Calculating minimum energy urban layouts with mathematical programming and Monte Carlo analysis techniques

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Abstract

Cities consume significant amounts of energy and improving their efficiency is an integral part of tackling global climate change. As existing urban layouts can be difficult to change, new developments offer significant opportunities for demonstrating low energy urban forms. However the limits of these improvements are not well known and so this paper presents an optimization tool for designing minimum energy urban layouts, considering both the transport and building sectors. Our work builds upon the excess commuting and sketch modelling literatures, with a greater focus on energy consumption as a specific objective and improved model performance. After validating the model with the case of a UK eco-town, Monte Carlo analysis is used to assess how the minimum energy benchmark varies in response to planning constraints on high-density housing and car ownership. The methodology presented here therefore offers a new tool for calculating absolute minimum urban energy benchmarks, which can be used early in the planning process to complement more behaviourally-realistic land-use transportation models.

Keywords: optimization, Monte Carlo, urban layout, energy, benchmark

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1. Introduction

Cities are major energy consumers, accounting for an estimated 67% of global primary energy demand and 71% of energy-related greenhouse gas emissions (IEA, 2008). While there is some dispute about the precise allocation (Satterthwaite, 2008), it is clear that urban energy efficiency must be improved if economic and cultural opportunities are to be maintained while avoiding the worst environmental effects.

Urban energy consumption is the consequence of decisions taken at a variety of spatial and temporal scales. Using domestic energy consumption as an example, the temporal scale spans from short-term decisions such as when to use appliances (seconds to days), medium-term choices about which appliances to purchase (months to years), and long-term decisions about the built fabric of the home (decades to centuries). For example Boardman et al. (2005) note that, at current rates, the UK's housing stock could take approximately 1300 years to be replaced completely. Variations in spatial scale also have a strong influence on energy consumption. End-use energy conversion technologies such as household gas boilers may be relatively easy to reposition or replace for improved efficiency but large infrastructure systems, such as resource distribution and transportation networks or the location of buildings and activities, are more persistent (e.g Morris, 1994). Consequently the layout of urban environments is perhaps the most difficult aspect of improving urban energy efficiency.

For existing cities, urban expansion and retrofit projects can lead to improvements in energy consumption as seen in London's Canary Wharf and La Défense in Paris (de Wilde and van Den Dobbelsteen, 2004). However working within the constraints of existing infrastructures is expensive and difficult and so new construction offers the greatest opportunities to create energy-efficient cities. In recent years the eco-cities movement, both in the UK and abroad, has created visions of new sustainable urban areas (Joss,

2010), such as Masdar, the world’s first “carbon-neutral zero-waste city” near Abu Dhabi in the United Arab Emirates (Masdar City, 2010). However these ambitions raise serious questions about the limits of low energy urban forms, boundaries which must be identified if new developments are to set realistic goals and existing cities are to understand their improvement potential.

This paper therefore focuses on the planning of new cities and considers the question: what is the minimum energy configuration for an urban area? In Section 2, we review the literature on urban modelling and highlight the potential of an optimization-based sketch modelling framework to design minimum energy layouts. The model itself is presented in Section 3 and validated against the case of a UK eco-town (Section 4). Section 5 then applies uncertainty analysis techniques to demonstrate the model’s behaviour under a range of inputs and to illustrate its potential use as a planning tool. The paper concludes by summarizing the key contributions and by commenting on the relevance of this minimum energy benchmark methodology to both new and existing cities.

2. Modelling urban layout and energy use

2.1. Theoretical considerations

The layout of an urban area – that is, the location of major land uses and the transportation networks that link them – is a major research theme in urban geography (for a review, see Pacione, 2009). Yet as Cope et al. (1984) note, “relatively little literature is available which attempts to investigate the linkages between energy and planning in a systematic fashion” (p. xi). There are a number of possible explanations for this shortcoming. First, there is a persistent lack of data. Owens’s 1984 call for more information to support both empirical “deductive” and more “normative” research on the topic remains valid, even though the spatial resolution of urban energy use data

is now quite good for many large cities (e.g. Parshall et al., 2009; GLA, 2003). Secondly, the goals of an analysis of urban form and energy consumption have changed over the years. Early work was driven by the energy supply crises of the 1970s and focused primarily on issues surrounding the location of large ex-urban generation plants. In contrast, modern energy policy has additional technologies and constraints that change the potential scope and scale of such analyses. In the UK, centralised energy policy issues like nuclear power and climate change are complemented by interest in local issues like fuel poverty and the connections between microgeneration technologies and consumer behaviour (e.g. Watson et al., 2006).

The links between energy use and urban layout are also complex. In the transport sector, it has been observed that higher density decreases travel distances and hence the need for motorised transport (though not without caveats, Newman and Kenworthy, 1989; Saunders et al., 2008; Mindali et al., 2004). Similarly, higher densities increase the feasibility of district heating and cooling systems, by reducing network lengths and concentrating demands (CABE, 2009). However at very high densities, some building-related energy uses increase – “vertical” transportation (lifts), air-conditioning (due to reduced natural ventilation), and lighting (due to overshading) (Givoni, 1998; Hui, 2001) – and the available space for certain renewable energy technologies decreases (e.g. roof-mounted solar energy technologies or urban arboricultural arisings). The density of the built environment also has a marked impact on the urban climate, affecting demand for energy services such as heating and cooling (Steeners, 2003; Coutts et al., 2007), and the lifecycle energy consumption of associated structures, such as skyscrapers made of concrete, steel and glass, can be substantial.

Urban energy consumption might therefore be divided into four components: the city’s form (its buildings, transport and resource distribution networks), its function (the use of infrastructure, e.g. activities and flows),

the energy supply system (e.g. the efficiency of centralized plant), and the lifecycle impacts of materials required to build capital stocks. In this paper, we will focus on the in-city elements of form and function which constitute the urban layout problem (Mackett, 1985). The other factors are beyond the scope of the present work as they are influenced primarily by processes outside the city boundary.

2.2. Modelling urban form and function

There is a substantial literature on the relationship between urban form and function, particularly in the area of land use and transportation (LUT) modelling. Review studies identify four major modelling approaches – regression, optimization, aggregate spatial models, and disaggregate individual models (including both random utility frameworks and activity-based models) – which collectively illustrate a clear trend towards increased behavioural realism (Mackett, 1985; Hunt et al., 2005; Chang, 2006). This can be explained by the operational use of these models, i.e. their deployment in real cities to answer policy questions such as how a city might expand over time (Wang and Wu, 2010), where people choose to live and work within cities (Waddell, 2000), and which modes of transportation will be used to facilitate urban travel (Hunt et al., 2005). Greater behavioural fidelity allows these models to test sophisticated policy interventions and increases confidence that the salient processes have been effectively represented: “the value of more complex, behaviourally valid, microscopic models is not that one obtains microscopic forecasts, but that one obtains macroscopic forecasts based on microscopic principles” (p. 239 Timmermans, 2003).

However these features are not necessarily advantageous when calculating a minimum energy layout. First, although LUT models are well-suited to assessing transportation energy consumption, the energy consumption of buildings is often omitted or treated as a minor contributor to location choice processes, represented by financial proxies rather than resource consump-

tion itself; an exception is the proposed integrated assessment framework by Dawson et al. (2009). Secondly, disaggregated LUT models are data intensive and, while some input values may be imputed from one location to the next, they require significant customization for each study location. Since planning decisions are made infrequently, these investments in data collection may not be worthwhile (Timmermans, 2003). Establishing minimum energy benchmarks for a proposed city is a similarly infrequent exercise, suggesting a flexible approach that can quickly assess a city’s potential rather than providing the full detail of an operational model.

For these reasons, earlier simplified LUT modelling approaches seem most relevant to the question of minimum energy configurations. Specifically, optimization modelling techniques have been previously identified as appropriate for finding “extreme solutions” and in the “preparation of plans” to meet a specific objective, rather than forecasting detailed behavioural patterns over time (Mackett, 1985, p. 330-331). The use of optimization models for designing hypothetical optimal configurations, but not in assessments of existing cities, is also supported by Hunt et al. (2005) and, while optimization models have fallen out of favour as stand-alone tools (Wegener, 2004), the techniques are still used in conjunction with more mainstream LUT modelling approaches, as so-called ‘combined’ models (e.g. Pfaffenbichler and Shepherd, 2002; Chang, 2006; Lowry and Balling, 2009).

One application of optimization which is particularly relevant here is the question of excess commuting, i.e. commuting longer or further than the actual spatial layout of a city suggests (Horner, 2002). Of the two main approaches to calculate excess commuting, optimization models after White (1988) are the most prevalent (Ma and Banister, 2006). This literature uses mathematical programming to calculate benchmark minimum and maximum commutes for a given spatial layout which can then be compared to existing commuting patterns. These results can be interesting in their own

right, for example to compare the jobs-housing balance of different cities, but a number of studies have also considered how such models could be of practical use to urban planners. An early application was demonstrated by Merriman et al. (1995), who calculated excess commuting levels for Tokyo under exogenously-provided alternative land use configurations. However when trying to assess minimum energy configurations for a new city, these residential and activity locations should be endogenous to the model so that transportation energy can be minimized by jointly establishing the relative position of individuals and activities (not just work). A similar concern exists in the excess commuting literature, where the location of jobs and workers has been built into models as a means of providing more realistic policy advice. This can be implemented via a heuristic approach for planning new developments (Scott and Getis, 1998) or, more recently, with optimization formulations that address how excess commuting might be reduced either by relocating jobs and workers (Horner and Murray, 2003) or by targetted “in-filling” with new development (Horner, 2008). Although much of this work is based on an aggregate approach, several studies have looked to disaggregate individuals and households within these calculations, for example to account for households with two working persons who need to live together but work in separate locations (Buliung and Kanaroglou, 2002; O’Kelly and Lee, 2005).

Optimization modelling is therefore a promising methodology for establishing a minimum energy benchmark for a new city. These models might be termed ‘sketch’ models, in the sense that they provide an incomplete representation of urban land use and transportation dynamics. When used in the context of master planning and benchmarking processes however, their reduced data requirements enables models to be quickly tested against multiple scenarios. The results can be then tested within a more rigorous LUT modelling framework as required.

2.3. The role of sketch models

Optimization-based sketch models have strong links with the field of operations research and represent a hybrid of two problems from that field. The first is the *transportation problem*, as used in the excess commuting literature, where the goal is to minimize the cost of moving people or resources through a network. The second problem is the *assignment problem*, i.e. finding an optimal combination of tasks and agents where each pairing incurs a given cost. The hybrid formulation is known as the *facility layout problem* and while there are multiple forms of the problem’s definition, the general aim is to determine the position of processes within a factory so that the combined costs of performing a task at a given work station and moving materials between each work station are minimized. Recent examples of this literature include Lahmar and Benjaafar (2005), Castillo and Peters (2003) and Urban et al. (2000).

The facility layout problem is analogous to urban layout: the question is where to position urban facilities (i.e. housing, work, education and other activities) so that the transportation costs of moving between these activities are minimized. Although Barber (1976) was not the first to use these techniques, his paper provides an excellent overview of the approach and illustrates a typical application wherein three goals are pursued in a multi-objective framework: minimization of land development costs, maximization of residential accessibility and minimization of transportation energy costs. This work has been extended and revised by others, most notably in the Sketch Layout Model (Feng and Lin, 1999; Lin and Feng, 2003). In its various incarnations, this model seeks to generate alternative master plan sketches as an input to participatory planning processes. However rather than focusing purely on energy, the model considers “harmony” (the similarity of adjacent land uses), “relevance” (the compatibility of adjacent land uses), and “traffic accessibility” (the shortest path between two cells). Re-

lated work in this area has examined the positioning of individual facilities within a city (e.g. shopping malls, Yu et al., 2007), laying out space within buildings (Sharpe et al., 1985), the planning of development densities around transit stations (Lin and Gau, 2006), and using multi-objective optimization to generate a range of Pareto optimal layouts for discussion with planners (Jiang-Ping and Qun, 2009).

These models reinforce the viability of mathematical programming techniques for urban design problems, but there are at least two areas for improvement. First, these sketch models only include energy in the form of transportation energy (if at all); the energy use associated with buildings is not considered. Secondly, previous studies used combinations of computer hardware and software (i.e. model formulations) that restricted the size and fidelity of model representations (see Table 1). For example, Barber’s early study used linear programming and a coarse spatial scale (≈ 1000 ha per cell), which meant that land-use possibilities were represented by continuous variables (e.g. a single cell could be occupied by 0.136 of a school). At the other extreme, Feng and Lin’s Sketch Layout Model uses a mixed-integer non-linear formulation that both requires customized solution routines (based on genetic algorithms) and imposes significant limits on the maximum problem size. This suggests that a middle-ground approach, based on a mixed-integer linear model, could facilitate a finer spatial discretization (≈ 1 ha per cell) and faster solve times. This in turn would enable the use of Monte Carlo simulation to explore the behaviour of these models under a range of scenarios, rather than solving for a single case study as in previous work.

It should be noted that these earlier sketch models adopted a single spatial scale (i.e. a non-hierarchical spatial resolution) and assessed a single point in time. We also use these simplifications.

3. Model description

We have developed an optimization model to identify minimum energy configurations for an urban area. Since the establishment of a minimum energy benchmark at the planning stage does not require the same level of behavioural fidelity as an operational LUT model, a simpler model formulation can be used. In practice this means that an aggregate representation of activity supply and demand is used, rather than the disaggregate discrete choice models that might be seen in traditional LUT models (e.g. for mode or location choice).

The model is implemented as a mixed-integer linear programming problem, using the General Algebraic Modelling System (GAMS) and the CPLEX 9.0.0 solver. Figure 1 gives a schematic overview of the model. Briefly, the model takes as input a description of the city to be planned including its spatial discretization, the population characteristics and the available building and infrastructure types. The optimization then seeks to minimize the total “cost” of the layout (energy in this case, although cost or carbon objectives are also possible) by positioning buildings, activities and network connections. The following section describes the model in greater detail.

3.1. Notation

The model data and equations are indexed over the following sets:

Set	Index	Description
$\mathcal{S}$	s	discrete spaces within the city
$\mathcal{I}$	i	infrastructure types
$\mathcal{B} \subset \mathcal{I}$	b	building types
$\mathcal{R} \subset \mathcal{B}$	r	residential building types
$\mathcal{C} \subset \mathcal{B}$	c	commercial building types
$\mathcal{M} \subset \mathcal{I}$	m	transport modes
$\mathcal{N} \subset \mathcal{I}$	n	transport networks
$\mathcal{A}$	a	activities

The spaces within the model, $\mathcal{S}$, can be subdivided into zones within the city and external hinterlands; however only isolated cities are considered in the present study (i.e. no hinterlands). This implies that there is no net daily inflow or outflow of visitors to and from the city. Joint sets are also defined to indicate the allowable pairings of activities and commercial buildings ($\mathcal{AC}$), and transportation modes and networks ($\mathcal{MN}$).

The properties of each set member are described below. Set indices are written in roman font (e.g. i), while exogenous user-specified parameters are monospaced (e.g. TOTAL_POP) and model decision variables are in sans-serif (e.g. XA).

Spaces, $\mathcal{S}$

AREA _{s}	area of space s (hectares)
EXCL _{s}	is building permitted on space s ?
D _{s,s'}	distance between space s and s' (km)

The distance, D _{s,s'} , is assumed here to be the Euclidean distance but

its value is only defined between neighbouring spaces (exogenously defined). Network flow equations are then constrained to only apply between cells with non-zero distances. For example, if s_1 and s_2 are neighbours, and s_2 and s_3 are neighbours, but s_1 and s_3 are not neighbours, then the model will ensure that flows from s_1 to s_3 must proceed through s_2 first. In this way, total distances travelled reflect the route distance through intermediate points, rather than the straight-line distance between end points.

Infrastructures, $\mathcal{I}$

EN_i annual energy consumption of infrastructure i

CRB_i annual carbon emissions of infrastructure i

These parameters are specified on a per unit basis, e.g. per dwelling for $\mathcal{R}$, per hectare for $\mathcal{C}$, per kilometre for $\mathcal{N}$, and per passenger-kilometre for $\mathcal{M}$.

Buildings, $\mathcal{B}$

$\text{A}_b^{\min}, \text{A}_b^{\max}$ min and max area for a single building of type b (hectares)

$\text{TOT_A}_b^{\min}, \text{TOT_A}_b^{\max}$ min and max total area for all buildings of type b (hectares)

DEN_r density of housing type r (dwellings per hectare)

Transportation, $\mathcal{M}$ and $\mathcal{N}$

OCC_m	average occupancy of mode m (ppl per vehicle)
USE_m	average use of mode m (uses per day)
$\text{CAP}_n^{\min}, \text{CAP}_n^{\max}$	min and max capacity of network n (vehicles per day)

Activities, $\mathcal{A}$

The model estimates transportation demands based on the need for activities. For example, a citizen might want to travel to their office for work or to a supermarket for shopping. The following parameters define the average characteristics of each activity.

$V_{a,b}$	daily visits to activity a by residents of building type b (fraction of total occupancy)
$V_a^{\min}, V_a^{\max}$	min and max daily visits to a single provider of activity a
$N_a^{\min}, N_a^{\max}$	min and max number of spaces that can provide activity a

The model also includes two scalar constants: the total population to be housed in the city, `TOTAL_POP`; and the average household size, `HHSIZE`.

3.2. Decision variables

The model includes a number of variables whose values are determined by the solver:

$SUP_{s,a}$	supply of activity a at location s (daily visits)
$DEM_{s,a}$	demand for activity a at location s (daily visits)
$TR_{s,s',a}$	travel from location s to s' for activity a (daily person-trips)
TOT_A_b	total area covered by building type b (hectares)
N_ACT_a	total number of providers of activity a
$VEH_{m,n,s,s'}$	total number of vehicles of mode m on network n between s and s'
$PASSKM_m$	total number of passenger-kilometres on mode m
$TOTVEH_m$	total number of vehicles of mode m
$NETLEN_n$	total length of network n (km)
$XA_{a,s}$	binary indicating if activity a is provided at space s
$XB_{b,s}$	binary indicating if building type b is located at space s
$XN_{n,s,s'}$	binary indicating a connection between s and s' on network n

3.3. Objective function

For the present analysis, the model's objective function represents the total energy consumption of the city, TOT_EN . This value is calculated from the constituent costs for buildings ($build$) and transportation ($trans$) infrastructure:

$$\begin{aligned}
TOT_EN_{build} &= \sum_{r,s} XB_{r,s} EN_r DEN_r AREA_s + \sum_{c,s} XB_{c,s} EN_c AREA_s \\
TOT_EN_{trans} &= \sum_n NETLEN_n EN_n + \sum_m PASSKM_m EN_m \\
TOT_EN &= TOT_EN_{build} + TOT_EN_{trans}
\end{aligned}$$

The model is divided into two separate optimization problems to improve performance. The first focuses on the location of buildings and the assignment of activities to each building type:

$$\min Z_{build} = \text{TOT_EN}_{build} + \text{PEN_FAC} \sum_m \text{PASSKM}_m$$

where PEN_FAC is a constant, determined by the model using an estimate of the average building and transportation costs so that the two objective function terms (i.e. building cost and the transport penalty function) are of similar magnitude. Once this model has solved, the positions of each building and activity are then fixed, and the network design model is solved with the following objective function:

$$\min Z_{trans} = \text{TOT_EN}_{trans}$$

3.4. Constraints

For both models, the optimization is subject to the following constraints:

Buildings and activities

- Each space s can be occupied by at most one building type (subject to building exclusions, see section 3.1 above). Similarly, each space can only offer at most one activity.

$$\begin{aligned} \sum_b \text{XB}_{b,s} + \text{EXCL}_s &\leq 1 & \forall s \\ \sum_a \text{XA}_{a,s} + \text{EXCL}_s &\leq 1 & \forall s \end{aligned}$$

- There must be sufficient housing for the total population. Note that this equation effectively assigns the population to their ‘optimal’ residential location; as a sketch model, a detailed location choice framework is not used.

$$\sum_{r,s} \text{XB}_{r,s} \text{AREA}_s \text{DEN}_r \text{HHSIZE} \geq \text{TOTAL_POP}$$

- The total area for each building type b must be within the allowed bounds

$$\text{TOT_A}_b = \sum_s \text{XB}_{b,s} \text{AREA}_s$$

$$\text{TOT_A}_b \geq \text{TOT_A}_b^{\min}$$

$$\text{TOT_A}_b \leq \text{TOT_A}_b^{\max}$$

- The total number of activity providers must be within the allowed bounds.

$$\text{N_ACT}_a = \sum_s \text{XA}_{a,s}$$

$$\text{N_ACT}_a \geq \text{N}_a^{\min}$$

$$\text{N_ACT}_a \leq \text{N}_a^{\max}$$

- Activities must be located in a commercial building of the allowed type.

$$\sum_{c \in \mathcal{AC}} \text{XB}_{c,s} \geq \text{XA}_{a,s} \quad \forall s, a$$

$$\sum_{a \in \mathcal{AC}} \text{XA}_{a,s} \geq \text{XB}_{c,s} \quad \forall s, c$$

Transportation flows and networks

- The demand for an activity at a given location ($\text{DEM}_{s,a}$) is determined by the number of people living at that location.

$$\text{DEM}_{s,a} = \sum_r \text{XB}_{r,s} \text{AREA}_s \text{DEN}_r \text{HHSIZE} \text{V}_{a,r}$$

- The supply of an activity at a given location ($\text{SUP}_{s,a}$) is determined by the presence of an activity facility. The **FACBAL** parameter, arbitrarily set at 1.1, ensures that no single location receives more than

10% above the average total demand for activity a . There are also explicit constraints on the maximum and minimum size of any activity provider; these parameters can be adjusted to control the degree of centralization within the city.

$$\text{SUP}_{s,a} \leq \text{FACBAL_TOTAL_POP} \max_b V_{a,b} X A_{a,s} / \max [1, N_a^{\min}]$$

$$\text{SUP}_{s,a} \geq V_a^{\min} \quad \forall s$$

$$\text{SUP}_{s,a} \leq V_a^{\max} \quad \forall s$$

- For each space s , there exists some demand for activity a (can be zero). This demand can be satisfied either by local provision of the activity or by citizens travelling to adjacent spaces. The number of inward trips plus local demand for activity a must therefore balance the number of outward trips plus local supply. This flow balance, in combination with the equation defining PASSKM_m , ensures that travel distances are minimized.

$$\sum_{s'} \text{TR}_{s',s,a} + \text{DEM}_{s,a} = \sum_{s'} \text{TR}_{s,s',a} + \text{SUP}_{s,a} \quad \forall s, a$$

- The total number of trips between s and s' (in both directions) must be within the capacity limits of the installed infrastructure. The right-hand side of the constraints below is equal to the sum, over all infrastructures and modes, of: the average occupancy of mode m (people per vehicle), multiplied by the minimum or maximum capacity of infrastructure n (vehicles per day), multiplied by $XN_{n,s,s'}$, which is 1 if there is a connection between s and s' . The NETBAL parameter is a constant (set to 1.2) that ensures that average flows will be smaller

than peak flows. In most cases, the $\text{CAP}_n^{\min}$ parameter is zero.

$$\begin{aligned} \text{NETBAL} \sum_a (\text{TR}_{s,s',a} + \text{TR}_{s',s,a}) &\geq \sum_{n,m} \text{XN}_{n,s,s'} \text{CAP}_n^{\min} \text{OCC}_m \quad \forall s, s' \\ \text{NETBAL} \sum_a (\text{TR}_{s,s',a} + \text{TR}_{s',s,a}) &\leq \sum_{n,m} \text{XN}_{n,s,s'} \text{CAP}_n^{\max} \text{OCC}_m \quad \forall s, s' \end{aligned}$$

- Enforce network symmetry, i.e. all roads can be travelled on in both directions.

$$\text{XN}_{n,s,s'} = \text{XN}_{n,s',s}$$

- The number of vehicles used between s and s' , and vice versa, must be sufficient to meet the activity trips but bounded by the maximum capacity of the available infrastructure. These equations, combined with the definition of PASSKM_m , ensure that the most energy efficient mode of transportation is used. A mode choice model is not used but users can place constraints on TOTVEH_m to simulate observed levels of car ownership; this is illustrated in Section 5.

$$\begin{aligned} \sum_{m|(m,n) \in \mathcal{MN}} (\text{VEH}_{m,n,s,s'} + \text{VEH}_{m,n,s',s}) &\leq \text{XN}_{n,s,s'} \text{CAP}_n^{\max} \quad \forall n \\ \sum_{m,n \in \mathcal{MN}} (\text{VEH}_{m,n,s,s'} + \text{VEH}_{m,n,s',s}) \text{OCC}_m &\geq \sum_a (\text{TR}_{s,s',a} + \text{TR}_{s',s,a}) \end{aligned}$$

- The total number of vehicles accounts for multiple daily uses per vehicle and the variable is rounded up to the nearest integer for the summary report, but not in the equations to improve performance.

$$\text{TOTVEH}_m = \lceil \sum_{n|(m,n) \in \mathcal{MN}} \sum_{s,s'} \text{VEH}_{m,n,s,s'} / \text{USE}_m \rceil$$

- The total network length is defined by summation, dividing by two because the network is symmetric.

$$\text{NETLEN}_n = \sum_{s,s'} \text{XN}_{n,s,s'} \text{D}_{s,s'} / 2$$

- Calculate the number of passenger kilometres travelled on each mode m . Since $\text{VEH}_{m,n,s,s'}$ is constrained by $\text{XN}_{n,s,s'}$, the total passenger kilometres will only include those $s \rightarrow s'$ links connected by a network. In other words, PASSKM_m represents the sum of all flows on valid route segments.

$$\text{PASSKM}_m = \sum_{n|(m,n) \in \mathcal{MN}} \sum_{s,s'} \text{VEH}_{m,n,s,s'} \text{OCC}_m \text{USE}_m \text{D}_{s,s'}$$

4. Applying the model to a UK eco-town

The validity of the model was assessed by applying it to the case of a UK eco-town. The eco-towns movement is a government-promoted initiative to demonstrate the best in sustainable community design and in its early stages, 12 potential sites were proposed (DCLG, 2009). Working with the developer of one of these sites, we were provided with details of the site design as would typically be available early in the master planning process: i.e. total population (6500), anticipated housing densities, activity provision, and planned transport networks. These input data are summarised in Table 2 and the results shown in Table 3.

The model was first run using the developer’s master plan in order to establish a benchmark for analysis; this layout is shown in Figure 2(a). When compared to the average consumption and emission figures for London, the results here are approximately 70% lower but this can be explained by the following differences. First, the developer’s master plan calls for a SAP rating¹ of 100 in domestic buildings, approximately twice as efficient as the current UK housing stock. Secondly, the development is small and features only minor commercial activity. Third, the model considers only transport within the city and no additional travel to or from outlying areas. The

¹The SAP rating system, or Standard Assessment Procedure, is widely used in the UK to assess the energy efficiency of buildings (BRE, 2009).

master plan therefore comes close to achieving the 80% carbon savings target envisioned for eco-towns (CABE and BioRegional, 2008).

The model was then run with the goal of establishing a minimum energy benchmark and the resulting layout is shown in Figure 2(b). The solution uses high-density housing exclusively and as a result, a large number of spaces in the city are left empty. Indeed the layout is fragmented into two distinct clusters, each containing housing and the required activities (school, leisure and mixed use). Since each cluster is self-sufficient, there is no need for transportation between clusters. Further savings are also achieved by the exclusive use of buses for transportation (the reference case has assumed that 50% of households own a car). When compared with the original master plan, this minimum energy benchmark represents a further 52% reduction in energy consumption and a 43% reduction in carbon emissions.

A distinction must be drawn between this absolute minimum energy benchmark and a feasible or realistic minimum benchmark. The initial results were discussed with the developer and they noted that such an extremely optimized case would be unlikely to be constructed in practice as the layout lacked the heterogeneity that makes a development attractive and liveable; the absence of cars was also deemed to be unrealistic. The question was therefore whether the structure of the model could be adapted to incorporate further constraints that would simulate a more realistic development environment. The following conditions were therefore added and the model re-run:

- a restriction on the minimum and maximum number of building types and activity providers within the development (e.g. a limit on how much high-density housing can exist);
- a restriction on the minimum and maximum allowable area for each single instance of an activity (e.g. a school requires a minimum of 1 hectare);

- a restriction on the minimum and maximum total area for each activity type (e.g. there must be at least 10 hectares of parkland for the entire development); and
- a restriction that vehicle ownership must be in-line with the original master plan, i.e. 50% of households own a car.

This revised optimization with constraints is shown in Figure 2(c). Again, there is a preference for high-density dwellings and so not all of the available land is used. However the urban form is remarkably similar to the original design, with mixed use buildings falling on the small zones in the southwest and northeast of the development and parkland spread throughout the middle of the site. Furthermore the transportation network now connects the entire site as the activity demands of the central housing can only be satisfied by the combination of activity provision at each end of the site. While this scenario still improves upon the original master plan, the additional savings are more modest: a 28% reduction in energy consumption and an 19% reduction in carbon emissions relative to the baseline planned case. The computational performance of the model was significantly better however, solving in approximately 5 seconds versus 5 minutes for the unconstrained case.

5. Monte Carlo analyses

Previous sketch layout models have been applied largely to single case studies, owing to hardware limitations or non-linear programming formulations. In this section, we demonstrate how the improved performance of the present mixed-integer model can facilitate its use within a Monte Carlo framework, in order to determine the variation of the minimum energy layout benchmark under a range of scenarios.

Monte Carlo analyses are important in the “post-normal” science debates

that surround contentious public issues where there is both significant uncertainty surrounding the parameters of debate and significant consequences of poor decisions (Ravetz, 1999). This has direct relevance to the planning of energy-efficient cities as developers and policy makers may wish to understand how different assumptions and imperfect project knowledge affect their development goals.

Two broad categories of analysis relevant here. The first is sensitivity analysis, “the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input” (Saltelli et al., 2008, p. 1). There are many different applications of sensitivity analysis (or *settings*) and in this case, we may wish to understand which of the model’s input variables has the most significant effect on the minimum energy consumption benchmark of a city. This suggests the use of the *factor prioritization* setting, i.e. the identification of input variables which, if fixed at their ‘true’ value (e.g. through better research or problem specification), would lead to the greatest reductions in the variability of the model results. However sensitivity analysis requires that a model’s output be defined at all points and certain parameter combinations in optimization models often result in an infeasible problem. A factor prioritization analysis was run for the present model but, owing to these infeasibility issues, only indicative results were possible (namely, that total population and the densities of housing and activity provision must all be well-defined – i.e. characterized by low variability – to calculate an effective benchmark). Instead we have therefore chosen to apply uncertainty analysis. This requires running the model multiple times with variable input data and simply describing the observed variation in output, rather than trying to attribute it to the variability of specific inputs.

It was noted above that cities require a certain degree of heterogeneity in order to make them sufficiently attractive to residents, but the eco-town

scenario showed that the absolute minimum energy configuration is based on the exclusive use of high-density housing and public transportation. This finding raises questions about how the minimum energy configuration benchmark might change in light of these liveability constraints.

5.1. Experimental design

The analysis proceeded as follows. First a stylised city was created, based on a grid of 100 16-hectare cells. There are only two activities within the city: work and shopping. More detailed examples, for instance with education, leisure or other activities, could be simulated but this adds computational complexity without generating significant additional insights. The three housing types (low, medium, and high density) and two transport modes (car and bus) from the eco-town case study are used and as before, each model run calculates the minimum energy layout of buildings, activities and transport networks in the city.

There are three sources of input variability to consider. First there is a range of possible population sizes. Since at least two cells must be set aside for the provision of work and shopping, at least one cell and at most ninety-eight cells can be occupied by residential housing of the varying densities described above. Calculating the low- and high-density extremes, this means that the model city might reasonably hold a population of between 180 and 360 000 residents (assuming a uniform distribution, U).

Next we assumed that planners had two policy decisions to make for their new eco-town. The first is a constraint on the maximum amount of high-density housing since developments of uniformly high-density may be undesirable places to live. This limit is expressed as a fraction of the total population housed, $U(0, 1)$. Similarly, a second constraint was added to restrict the minimum car ownership rate as a fraction of all households, $U(0, 1)$.

With this problem configuration, a single model run takes approximately

30 seconds to solve. While this is certainly acceptable for a single case study, running multiple scenarios can be time-consuming and it is therefore important that the input parameter distributions are sampled efficiently so as not to waste computational effort. The parameter values for each model run were therefore generated using Sobol’ low-discrepancy quasi-random sequences found in the `randtoolbox` package of R (R Development Core Team, 2009). Performance could be further improved by parallel computation but this strategy was not used here.

5.2. Results

The model was run 1000 times and 539 of the parameter combinations resulted in an infeasible problem, due to combinations of large population and insufficient housing provision. These infeasible results were removed from the data set before conducting the analyses.

Impact of density restrictions

To assess the impact of high-density housing restrictions on the minimum energy consumption benchmark, a baseline case was first defined by calculating the average minimum energy consumption benchmark for all feasible model runs with a high-density housing fraction of $10\% \pm 2.5\%$; this gives a value of 76 GJ per capita per year. A linear model was then fitted to the data as shown in Figure 3. This indicates that, relative to the baseline, every 1% point increase in the allowed maximum fraction of high-density housing will reduce minimum total energy consumption benchmark per capita by 0.35%. In other words, if a city decided to allow 20% high-density housing instead of only 10%, its minimum total energy consumption benchmark per capita would be reduced by approximately 3.5%. The figure also shows a hard lower bound, illustrating the threshold for feasible population and housing density combinations.

Impact of car ownership restrictions

The analysis was then repeated but this time examining the effect of restrictions on car ownership. We assumed that current practice is an unrestricted case, corresponding to 85 to 90% of households owning a car (the UK average is 88%). This gives a baseline transport energy consumption of 9.1 GJ per capita per year.

No significant linear regression was found between the total energy consumption per capita and the level of car ownership ($p = 0.85, df = 343$). However if transport energy consumption is considered on its own, there is a significant relationship as shown in Figure 4; for every additional 1% of households that own a car (above the 87.5% baseline), transport energy consumption increases by 0.24%. Therefore by promoting car sharing and other measures to encourage a 50% car ownership level, as in the eco-town, the minimum transport energy consumption per capita could be reduced by approximately 10%.

6. Discussion and conclusion

This paper has explored the use of an optimization-based sketch model to calculate minimum energy consumption benchmarks for new urban developments. In this concluding section, we highlight the major findings of the paper, consider the role of an “absolute minimum” benchmark, and highlight areas for future work.

6.1. Major findings

The research was motivated by an interest in the potential limits to low energy urban design, with regard to current eco-town projects. Conceptually, this question is very similar to the issue of excess commuting found in the land use and transportation modelling literature; however these approaches are typically used for existing cities and crucially lack building energy consumption. The present model extends upon this by jointly

optimizing building and activity location, as well as transportation flows. Methodologically, the developed model is similar to earlier work on sketch models but uses a mixed-integer linear formulation that balances higher spatial resolution and fast solution times.

The case of a UK eco-town provided a partial validation of the method. It was noted that there is a gap between the theoretical minimum energy urban layout and the layouts that developers are willing to construct. However it was shown that by adding a few additional constraints layouts can be generated that deliver energy savings but are still practicable for developers. Therefore while a modern disaggregated land use and transportation model would be valuable to assess the detailed use of a proposed urban plan, the results of the present model were recognized by practitioners as being relevant to their design processes.

A key benefit of the model’s formulation is its relative speed, which facilitates Monte Carlo-based uncertainty analysis. This technique was applied to examine the consequences of two planning decisions on the minimum energy consumption benchmark. The results found that, relative to a baseline of 10% of housing provided by high-density stock, each additional 1% of allowed high-density housing reduces the benchmark by approximately 0.35%. Similarly each percentage reduction in household car ownership, relative to the UK average of approximately 88%, results in a reduction in the transport energy benchmark of 0.24%. These analyses demonstrate how planners could use this modelling approach to assess the energy consequences of their decisions early in the master planning process.

6.2. How simple is too simple?

When unconstrained, the methodology presented here calculates a minimum possible, not feasible, energy configuration. However in a discussion of excess commuting benchmarks, Buliung and Kanaroglou (2002) makes the point that such “absolute” benchmark figures can be troublesome as they

may overstate achievable reductions. A similar question can be raised of the current work: to what extent are the benchmark figures presented here too simplified to be meaningful?

First, the validation study has shown that this framework is usable in practice as master planners can generate realistic designs by introducing site-specific constraints, such as limitations on the number of individual facilities, their sizes (for individual sites and total provision), or vehicle ownership. These types of modifications are well-suited to the optimization methodology and can be used to generate site-specific feasible benchmarks.

However, if the goal is to use the benchmark to assess the potential for improvements in existing cities (as in the excess commuting literature), then the effectiveness of the current model is less certain. One obstacle is normalization. For example the Monte Carlo analyses performed here assumed a UK context, as embodied in the energy consumption figures of each building type and transportation mode. If comparisons were to be made across countries (or even across climatic zones within a country), these values might be significantly different. This suggests that calculated benchmark values should be normalised on per capita basis, but also correcting for degree days of heating and cooling. A second issue is defining the precise meaning of a “feasible” benchmark. In the excess commuting literature, this has driven a shift from coarse aggregate models to more detailed representations of realistic behavioural constraints in order to better explain observed transportation patterns. Improving urban energy consumption more generally will also require a better understand of the acceptable level of service standards within buildings. To some extent, this is captured by the benchmark energy consumption values used here but again, these are UK specific and for international comparisons, a more general standard would be required.

These issues suggest that the present methodology is best used to calculate “absolute” minimum energy urban layouts. Given the ambitious targets

of proposed low energy urban developments, the greater savings indicated by such benchmarks can be used as a sanity check, to flag up the limitations of changes in urban form and the need for complementary changes in the energy system beyond the urban boundary. These values may also be used for comparing developments, so long as they are properly normalized and the general context is similar. To some extent, “feasible” minimum energy benchmarks can be calculated with this tool on a site-specific basis using customized constraints as shown earlier. However in general, a more behaviourally realistic approach should be adopted for such assessments.

6.3. Directions for future research

Even if restricted to the role of calculating absolute minimum energy layouts, there are a number of areas in which the current model could be improved. First, the model has only considered a single spatial scale. Using multi-scale models, for example by solving the assignment problem in a relaxed form at the borough level and then in discrete detail at a block level, would link with existing work on the fractal nature of cities (Batty and Longley, 1994) and improve model performance. Similarly, we have explored only a single time period. As the structure of cities evolves, it would be interesting to modify the model so that it could plan a sequence of infrastructure investments in order to maximize performance and minimize “regret” over decadal time periods. This would be a useful framework to assess the potential carbon liabilities of an urban infrastructure over time. Thirdly, the model was concerned only with minimizing energy consumption but there are many other objectives – both single and multi-criteria – which could be explored, such as using a minimum financial cost benchmark as a tool for developers. Finally, the model used simplified estimates of building energy demand. A more sophisticated model that could incorporate vertical transportation energy (e.g. from lifts), the effects of density on urban climate and hence heating and cooling demands, would be valuable.

Both the validation study and the Monte Carlo analyses focused on new build cities but the model formulation does not preclude its use in existing cities. Specifically, the model formulation allows parts of the city to be fixed and a limited area of redevelopment to be optimised within those constraints. This allows developers to calculate minimum energy benchmarks for a single project, while still capturing some interactions with existing infrastructures.

The minimum urban energy benchmark methodology presented in this paper is therefore not a comprehensive solution to the design of energy efficient urban layouts. However it does offer a new approach, incorporating both transport and building energy consumption, that can complement existing urban modelling and assessment tools. Future work should therefore focus on refining the technique for use in specific application domains, such as the planning of new eco-towns and the comparison of existing urban energy consumption against an absolute benchmark.

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Study	Case study	Computer set-up	Formulation	n_c	n_a
Barber (1976)	Germantown, WI, USA (7000 people)	–	LP	9	11
Feng and Lin (1999)	Tanhai, Taiwan (300k people)	IBM/PC 486-33 with Turbo Pascal 6.0	MINLP	13	10
Lin and Feng (2003)	–	IBM Pentium II 300 MHz with Turbo Pascal 6.0	MINLP	13	7
Lin and Gau (2006)	Taipei central business district	–	LP	–	–
Yu et al. (2007)	Dalian, China (2.23 million)	8 CPU cluster with C++	NLP	300	2
This paper	various UK	500 CPU cluster with GAMS/CPLEX 9.0	MILP	100	35

Table 1: Comparison of previous urban layout optimization studies. Other variables are used in these models but only the core assignment problem variables, i.e. the number of cells (i.e. discrete zones within the model, n_c) and number of activities (i.e. land use categories, n_a) are shown here for an indicative comparison. Model formulations: LP = linear programming, NLP = non-linear programming, MILP = mixed integer linear programming, MINLP = mixed integer non-linear programming.

<i>Residential buildings</i>			
Type	Density (dw/ha)	Energy demand (GJ/dw)	Carbon emissions (tC/dw)
Low density	20	35.6	0.947
Medium density 1	35	28.9	0.796
Medium density 2	50	22.3	0.644
High density	65	17.0	0.522
<i>Commercial buildings</i>			
Type		Energy demand (GJ/ha)	Carbon emissions (tC/ha)
Open space		0	0
Mixed use		7600	195
School		5700	121
<i>Transport modes</i>			
Type		Energy demand (MJ/ps-km)	Carbon emissions (kg C/ps-km)
Car		5.00	0.128
Bus		1.11	0.089

Table 2: Input data for the eco-town case study. Housing densities were provided by the developer, the energy consumption of residential buildings is estimated from the planned SAP rating of 100 (BRE, 2009), commercial building loads are estimated, transport energy from Hobson (2003); maintenance energy for roads is assumed to be negligible. Energy and carbon values also represent Scope 1 and 2 emissions, i.e. no upstream emissions (Kennedy et al., 2009).

Scenario	Total final energy consump. (GJ/cap)	Total carbon emissions (tC/cap)	Optimality gap (%)	Solve time (s)
London	78.6	1.62	–	–
As planned	23.8	0.81	1.2	0.2
Min. energy (no constraints)	11.4	0.46	10.2	300.2
Min. energy (w/ constraints)	17.1	0.66	4.5	5.5

Table 3: Summary of modelled layouts for a UK eco-town. London data from GLA (2003). The optimization was set to terminate at a 5% optimality gap or after 5 minutes.

Figure 1: Schematic for the layout model.

(a) As planned

(b) Energy optimized

(c) Energy optimized (with constraints)

Figure 2: Alternative optimized layouts for a UK eco-town. White cells with black borders are labelled with the housing density in dwellings per hectare. Activity cells are shaded as follows: light gray, schools (S1 and S2); medium gray, open space/leisure (L); black, mixed use (MX). Road networks are shown with dashed line (omitting connections to individual cells for clarity).

Figure 3: Linear regression showing total energy consumption savings achieved as allowable high-density fraction increases. The dashed line indicates the reference case with 10% maximum high-density housing; the shaded area and line represent the linear regression model with 95% confidence intervals ($r^2 = 0.32, p \ll 0.0001, df = 343$ after removing outliers).

Figure 4: Linear regression showing transport energy savings as allowable car-ownership levels decrease. The dashed line indicates the reference case of 87.5% car-ownership; the shaded area and line represent the linear regression model with 95% confidence intervals ($r^2 = 0.15, p \ll 0.0001, df = 343$).