). It is also worth noting that the value is 2.6 times larger than the coefficient found for the 12 mm OD prototype in [4] and that the maximum curvature for a 25 mm offset steering, 0.0142 mm^-1, is comparable to the largest values in literature, a detailed review of which can be found in [4].

C. Feedback Control Results in Phantoms

This section reports on the experimental results obtained with the flexible needle described in Section II and 2D trajectory following controller described in [4]. The steering coefficient used in these experiments was set to 0.0004824 mm^-2 as obtained in Section IV.B. The linear forward velocity was chosen to be 0.5 mm/sec since it was recommended that the advancing or withdrawing of microelectrodes or other instruments should be no greater than 0.5 mm/sec to reduce the risk of hemorrhage in brain surgery [31]. The shape compensation coefficient κ, which is a coefficient used to compensate for the difference in steering offset between the tip and the base of the needle and is modeled as the distance between the centroids of the two segments, i.e. 8R/3π, was set to 1.698 mm, and the time constant of the low pass filter used to smoothen the EM sensor data and the control gains were kept the same as in our previous work [4].

To evaluate the controller performance, twelve tests were carried out – eight with the double bend trajectory (15) and four with the circular trajectory (16).

The parameters pertaining to the trajectory of each experiment, alongside the experimental results obtained, are listed in Table 1. The last four columns of Table 1 report on the overall steering results, expressed as the mean, standard deviation, RMS and maximum positional error between the needle tip and predefined trajectory. The maximum curvature ρ_{max path} of each trajectory is also reported in Table 1. For example, the curvature of the path used for Ex08 is 10.28×10^{-3} mm^{-1}, which corresponds to a radius of curvature of 97.28 mm.

Fig. 5 graphically illustrates the results obtained in two out of the twelve experiments: Ex08 shows the results obtained for the double bend trajectory and Ex10 shows one example of single bend trajectory.

Comparing the 2D tracking errors found here with those

![Table 1. Experimental Result of Needle Steering (Errors to δP)](https://example.com/table1)

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>A (mm)</th>
<th>ρ_{max path} (mm^{-1})</th>
<th>P_{error} (mm)</th>
<th>standard deviation</th>
<th>RMS</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex01</td>
<td>120</td>
<td>15</td>
<td>5.14×10^{-3}</td>
<td>-0.52</td>
<td>0.53</td>
<td>0.74</td>
</tr>
<tr>
<td>Ex02</td>
<td>120</td>
<td>15</td>
<td>5.14×10^{-3}</td>
<td>-0.45</td>
<td>0.63</td>
<td>0.77</td>
</tr>
<tr>
<td>Ex03</td>
<td>120</td>
<td>20</td>
<td>6.85×10^{-3}</td>
<td>-0.25</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>Ex04</td>
<td>120</td>
<td>20</td>
<td>6.85×10^{-3}</td>
<td>-0.32</td>
<td>0.59</td>
<td>0.67</td>
</tr>
<tr>
<td>Ex05</td>
<td>120</td>
<td>25</td>
<td>8.57×10^{-3}</td>
<td>-0.23</td>
<td>0.64</td>
<td>0.68</td>
</tr>
<tr>
<td>Ex06</td>
<td>120</td>
<td>25</td>
<td>8.57×10^{-3}</td>
<td>-0.35</td>
<td>0.80</td>
<td>0.87</td>
</tr>
<tr>
<td>Ex07</td>
<td>120</td>
<td>30</td>
<td>10.28×10^{-3}</td>
<td>-0.35</td>
<td>0.70</td>
<td>0.78</td>
</tr>
<tr>
<td>Ex08</td>
<td>120</td>
<td>30</td>
<td>10.28×10^{-3}</td>
<td>-0.17</td>
<td>0.88</td>
<td>0.90</td>
</tr>
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</table>

Double bend

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>ρ_{max path} (mm^{-1})</th>
<th>P_{error} (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex09</td>
<td>150</td>
<td>6.67×10^{-3}</td>
</tr>
<tr>
<td>Single</td>
<td>150</td>
<td>6.67×10^{-3}</td>
</tr>
<tr>
<td>bend</td>
<td>120</td>
<td>8.33×10^{-3}</td>
</tr>
<tr>
<td>Ex12</td>
<td>120</td>
<td>8.33×10^{-3}</td>
</tr>
</tbody>
</table>

Overall Results of P_{error} (mm) 0.03 0.85 0.85 1.89

(a) Camera view of experiments  (b) Relation b/w offset and curvature

Fig. 4. Experimental results describing the relationship between steering offsets and the measured curvature

(a) Ex08  (b) Ex10

Fig. 5. Experimental results of needle steering: The images depict actual trajectories of the flexible needle.
measured for the 12 mm OD prototype (0.68 mm tracking error with 1.45 mm standard deviation [4]), the tracking performance of the 4 mm needle is substantially better, with a 0.03 mm tracking error with 0.85 mm standard deviation. One of the major factors reducing the tracking errors reported in Table 1 seems to stem from a larger value of $\kappa$ because it results in quicker response, that is, the needle tip can be reoriented over a shorter distance when compared to the larger prototype described in [4]. In addition, the two-segment STING showed better planar behavior, as intuitively, four-segment needles tend to twist more than two-segment needles.

D. Path Planner Performance in Simulation

First, four valid exemplars of possible 2D maps were produced to illustrate the capabilities of our needle system and path planner. “Map 01” features in Fig. 7; “Map 02” and “Map 03” are shown in Fig. 8 (a) and (b) respectively; Fig. 6 graphically illustrates Map 04. The path planning problem was defined as follows: to find a parameter vector $P$ to produce a trajectory from the start position $[x_0, y_0]$ to the target position $[x_f, y_f]$, which satisfies the given constraints. The target state for the optimization process is set to $X_f=[x_f, y_f, L_f(=0), M_f(=0), N_f(=0)]$. The clearance distance from obstacles was set to 4 mm to account for the radius of the flexible needle and any control inaccuracies (~1 mm RMS). The maximum curvature $\rho_{\text{max}}$ is approximately 0.01 mm$^{-1}$, considering Fig. 4, where the coefficient $\kappa$ measures $4.824 \times 10^4$ (mm$^2$). In the simulation, the maximum curvature of the planned path was conservatively chosen to be 80% of the real maximum curvature to make sure some range is always available to correct tracking inaccuracies. Since the maximum derivative of curvature, $9.648 \times 10^4$ (mm$^2$), of the current prototype is quite

\[ \rho = \kappa \delta \]

\[ \dot{\rho} = \frac{d\rho}{ds} = \kappa \frac{d\delta}{ds} = \kappa \frac{d\delta}{dt} \frac{dt}{ds} = \kappa \frac{\omega}{v} \]  

Assuming that the linear forward velocity $v$ is 0.5 mm/sec and that the maximum rate of change of the steering offset $\omega$ is twice larger than the forward velocity [5], the maximum derivative of curvature $\rho_{\text{max, result}}$ becomes $9.648 \times 10^4$ (mm$^2$). In the simulation, the maximum curvature of the planned path was conservatively chosen to be 80% of the real maximum curvature to make sure some range is always available to correct tracking inaccuracies. Since the maximum derivative of curvature, $9.648 \times 10^4$ (mm$^2$), of the current prototype is quite

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Table 2. Parameters used for path planning simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space Size (mm) $H \times W$</td>
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</tr>
<tr>
<td>Start state $[x_0, y_0, \theta_0, \rho_0]^T$</td>
<td>[10, 65, 0, 0]</td>
</tr>
<tr>
<td>Target Position $[x_f, y_f]^T$</td>
<td>[130, 80]</td>
</tr>
<tr>
<td>Clearance Distance $D_c$</td>
<td>4</td>
</tr>
<tr>
<td>Max. Curvature $\rho_{\text{max}}$</td>
<td>0.01 mm$^{-1}$</td>
</tr>
<tr>
<td>Max. Curvature in Simulation $\rho_{\text{max, simul}}$</td>
<td>0.008 mm$^{-1}$</td>
</tr>
<tr>
<td>Max. Derv. Curvature $\rho_{\text{max, simul}}$</td>
<td>9.6x10$^4$ mm$^{-2}$</td>
</tr>
<tr>
<td>Max. Derv. Curvature in Simulation $\rho_{\text{max, result}}$</td>
<td>4.8x10$^4$ mm$^{-2}$</td>
</tr>
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</table>

Table 3. Simulation results of path planning: Map 01

<table>
<thead>
<tr>
<th>Constraints</th>
<th>CC</th>
<th>OA + CC</th>
<th>OA + CC + DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\text{max, result}}$</td>
<td>0.0067</td>
<td>0.0080</td>
<td>0.0080</td>
</tr>
<tr>
<td>$\rho_{\text{max, result}}$</td>
<td>0.695x10$^{-4}$</td>
<td>6.22x10$^{-4}$</td>
<td>4.8x10$^{-4}$</td>
</tr>
<tr>
<td>Polynomial Order</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
</tr>
<tr>
<td>Final Parameter</td>
<td>4.01x10$^{-5}$</td>
<td>-4.93x10$^{-5}$</td>
<td>-4.8x10$^{-4}$</td>
</tr>
<tr>
<td>$P=[a, b, \ldots, s]^T$</td>
<td>1.19x10$^{-5}$</td>
<td>4.3x10$^{-6}$</td>
<td>2.62x10$^{-5}$</td>
</tr>
<tr>
<td>$123.0$</td>
<td>121.6</td>
<td>122.1</td>
<td></td>
</tr>
</tbody>
</table>

* Abbreviations 
$\rho_{\text{max, result}}$: Resultant Maximum Curvature 
$\rho_{\text{max, result}}$: Resultant Maximum Derivative of Curvature
large compared to its maximum curvature, the curvature derivative in the simulation was chosen to be arbitrarily smaller, with a value of $4.8 \times 10^4$ (mm$^{-2}$), to assess whether the constraint is satisfied properly. The parameters used in simulations are listed in Table 2.

Given a map, three simulations were performed: (a) path planning using only the curvature constraint (CC), (b) path planning using obstacle avoidance and curvature constraint (OA+CC), and (c) path planning using obstacle avoidance, curvature constraint and a constraint on the rate of change of curvature (OA+CC+DC). Fig. 7 and Table 3 summarize the simulation results of the proposed path planning method applied to Map01. Fig. 7(a) shows the resulting trajectories. As expected, the first simulation, i.e. CC, showed the optimum path disregarding obstacles. When obstacle avoidance is considered during path planning (OA+CC and OA+CC+DC), the trajectories avoid the obstacles by the amount of clearance imposed by the user (4 mm). Fig. 7(b) shows the curvature functions and their derivatives. In the first simulation (i.e. CC), the maximum curvature measured 0.0067, which satisfies the curvature constraint. When obstacle-avoidance is applied (OA+CC), the curvature value and its derivative become larger. The curvature along the entire path is bounded, but the derivative of the curvature becomes larger than the limit. However, as shown in Fig. 7(b), in the case where all three constraints (OA+CC+DC) are enforced, the curvature and its derivative are both bounded within the range. Results of the simulations are summarized in Table 3. The order of the curvature polynomial increases as the number of constrains increases. Note that the last parameter $s_i$ indicates the overall length of the trajectory.

E. Evaluation of the Integrated System in Phantoms

In this section, the integrated system was evaluated. First, trajectories were generated using the smooth path planner described in Section III.C. These were then saved at 0.5 mm intervals in the base (initial) frame of the flexible needle. These trajectories are subsequently interpolated down to 0.0125 mm intervals, since the closed-loop path following algorithm updates at 40 Hz. The low-level controller runs at 1 kHz to control each segment of the flexible needle.

For the evaluation, the aforementioned four maps were used: two (Map01, Map02) produce trajectories with multiple bends while the others (Map03, Map04) produce simpler trajectories. The evaluation results are reported in Fig. 8 and quantitatively in Table 4. Fig. 8 shows the planned (solid yellow line) and the measured trajectories (dashed magenta line) using Map02 and Map03 in the first row. The obstacles and the clearance distance are marked in the maps with white dots and green circles, respectively. Fig. 8 also shows the final configuration of the needle in the middle row to aid comparison between the measured trajectories and the final shape of the flexible needle. The mean tracking errors and their standard deviation measured 0.1 mm and 0.64 mm respectively, as reported in Table 4. RMS and maximum errors measured 0.65 mm and 1.71 mm, respectively.

V. CONCLUSION AND FUTURE WORK

This paper describes our latest work on the development of a miniaturized soft tissue needle for percutaneous intervention. It details the design and manufacture of a 4 mm two-part prototype that includes unique features, such as a bespoke trocar and insertion mechanism, which ensure that the segments do not come apart or buckle during the insertion process. With a reduced cross sectional area and a more favorable width to length ratio, it was shown that this prototype can steer around tight bends (down to a radius of curvature of ~70 mm) without the protruding segment buckling (as evidenced in [5] with the larger prototype), a performance which is comparable to the best needle steering systems in this class.

This paper also introduces a novel path planner which can produce a differentiable trajectory which satisfies constraints on the maximum curvature of the final trajectory and its derivative. For this purpose, a gradient-based optimization using a curvature polynomial is adopted. To deal with the constraints, three special parameters are introduced for avoiding obstacles, restricting the maximum curvature, and restricting the rate of change of the curvature. While tailored to

<table>
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<tr>
<th>Table 4. Experimental Results for Integrated Flexible Needle</th>
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<tbody>
<tr>
<td>$\rho_{max}$ (mm)</td>
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<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Map01</td>
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<td>Map02</td>
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<td>Overall</td>
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![Fig. 8. Experimental results for the integrated flexible needle platform and path planner. In Figures (a) and (b), solid yellow lines indicate the path planned by the proposed path planner and the dashed magenta lines indicate the trajectories measured during the experiments. White dots are obstacles (or no-go areas) and green circles describe the same obstacles expanded by a predefined clearance margin. Figures (c) and (d) show the final shapes of the needle, while figures (e) and (f) depict positional errors from the desired paths against the needle’s length.](image-url)
the specific requirements of our system, the novel path planner described here improves upon the available methods in the literature.

Finally, the paper describes the results of our first integrated planning and execution trial, where the performance of the 4 mm OD STING was tested on curvilinear trajectories generated with the new path planner, using the closed loop control strategy described in [4]. The results demonstrate that the choice of constraints, the actuation mechanism and control strategy are satisfactory and that future work on a clinically viable system is warranted.

Although the experimental results described here show promise, research to date offers significant scope for future work. Firstly, ex vivo tests on biological tissue will be carried out to evaluate the performance of the needle in a more realistic substrate. Secondly, three- and four-segment flexible needles with wing-shaped guides will be developed and tested to explore the extension of our biologically inspired steering concept to three dimensions. Thirdly, a better, more accurate model of steering, which describes the complex relationship between steering offset and curvature by taking into account interaction forces, friction and substrate properties, will be developed. Following on from our first attempt to model tool-tissue interaction forces with finite elements [30], work will also continue on tissue deformation modeling and tracking, online path re-planning and better predictive models of tool tissue interaction and damage initiation. Lastly, research will progress of the identification of suitable materials and manufacturing processes which will aid in the development of a biocompatible, scaled down prototype suitable for clinical use.

VI. REFERENCES