A Generalised Component-Based Model for Semi-Rigid Beam-to-Column Connections Including the Axial versus Bending Moment Interaction

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Abstract

This paper presents a generalised component-based model for semi-rigid beam-to-column connections including the axial force versus bending moment interaction. The detailed formulation of the proposed analytical model is fully described in this paper, as well as all the analytical expressions used to evaluate the model properties. This paper also illustrates detailed examples of how to use this model to predict moment-rotation curves for any axial force level, the numerical results that were generated and validated against experimental tests, and a tri-linear approach to characterise the force-displacement relationship of the joint components. A bibliographical review containing a brief description of the most important available techniques to predict the joint structural behaviour, using mechanical models, and some experimental tests is also presented.

Keywords: Steel structures; Semi-rigid joints; Joint behaviour; Axial versus bending moment interaction; Component models; Component method.

1. Introduction

The continuous search for the most accurate representation of structural behaviour depends directly on a detailed structural modelling, including the interactions between

all the structural elements, linked to the structural analysis procedures, such as a material and geometric non-linear analysis. This strategy permits a more realistic modelling of connections, instead of the usual pinned or rigid assumptions. This idea is crucial to advance towards a better overall structural behavioural understanding, since the joint response is well-described by the moment-rotation curve. However, this approach involves a complete knowledge of semi-rigid joint behaviour, which is, for some situations, beyond the scope presents in the design codes, such as the influence of axial forces on the joint bending moment versus rotation characteristic curve. In addition to the most accurate structural modelling, the use of semi-rigid joints have several advantages such as the ones identified in SCI Publication 183 (1997): economy of both design effort and fabrication costs; beams may be lighter than in simple construction; reduction of mid-span deflection due to the inherent stiffness of the joint; connections are less complicated than in continuous construction; frames are more robust than in simple construction; and for an unbraced frame, additional benefit may be gained from semi-continuous joints in resisting wind loading without the extra fabrication costs incurred when full continuity is adopted.

Under certain circumstances, beam-to-column joints can be subjected to the simultaneous action of bending moments and axial forces. Although, the axial force transferred from the beam is usually low, it may, in some situations attain values that significantly reduce the joint flexural capacity. These conditions may be found in: structures under fire situations where the effects of beam thermal expansion and membrane action can induce significant axial forces in the connection (Ramli-Sulong et al., 2007); Vierendeel girder systems (widely used in building construction because they take advantage of the member flexural and compression resistances eliminating the need for extra diagonal members); regular sway frames under significant horizontal loading (seismic or extreme wind); irregular frames (especially with incomplete storeys) under gravity/horizontal loading; and pitched-roof frames. On the other hand, due to the recent escalation of terrorist attacks on buildings, the investigation of progressive collapse of steel framed building has been highlighted, as can be seen in Vlassis et al. (2006). Examples of these exceptional conditions are the cases where structural elements, such as central and/or peripheral columns and/or main beams, are suddenly removed, abruptly increasing the joint axial forces. In these situations the structural system, mainly the connections, should be sufficiently robust to prevent the premature failure modes that may lead to a progressive structural collapse.

Unfortunately, few experiments considering the bending moment versus axial force interactions have been reported. Additionally, the available experiments are related to a small number of axial force levels and associated bending moment versus rotation curves. Recently, some mechanical models have been developed, see section 1.2, to deal with the bending moment-axial force interaction. However these models are still not able to accurately predict the joint moment-rotation curves disabling their incorporation in the current design codes. There is, therefore, the need to develop the mechanical model for semi-rigid beam-to-column joints including the axial force versus bending moment interaction, based on the principles of the component method, Eurocode 3 (2005). The next sections present a detailed formulation of this generalised mechanical model including a proposal for joint component characterisation, as well as examples of its application and validation against experimental tests. However, before this is accomplished a bibliographical review containing a brief description of the most

important available techniques to predict the joint structural behaviour and some experimental tests is presented.

1.1. Component method

The component method entails the use of relatively simple joint mechanical models, based on a set of rigid links and spring components. The component method – introduced in Eurocode 3 (2005) – can be used to determine the joint's resistance and initial stiffness. Its application requires the identification of active components, the evaluation of the force-deformation response of each component (which depends on mechanical and geometrical properties of the joint) and the subsequent assembly of the active components for the evaluation of the joint moment versus rotation response.

Nowadays, using the Eurocode 3 (2005) component method, it is possible to evaluate the rotational stiffness and moment capacity of semi-rigid joints when subject to pure bending. However, this component method is not yet able to calculate these properties when, in addition to the applied moment, an axial force is also present. Eurocode 3 (2005) suggests that the axial load may be disregarded in the analysis when its value is less than 5% of the beam's axial plastic resistance, but provides no information for cases involving larger axial forces. Although, the component method has not considered the axial force, its general principles could be used to cover this situation, since it is based on the use of a series of force versus displacement relationships, which only depend on the component's axial force level, to characterize any individual component behaviour.

1.2. Background

The study of the semi-rigid characteristics of beam to column connections and their effects on frame behaviour can be traced back to the 1930s, Li et al. (1995). Since then, a large amount of experimental and theoretical work has been conducted both on the behaviour of the connections and on their effects on complete frame performance. Despite the large number of experiments, few of them consider the bending moment versus axial force interactions.

This section has attempted to provide a summary of the techniques currently available to predict the joint structural behaviour, as well as a brief discussion of some experimental tests, focusing on the study of the joint behaviour under combined bending moment and axial force using mechanical models.

1.2.1. Experimental

A detailed discussion of all available experimental tests is beyond the scope of this paper; a compilation of the experiments is, however, available in Nethercot (1985); Weynand (SERICON I, 1992) and Cruz et al. (SERICON II, 1998). Recently, several researchers have paid special attention to joint behaviour under combined bending moment and axial force. Guisse et al. (1996) carried out experiments on twelve column bases, six with extended and six with flush endplates. Wald and Svarc (2001) tested three flush endplate beam-to-beam joints and two extended endplate beam-to-column joints; however there is no reference to tests made with only bending moment, which is important to study the influence of the axial force in the joint response. Lima et al. (2004) and Simões da Silva et al. (2004) performed tests on eight flush endplate joints and seven extended endplate joints. The investigators concluded that the presence of the

axial force in the joints modifies their structural response and, therefore, should be considered in the joint structural design.

1.2.2. Theoretical models

As an alternative to tests, other methods have been proposed to predict bending moment versus rotation curves. These procedures range from a purely empirical curve fitting of test data, passing through ingenious behavioural, analogy and semi-empirical techniques, to comprehensive finite element analysis, Nethercot & Zandonini (1989).

Mathematical formulations (empirical models)

The first attempt of fitting a mathematical representation to connection moment-rotation curves dates back to the work of Baker (1934) and Rathbun (1936), who used a single straight-line tangent to the initial slope, thereby overestimating connection stiffness at finite rotations. In the 1970s the use of bilinear representations was introduced by Lionberger & Weaver (1969) and Romstad & Subramanian (1970). These recognised the reduced stiffness at higher rotations, however it was only acceptable for certain joint types and for applications where only small joint rotations are likely. Kennedy (1969), Sommer (1969), Frye & Morris (1975) proposed polynomial representations that recognised the curved nature, but required mathematical curve fitting and consideration of a family of experimental moment-rotation curves. Ang & Morris (1984) replaced the polynomial representation by a Ramberg-Osgood (1943) type of exponential function that has the advantage of always yielding a positive slope, but is also dependent on mathematical curve fitting. Multi-linear representations were proposed by Moncarz & Gerstle (1981) and Poggi & Zandonini (1985) to overcome the obvious limitation of the bilinear model in that it could not deal with continuous changes in stiffness in the knee region. B-spline techniques were suggested by Jones et al. (1981) as an alternative to polynomials as a means of avoiding possible negative slopes. Lui & Chen (1986) used an exponential representation that despite being complex could readily be incorporated in analytical computer programs (Nethercot et al., 1987). Although it is possible to closely fit virtually any shape of moment-rotation curve, purely empirical methods possess the disadvantage that they cannot be extended outside the range of the calibration data. This is particularly important for joints such as endplates where the change in geometrical and mechanical properties of the connection may lead to substantially different behaviours and collapse mechanisms (Nethercot & Zandonini, 1989). Aiming to overcome this limitation, Yee & Melchers (1986), Kishi et al. (1988a,b) and Chen & Kishi (1987) proposed models linking curve fitting approaches to some form of behavioural model, but these were still dependent on a mathematical curve fitting.

Focusing on finite element analysis, Richard et al. (1980) used a type of formula already developed by Richard & Abbott (1975) to represent data generated by finite element analyses in which the constitutive relations of certain of the joint components, e.g. bolts in shear, were directly obtained from subsidiary tests.

Each of the models discussed so far may only be used to describe the joint behaviour under a single application of a monotonically increasing load. However, some of them were modified and/or adapted to represent the performance of certain connection types under cyclic loading, as can be seen in the work done by Moncarz & Gerstle (1981), Altman et al. (1982) and Mazzolani (1988).

Aiming to incorporate a limited set of experiments including the axial versus bending moment interaction into a structural analysis, Del Savio et al. (2007b) developed a consistent and simple approach to determine moment-rotation curves for any axial force level. Basically, this method works by finding moment-rotation curves through interpolations executed between three required moment-rotation curves, one disregarding the axial force effect and two considering the compressive and tensile axial force effects. This approach can be easily incorporated into a nonlinear joint finite element formulation since it does not change the finite element basic formulation, only requiring a rotational stiffness update procedure.

Simplified analytical models

Several authors have applied the basic concepts of structural analysis (equilibrium, compatibility and material constitutive relations) to simplified models of the key components in various types of beam-to-column connections (Nethercot & Zandonini, 1989). Lewitt et al. (1969) provided formulae for the load-deformation behaviour of double web cleat connections in both the initial and the final plastic phases; however these models needed to be used in conjunction with knowledge of the connection rotation centre. Chen & Kishi (1987) and Kishi et al. (1988a,b) considered the behaviour of web cleats, flange cleats and combined web and flange cleat connections where their resulting values of initial connection stiffness and ultimate moment capacity were utilized in a Richard type of power expression (Richard & Abbott, 1975) to represent the resulting moment-rotation curve. Assuming that the behaviour of the whole joint may be obtained simply by superimposing the flexibilities of the joint components (member elements, connecting elements, fasteners) Johnson & Law (1981) proposed a method for the prediction of the initial stiffness and plastic moment capacity of flush endplate connections, however no comparison was conducted against experimental results. Based on the same philosophy, Yee & Melchers (1986) developed a method for bolted endplate eaves connections in which an exponential representation was assumed, which depends on four parameters where only one is dependent on test data. Richard et al. (1988) proposed a four-parameter formula to describe the load-deformation and moment-rotation relationship for bolted double framing angle connections. This model is composed of a rigid bar and nonlinear spring, representing the angle segments in either tension or compression. The moment-rotation behaviour of the connections is determined through an iterative procedure by satisfying equilibrium and compatibility conditions. A similar approach was developed and used by Elsati & Richard (1996) in a computer-based programme to validate the model against the test results of a variety of connection types for both composite and steel beam connections. A three-parameter exponential model was suggested by Wu & Chen (1990) to model top and seat angles with and without double web angle connection and due to its simplicity it could be implemented in the analysis of semi-rigid frames. In the same year, Kishi & Chen (1990) proposed a semi-analytical model to predict moment-rotation curves of angle connections, which later was extended by Foley & Vinnakota (1994) for unstiffened extended endplate connections. Although these methods require a few key parameters, the use of test data is normally necessary to calibrate some of its coefficients. A wider discussion about some of these methods can be found in Nethercot & Zandonini (1989) and Faella et al. (2000).

Finite element analysis

The numerical simulation started being used as a way to overcome the lack of experimental results; to understand important local effects that are difficult to be measured with sufficient accuracy, e.g. prying forces and extension of the contact zone, contact forces between the bolt and the connection components; and to supply extensive parametric studies. The first study into joint behaviour making use of the FEM was executed by Bose et al. (1972) related to welded beam-to-column connections, where an incremental analysis was performed, including in the formulation plasticity, with strain hardening, and buckling. The comparison with available experimental results showed satisfactory agreement, but only the critical load levels were considered. Since then, several researchers have been using the FEM to investigate the joint behaviour, such as: Lipson & Hague (1978) – single-angle bolted-welded connection; Krishnamurthy et al. (1979) – extended endplate connections; Richard et al. (1983) – double-angle connection; Patel & Chen (1984) – welded two-side connections; Patel & Chen (1985) – bolted moment connection; Kukreti et al. (1987) – flush endplate connections; Beaulieu & Picard (1988) – bolted moment connection; Atamiaz Sibai & Frey (1988) – welded one-side unstiffened joint configuration. More recently, focusing on 3D finite element models the following works can be mentioned: Sherbourne & Bahaari (1994); Bursi & Jaspart (1997, 1998); Yang et al. (2000); Citipitioglu et al. (2002); Coelho et al. (2002); and Maggi et al. (2002).

Mechanical models (component-based approaches)

Mechanical models have been developed by several researchers for the prediction of moment-rotation curves for the whole range of connections/joints, where the number of physical governing parameters is rather limited. These models have also been confirmed as an adequate tool for the study of steel connections; however their accuracy relies on the degree of refinement and accuracy of the assumed load-deformation laws for the principal components. The determination of such characteristics requires a complete understanding of the behaviour of single components, as well as of the way in which they interact, as a function of the geometrical and mechanical factors of the complete connections, Nethercot & Zandonini (1989).

Wales & Rossow (1983) practically introduced the use of mechanical models, or rather, a component-based method, when they developed a model for double web cleat connections, Fig. 1, in which the joint was idealised as two rigid bars connected by a homogeneous continuum of independent nonlinear springs. Each nonlinear spring was defined by a tri-linear load-deformation law obtained via the analysis of numerical models for the whole connection. Both bending moment and axial force were considered to act on the connection and coupling effects between the two stress resultants were then included in the joint stiffness matrix. Comparisons were made with a single test by Lewitt et al. (1969) aiming to validate the philosophy. An important feature of this model is to account for the presence of the axial force. Results obtained by Wales & Rossow (1983) indicate that greater attention should be given to such axial forces, as a factor affecting the response of beam-to-column connections.

Kennedy & Hafez (1984) used a technique of connection discretisation to describe the behaviour of header plate connections. T-stub models were used to represent the tension and compression parts of the connection. Although this model had provided good agreement with comparisons done against the author's own tests for ultimate moment capacity, the prediction of the corresponding rotations were not as accurate.

Chmielowiec & Richard (1987) extended the model proposed by Wales & Rossow (1983) to predict the behaviour of all types of cleated connections only subjected to bending and shear, Fig. 2. Mathematical expressions were adopted for the force-deformation relationships of the double angle segments and later calibrated by curve fitting against experimental results obtained by the same author. Comparisons with experimental data from a different series of connection tests in general confirmed the accuracy of the method.

An extensive investigation into the response of fully welded connections was conducted by Tschemmerneegg (1988), where the mechanical model of Fig. 3 was proposed. In this model, springs A are meant to account for the load introduction effect from the beam to the column, while springs B simulate the shear flexibility of the column web panel zone. Thirty tests were carried out, using a wide range of beam and column sections, making possible a calibration of the mathematical models assumed to describe the spring element properties. The moment-rotation curves for fully welded connections were determined via the model for all possible combinations of beams and columns made of European rolled sections IPE, HEA and HEB. This model was extended by Tschemmerneegg & Humer (1988) for endplate bolted connections by adding new springs (Fig. 4, springs C), to take into account the new sources of deformation. This model was also calibrated against experimental tests with good results.

For 10 years, since the proposed model by Wales & Rossow (1983) considering the bending moment and axial force interaction, nothing had been done in terms of these coupled effects until Madas (1993). Despite the fact that Wales & Rossow in 1983 noted that greater attention should be given to such axial forces, as a factor affecting the response of beam-to-column connections. Madas (1993) extended the mechanical model proposed by Wales & Rossow (1983) to flexible endplate, double web angle and top and seat angle connections including both bare steel and composite connections. Fig. 5 shows the idealized beam-to-column connection used by Madas (1993). This model presented good agreement with experimental results; however it was not evaluated against experiments including the axial force versus bending moment interaction.

Based on preliminary studies carried out by Finet (1994), Jaspart et al. (1999) and Cerfontaine (2003) developed a numerical approach aiming at analysing the joint behaviour from the first loading steps up to collapse, Fig. 6, subjected to bending moment and axial force. This approach is idealised by a mechanical model comprising extensional springs, Fig. 6(b). Each spring represents a joint component by exhibiting non-linear force-displacement behaviour, Fig. 6(c). Nunes (2006) compared the experimental results obtained by Lima (2003) for flush and extended endplate joints to the analytical results using the Cerfontaine (2003) analytical model. This study pointed out some problems in the joint behaviour prediction using this analytical model, such as an overestimation of the initial stiffness in the majority of the cases, as well as variations between over and underestimation of the final moment capacity for some cases. These discrepancies were more pronounced for the cases in which the joints were subjected to bending moments and tensile axial forces.

A simplified mechanical model was suggested by Pucinotti (2001) for top-and-seat and web angle connections as an extension of Eurocode 3 (1998) to take into account the web cleats and hardening contributions. Comparisons against experimental tests showed that this model is able to estimate the initial stiffness accurately; however the final flexural capacity prediction is slightly erratic.

Using the same general principles, Simões da Silva & Coelho (2001) formulated analytical expressions for the full non-linear response of a welded beam-to-column joint under combined bending moment and axial force. Each bi-linear spring of this model was replaced by two equivalent elastic springs using an energy formulation and a post-buckling stability analysis. A comparison was made against a welded joint only subjected to bending moments and the results presented a good agreement with the experiments.

Sokol et al. (2002) developed an analytical model to predict the endplate joint behaviour subjected to bending moment and axial force interaction. This model was tested against two sets of experiments with flush endplate beam-to-beam joints and extended endplate beam-to-column joints carried out by Wald and Svarc (2001). In general, the results involving moment-rotation comparisons provided rather close agreement with the experimental tests, however, for all the analysed cases, the initial stiffness was overestimated whilst the final moment capacity was underestimated.

Lima (2003) and Simões da Silva et al. (2004) proposed mechanical models for extended (Fig. 7) and flush (Fig. 8) endplate joints, respectively. Following, basically, the same idea and also based on Madas (1993), Ramli-Sulong (2005) also developed a component-based connection model, Fig. 9, for flush and extended endplate, top-and-seat and/or web angles, and fin-plate connections. These models basically consist of two rigid bars representing the column centreline and the beam end, connected by non-linear springs that model the joint components. Furthermore, these authors included the compressive components (for instance, cwc - column web in compression, Figs. 7, 8 and 9 at the same location as the bolt rows and the tensile components (column web and beam web in tension, column flange and endplate in bending, and bolts in tension, Figs. 7 and 8) at the same location as the flanges (compressive rows). Proposed models by Lima (2003) and Simões da Silva et al. (2004) were tested against their own experimental tests. Although these models presented satisfactory results in terms of ultimate flexural capacity, the prediction of the initial stiffness, for the case of different axial load levels, was not accurate, predicting almost the same initial stiffness for the whole set up evaluated cases, Figs. 10 and 11. Regarding Ramli-Sulong's model (Ramli-Sulong, 2005, and Ramli-Sulong et al., 2007), neither comparison has been done against experimental moment-rotation curves nor parametric analysis involving different axial force levels, which are needed to evaluate this model in terms of quality of moment-rotation curve prediction for moment-axial interaction. On the other hand, this model was shown to be able to predict, with a good accuracy, the experimental moment-rotation curves, disregarding the axial effect. Comparisons made at elevated temperature with available tests also presented a good agreement.

Urbonas & Daniunas (2006) proposed a component method extension to endplate bolted beam-to-beam joints under bending and axial forces, however the procedure for joint moment-rotation curve prediction is only applicable and valid throughout the elastic regime of structural behaviour. Numerical tests were executed by the authors with a three-dimensional joint modelling, using finite element, with the goal to validate this model. The results obtained for the beam-to-beam joint initial stiffness were close to the finite element analysis.

Table 1 presents a summary of the mechanical models for predicting joint behaviour discussed in this section.

Despite the continuous development and improvement of analytical models to predict the behaviour of joints under bending moment and axial force, there are still

problems in the prediction of the moment-rotation curves, such as the joint initial stiffness for different axial force levels, as can be seen, for example, in Figs. 10 and 11 or in Nunes (2006). The magnitude of this problem increases when joints are subjected to tensile axial forces. This problem relates to the ability of these models to deal with moment-axial interaction, and consequently changes of the compressive centre, before the first component yields. If the model is working on the linear-elastic regime, without reaching any component yield (i.e. the component stiffness is also working linearly), the modification of the joint stiffness matrix, only due to the geometric stiffness changes, will be insignificant. From this point upwards to the onset of first component yield, these models are not able to represent accurately the joint initial stiffness for any level of axial load and bending moment while working on the linear-elastic regime. Aiming to overcome this limitation, a mechanical model is proposed in this paper, which allows modifications of the compressive centre position even before reaching the first component yield, i.e. in the linear-elastic regime.

2. Generalised mechanical model for beam-to-column joints including the axial-moment interaction

The generalised model proposed for semi-rigid beam-to-column joints including the axial force versus bending moment interaction is depicted in Fig. 12. This model, based on the component method, contains three rigid bars representing the column centreline (support bar), the column flange centreline (bar 2) and the beam end (bar 1). These rigid bars are connected by a series of springs that model the joint components. Due to the generalised formulation developed in this work, the model is able to represent any kind of connection, since the joint can be modelled according to the scheme shown in Fig. 12. The following sections present the adopted behaviour for each joint component as well as the complete formulation of this generalised mechanical model.

2.1. Characterisation of the joint components

The behaviour of each component of the joint is given by a force-deformation relationship, which may be characterised, for example, by a bi-linear, tri-linear or even a non-linear curve. Simões da Silva et al. (2002), based on Kuhlmann et al. (1998), classified the endplate joint components according to their ductility:

- Components with high ductility, Fig. 13(a): column web in shear (assuming no occurrence of local buckling), column flange in bending, endplate in bending and beam web in tension.
- Components with limited ductility, Fig. 13(b): column web in compression, column web in tension and beam flange in compression.
- Components with brittle failure, Fig. 13(c): bolts in tension and welds.

However, some comments are necessary regarding this classification:

- Eurocode 3 (2005) considers a rigid-plastic behaviour for beam web in tension.
- Lima (2003) verified a ductile behaviour for the beam flange in compression in his experiments.
- Welds are not considered in the joint rotation stiffness evaluation according to Eurocode 3 (2005).

In this work, a tri-linear approach for the force-deformation relationship is suggested and used for all the joint components as shown in Fig. 14. The component elastic stiffness, k_{cp}^e , and the component yield strength, f_{cp}^y , are calculated according to

the Eurocode 3 (2005) component method. On the other hand, for the component plastic stiffness, a strain hardening stiffness k_{cp}^p is evaluated as:

$$k_{cp}^p = \mu^p k_{cp}^e \quad (1)$$

The component reduced strain hardening stiffness, k_{cp}^u , referred to the component material fracture, is:

$$k_{cp}^u = \mu^u k_{cp}^e \quad (2)$$

where μ^p and μ^u are the strain hardening coefficients, respectively, for the plastic and ultimate stiffness, which depend on the component type. Based on the classification suggested by Simões da Silva et al. (2002), briefly discussed in this paper, and curve fitting executed on the experimental tests carried out by Lima (2003), Table 2 presents the values adopted for the strain hardening coefficient for each joint component.

The component ultimate capacity, f_{cp}^u , is determined by, for each component, using the ultimate stress instead of the yield stress in equations present in Eurocode 3 (2005).

For the case when the component related to the column web panel in shear is activated, i.e. when unbalanced moments exist in the connection, and the beam top flange and bottom flange of the connection are in compression, this component will be divided into two equal springs (one for the beam top flange and another for the beam bottom flange) characterised by its usual stiffness and yield and ultimate strengths divided by two.

The generalised mechanical model formulation, described in the next section, uses an effective stiffness for each model row/spring i referred to the bolts and beam flanges, which is evaluated as:

$$k_i = \left\{ k_i^e = \frac{1}{\sum_{j=1}^{nc} \frac{1}{k_{cp}^e}} \text{ or } k_i^p = \frac{1}{\sum_{j=1}^{nc} \frac{1}{k_{cp}^p}} \text{ or } k_i^u = \frac{1}{\sum_{j=1}^{nc} \frac{1}{k_{cp}^u}} \right\} \quad (3)$$

where nc is the component number that contributes to the stiffness k_i of the row/spring i . The spring/row stiffness depends on the force-deformation relationship of each joint component that is evaluated according to the proposed procedure described in this section.

2.2. Generalised mechanical model formulation

The stationery potential energy principle was used to formulate the model stiffness matrix and the corresponding equilibrium equation. The total potential energy functional, Π , is:

$$\Pi = U - W \quad (4)$$

where U is the system strain energy and W is the load total potential. The system strain energy can be expressed in terms of the tangent stiffness k_i , Eq. 3, of the spring i , and relative displacements, Δ_i , as:

$$U = \frac{1}{2} \sum_{i=1}^{ns} k_i \Delta_i^2 \quad (5)$$

where ns is the system spring number. Assuming small displacements, the relative (Δ_i) and absolute (u_i and ul_i) displacements for the system presented in Fig. 12 can be evaluated as:

$$\begin{array}{ll}
\text{Bar 1} & \text{Bar 2} \\
\Delta_i = u_i - ul_i & \Delta_i = u_i \\
u_i = u_{b1} - C_i \sin(\theta_{b1}) & u_i = u_{b2} - C_i \sin(\theta_{b2}) \\
ul_i = u_{b2} - C_i \sin(\theta_{b2}) &
\end{array} \tag{6}$$

where C_i is the spring vertical coordinate i regarding the load application line. The spring coordinates above the loading application line must have a positive sign while the springs located below the loading application line should attain a negative sign. θ_{b1} and u_{b1} , θ_{b2} and u_{b2} are the rotations (θ_{bi}) and displacements (u_{bi}) of bars 1 and 2, respectively.

The system load total potential is, Fig. 12:

$$W = P(u_{b1} - u_{b2}) + M\theta_{b1} \tag{7}$$

where P is the axial load and M is the bending moment. Using the total potential energy principle, the equilibrium equations can be derived from the functional stationary condition Π (Eq. 4),

$$\frac{\partial \Pi}{\partial d_i} = 0; \quad d_i = u_{b1}, \theta_{b1}, u_{b2}, \theta_{b2} \tag{8}$$

the stiffness matrix, K_{ij} , and internal load vector, F_i , can be derived using Eq. 5,

$$K_{ij} = \frac{\partial^2 U}{\partial d_i \partial d_j}; \quad d_i = u_{b1}, \theta_{b1}, u_{b2}, \theta_{b2} \tag{9}$$

$$F_i = \frac{\partial U}{\partial d_i}; \quad d_i = u_{b1}, \theta_{b1}, u_{b2}, \theta_{b2} \tag{10}$$

Approximating the trigonometric expressions in Eq. 6 to the first order, the model stiffness matrix, Fig. 12, for any spring number at any position can be evaluated as:

$$\left[\begin{array}{cccc}
K_{11} = \sum_{i=1}^{ns_{b1}} k_i & K_{12} = -\sum_{i=1}^{ns_{b1}} k_i C_i & K_{13} = -K_{11} & K_{14} = -K_{12} \\
& K_{22} = \sum_{i=1}^{ns_{b1}} k_i C_i^2 & K_{23} = -K_{12} & K_{24} = -K_{22} \\
\text{Symmetric} & K_{33} = \sum_{i=1}^{ns} k_i & K_{34} = -\sum_{i=1}^{ns} k_i C_i & \\
& & & K_{44} = \sum_{i=1}^{ns} k_i C_i^2
\end{array} \right] \tag{11}$$

where K_{11} and K_{33} are the matrix terms related to the axial deformations of the beam-to-column connection; K_{12} and K_{34} are associated with the interaction between the axial and the rotational deformations; K_{22} and K_{44} are correlated with the rotational deformations. The internal loading vector is:

$$F = [P \quad M \quad 0.0 \quad 0.0]^T \tag{12}$$

Due to the simplicity of this mechanical model formulation, it can be easily incorporated into a nonlinear semi-rigid joint finite element formulation, only requiring a tangent stiffness update procedure of each joint spring.

Regarding the first order approximations for the trigonometric expressions used on the generalised mechanical model formulation, in section 5 is presented the error evaluation of these approximations versus joint rotations.

2.2.1. Analytical expressions: displacements and rotations

This section presents the analytical expressions for the displacements and the rotations of the proposed generalised mechanical model, Fig. 12. The main goal is to generate equations for the evaluation of these properties without executing a mechanical model numerical analysis.

Rewriting the equilibrium equations, Eq. 8, based on the symmetric stiffness matrix, Eq. 11, provides the complete equilibrium equations as a function of six stiffness terms, K_{11} , K_{12} , K_{22} , K_{33} , K_{34} and K_{44} ,

$$K_{11}u_{b1} + K_{12}\theta_{b1} - K_{11}u_{b2} - K_{12}\theta_{b2} = P \quad (13)$$

$$K_{12}u_{b1} + K_{22}\theta_{b1} - K_{12}u_{b2} - K_{22}\theta_{b2} = M \quad (14)$$

$$-K_{11}u_{b1} - K_{12}\theta_{b1} + K_{33}u_{b2} - K_{34}\theta_{b2} = 0.0 \quad (15)$$

$$-K_{12}u_{b1} - K_{22}\theta_{b1} + K_{34}u_{b2} + K_{44}\theta_{b2} = 0.0 \quad (16)$$

Isolating θ_{b2} from the equilibrium Eq. 16,

$$\theta_{b2}(u_{b1}, \theta_{b1}, u_{b2}) = \kappa u_{b1} + \lambda \theta_{b1} - \rho u_{b2} \quad (17)$$

where,

$$\kappa = \frac{K_{12}}{K_{44}}; \quad \lambda = \frac{K_{22}}{K_{44}}; \quad \rho = \frac{K_{34}}{K_{44}} \quad (18)$$

Substituting θ_{b2} , Eq. 17, into the equilibrium Eq. 15, and isolating u_{b2} ,

$$u_{b2}(u_{b1}, \theta_{b1}) = \frac{\omega_2 u_{b1} + \chi_2 \theta_{b1}}{\psi} \quad (19)$$

where,

$$\psi = 1 - \frac{K_{34}^2}{K_{33}K_{44}}; \quad \omega_2 = \frac{K_{11}}{K_{33}} - \frac{K_{34}K_{12}}{K_{33}K_{44}}; \quad \chi_2 = \frac{K_{12}}{K_{33}} - \frac{K_{34}K_{22}}{K_{33}K_{44}}; \quad (20)$$

Substituting θ_{b2} , Eq. 17, into the equilibrium Eq. 14, and after substituting u_{b2} , Eq. 19, and subsequently isolating u_{b1} ,

$$u_{b1}(\theta_{b1}, M) = \frac{M\psi - \theta_{b1}X}{\Omega} \quad (21)$$

where,

$$\Omega = \omega_1 \psi + \omega_2 \xi; \quad X = \chi_1 \psi + \chi_2 \xi \quad (22)$$

$$\xi = \frac{K_{22}K_{34}}{K_{44}} - K_{12}; \quad \omega_1 = \frac{K_{12}}{K_{44}}(K_{44} - K_{22}); \quad \chi_1 = K_{22} - \frac{K_{22}^2}{K_{44}} \quad (23)$$

Substituting θ_{b2} (Eq. 17) into the equilibrium Eq. 13, u_{b2} (Eq. 19), then u_{b1} (Eq. 21), and subsequently isolating θ_{b1} generates the expression for the joint rotation (or first bar rotation), for any axial force and bending moment levels:

$$\theta_{b1}(P, M) = \frac{P\Omega - M\vartheta\psi}{Z} \quad (24)$$

where,

$$Z = \varphi\Omega - \vartheta X \quad (25)$$

$$\vartheta = K_{11} - \frac{K_{12}^2}{K_{44}} + \left(\frac{K_{11}K_{44} - K_{34}K_{12}}{K_{33}K_{44} - K_{34}^2} \right) \left(\frac{K_{12}K_{34}}{K_{44}} - K_{11} \right) \quad (26)$$

$$\varphi = K_{12} - \frac{K_{12}K_{22}}{K_{44}} + \left(\frac{K_{12}K_{44} - K_{34}K_{22}}{K_{33}K_{44} - K_{34}^2} \right) \left(\frac{K_{12}K_{34}}{K_{44}} - K_{11} \right) \quad (27)$$

Substituting θ_{b1} (Eq. 24) into Eq. 21 leads to the joint horizontal displacement (first bar horizontal displacement):

$$u_{b1}(P, M) = \frac{P}{\vartheta} \left(1 - \frac{\varphi\Omega}{Z} \right) + \frac{M\varphi\psi}{Z} \quad (28)$$

Substituting θ_{b1} (Eq. 24) and u_{b1} (Eq. 28) into Eq. 19 produces the following expression for the second bar horizontal displacement:

$$u_{b2}(P, M) = \frac{P}{\psi} \left[\frac{\omega_2}{\vartheta} \left(1 - \frac{\varphi\Omega}{Z} \right) + \frac{\chi_2\Omega}{Z} \right] + \frac{M}{Z} (\varphi\omega_2 - \vartheta\chi_2) \quad (29)$$

Finally, substituting θ_{b1} (Eq. 24), u_{b1} (Eq. 28) and u_{b2} (Eq. 29) into Eq. 17 leads to the expression for the second bar rotation:

$$\begin{aligned} \theta_{b2}(P, M) = & \frac{P}{Z} \left\{ \frac{\kappa}{\vartheta} (Z - \varphi\Omega) + \lambda\Omega - \frac{\rho}{\psi} \left[\frac{\omega_2}{\vartheta} (Z - \varphi\Omega) + \chi_2\Omega \right] \right\} \\ & + \frac{M}{Z} [\kappa\varphi\psi - \lambda\vartheta\psi - \rho(\varphi\omega_2 - \vartheta\chi_2)] \end{aligned} \quad (30)$$

With Eqs. 24, 28, 29 and 30 it is possible to evaluate all the joint displacements and rotations for any interaction level between axial force and bending moment, as well as, the forces in each spring:

$$f_i = k_i \Delta_i \quad (31)$$

where k_i and Δ_i are, respectively, the stiffness (Eq. 3) and the relative displacement (Eq. 6) of each spring.

2.2.2. Limit bending moments

For the correct use of the component method the prior knowledge of which model rows (bolts and flanges) are in tension and/or compression is needed due to their effect on the evaluation of the joint rotation and flexural capacity. In the usual Eurocode 3 (2005) mechanical model for joints subjected only to bending moment actions, a straightforward procedure is used to identify which rows are in compression and/or tension. However, when additional axial forces act on the joint, the identification whether each row is in tension or compression is not known in advance. This fact implies in the determination of the limit bending moment for the proposed mechanical model, Fig. 12, indicating the need to identify when the row forces change from compression to tension or vice-versa. With these results in hand, it is possible to adopt a consistent component distribution to be used following the Eurocode 3 (2005)

principles. The limiting bending moment, for each j -spring (component) located between the first and second bars, can be obtained by isolating, u_{b1} from Eq. 6,

$$u_{b1} = \Delta_j + C_j \sin(\theta_{b1}) + u_{b2} - C_j \sin(\theta_{b2}) \quad (32)$$

substituting u_{b1} into the two first equilibrium equations, of Eq. 8,

$$\frac{\partial \Pi}{\partial u_{b1}} = 0 \quad (33)$$

$$\frac{\partial \Pi}{\partial \theta_{b1}} = 0 \quad (34)$$

This is followed by isolating θ_{b1} from the first equilibrium equation Eq. 33, then substituting it into the second equilibrium equation Eq. 34 and making the relative displacement (Δ_j) equal to zero, and finally isolating the bending moment generates the following expression for the j -spring limit bending moment:

$$M_{j,\lim} = P \left(\frac{\sum_{i=1}^{ns_{b1}} k_i C_i^2 - C_j \left(\sum_{i=1}^{ns_{b1}} k_i C_i \right)}{C_j \left(\sum_{i=1}^{ns_{b1}} k_i \right) - \sum_{i=1}^{ns_{b1}} k_i C_i} \right) \quad (35)$$

It is worth noting that Eq. 35 depends only on the axial load applied to the connection, and the stiffness and the vertical coordinates of springs located between the first and second bars. There is no significant influence of springs located between the second bar and supports on the limit bending moment evaluation.

According to Eq. 35, for instance, for the first spring ($j = 1$), for $M < M_{1,\lim}$ all rows are compressed; $M = M_{1,\lim}$ first spring axial force is equal to zero; and $M > M_{1,\lim}$ there are both tensioned and compressed rows.

2.2.3. Moments that lead the joint rows and the joint to yield and failure

In this section analytical equations are derived, from the analytical expressions presented in section 2.2, for the evaluation of bending moments that lead the model springs/rows and the joint to both yield and failure, for any axial force level.

The displacement Δ_i^y that leads the model spring/row i to yield is obtained by isolating Δ_i from Eq. 31, and setting f_i equal to the weakest component yield strength of spring/row i , f_{cp}^y ,

$$\Delta_i^y = \frac{f_{cp}^y}{k_i^e} \quad (36)$$

Similarly, the displacement Δ_i^u that leads the model spring/row i to failure is,

$$\Delta_i^u = \frac{f_{cp}^u}{k_i^p} \quad (37)$$

where k_i^e and k_i^p are the elastic and the plastic stiffness of the spring/row i , respectively, given in Eq. 3. The relative displacement of spring/row i located between the first and second bars, from Eq. 6, is,

$$\Delta_{br,i} = u_{b1} - C_i \sin(\theta_{b1}) - (u_{b2} - C_i \sin(\theta_{b2})) \quad (38)$$

Approximating the trigonometric expressions in Eq. 38 to the first order; then substituting u_{b1} (Eq. 28), θ_{b1} (Eq. 24), u_{b2} (Eq. 29) and θ_{b2} (Eq. 30) into it; and making the relative displacement ($\Delta_{br,i}$) equal to Δ_i^y (Eq. 36) and subsequently isolating the bending moment generates the expression that leads the i -spring/row, located between the first and second bars, to yield:

$$M_{br,i}^y = \frac{\Delta_i^y - \alpha_1 + \alpha_2 - C_i(\alpha_3 - \alpha_4)}{\eta_1 - \eta_2 + C_i(\eta_3 + \eta_4)} \quad (39)$$

Similarly, making the relative displacement ($\Delta_{br,i}$) equal to Δ_i^u (Eq. 37), the expression for the bending moment that leads the i -spring/row, located between the first and second bars, to failure is produced:

$$M_{br,i}^u = \frac{\Delta_i^u - \alpha_1 + \alpha_2 - C_i(\alpha_3 - \alpha_4)}{\eta_1 - \eta_2 + C_i(\eta_3 + \eta_4)} \quad (40)$$

where the coefficients of Eqs. 39 and 40 are:

$$\begin{aligned} \alpha_1 &= \frac{P}{g} \left(1 - \frac{\varphi \Omega}{Z} \right) \\ \alpha_2 &= \frac{P}{\psi} \left[\frac{\omega_2}{g} \left(1 - \frac{\varphi \Omega}{Z} \right) + \frac{\chi_2 \Omega}{Z} \right] \\ \alpha_3 &= \frac{P}{Z} \left\{ \frac{\kappa}{g} (Z - \varphi \Omega) + \lambda \Omega - \frac{\rho}{\psi} \left[\frac{\omega_2}{g} (Z - \varphi \Omega) + \chi_2 \Omega \right] \right\} \\ \alpha_4 &= \frac{P \Omega}{Z} \\ \eta_1 &= \frac{\varphi \psi}{Z} \\ \eta_2 &= \frac{\varphi \omega_2 - g \chi_2}{Z} \\ \eta_3 &= \frac{\kappa \varphi \psi - \lambda g \psi - \rho (\varphi \omega_2 - g \chi_2)}{Z} \\ \eta_4 &= \frac{g \psi}{Z} \end{aligned} \quad (41)$$

Following the same idea, now, for spring/row i located between the second bar and supports, the relative displacement, from Eq. 6, is,

$$\Delta_{fr,i} = u_{b2} - C_i \sin(\theta_{b2}) \quad (42)$$

Approximating the trigonometric expressions in Eq. 42 to the first order; then substituting u_{b2} (Eq. 29) and θ_{b2} (Eq. 30) into it; and making the relative displacement ($\Delta_{fr,i}$) equal to Δ_i^y (Eq. 36) and subsequently isolating the bending moment produces the expression that leads the i -spring/row, located between the second bar and supports, to yield:

$$M_{fr,i}^y = \frac{\Delta_i^y - \alpha_2 + C_i \alpha_3}{\eta_2 - C_i \eta_3} \quad (43)$$

Similarly, making the relative displacement ($\Delta_{fr,i}$) equal to Δ_i^u (Eq. 37) leads to the following expression for the bending moment that leads the i -spring/row, located between the second bar and supports, to failure:

$$M_{fr,i}^u = \frac{\Delta_i^u - \alpha_2 + C_i \alpha_3}{\eta_2 - C_i \eta_3} \quad (44)$$

where the coefficients of Eqs. 43 and 44 given in Eq. 41.

Finally, the joint yield bending moment can be calculated as being the minimum yield bending moment given in Eqs. 39 and 43,

$$M^y = \min\{M_{br,i}^y \rightarrow Eq. 39, M_{fr,i}^y \rightarrow Eq. 43\} \quad (45)$$

and the joint plastic bending moment as being the minimum plastic bending moment evaluated by Eqs. 40 and 44,

$$M^u = \min\{M_{br,i}^u \rightarrow Eq. 40, M_{fr,i}^u \rightarrow Eq. 44\} \quad (46)$$

The joint rotational capacities, θ^y and θ^u , referred to the joint yield and plastic bending moments are, respectively,

$$\theta^y = \frac{P\Omega - M^y \mathcal{G}\psi}{Z} \quad (47)$$

$$\theta^u = \frac{P\Omega - M^u \mathcal{G}\psi}{Z} \quad (48)$$

For a given joint rotation (θ) and axial force (N), it is also possible to calculate the corresponding joint bending moment by isolating it from Eq. 24,

$$M = \frac{P\Omega - \theta Z}{\mathcal{G}\psi} \quad (49)$$

The analytical expressions developed in this section provide all the necessary information to predict bending moment versus rotation curves for any axial force level applied to the joint.

2.3. Prediction of bending moment versus rotation curve for any axial force level

Based on the equations previously developed, Fig. 15 presents an approach to characterise bending moment versus rotation curves considering the bending moment versus axial force interaction.

For each moment-rotation curve, the first point (θ^y, M^y) defines the joint initial stiffness corresponding to the attainment of the weakest component yield while the second point (θ^u, M^u) is obtained when the weakest component reaches its ultimate strength. The third point (θ^f, M^f) depends on the joint assumed final rotational capacity for the moment-rotation curve. In this work a 0.10-radian joint final rotation was adopted based on studies for both frames and individual restrained member. The joint rotations required at maximum load have shown that behaviour at rotations beyond 0.05 radians, often much less, has little practical significance, Nethercot & Zandonini (1989).

Summarising, the points of the moment-rotation curve are:

$$\begin{aligned} \text{Pnt 1} \quad \theta^y &\rightarrow Eq. 47; & M^y &\rightarrow Eq. 45 \\ \text{Pnt 2} \quad \theta^u &\rightarrow Eq. 48; & M^u &\rightarrow Eq. 46 \\ \text{Pnt 3} \quad \theta^f &= 0.10 \text{ radians}; & M^f &\rightarrow Eq. 49 \end{aligned} \quad (50)$$

It is worth highlighting that more points could have been used to describe the bending moment versus rotation curve because, for instance, before reaching the joint

plastic bending moment other joint rows might start yielding by generating new points between the first and second points (Eq. 50) changing the joint stiffness matrix. However, for simplicity of the approach and examples described in section 3, three points were adopted.

2.4. Lever arm d

The lever arm d represents the tensile rigid link position that unites the second bar to the supports, as can be seen, for instance, in Fig. 16. The evaluation of this lever arm d is needed when a mechanical model is adopted as in Fig. 16, where the first bolt rows are in tension, i.e., the beam top flange is not under compression. According to Del Savio et al. (2007a), the joint initial stiffness is strongly influenced by this lever arm d . Based on this fact, an approach is here presented for evaluation of this lever arm d which is divided into two equations: one for tensile forces and another for the complimentary cases disregarding axial forces and/or considering compressive forces applied to the joint.

2.4.1. Lever arm evaluation for tensile forces applied to the joint

Considering the support reactions and the applied loads, Fig. 16, the system force equilibrium can be evaluated as:

$$F_{bbf} - F_{linkt} = P \quad (51)$$

The system moment equilibrium at the beam bottom flange is:

$$F_{linkt}(d + e) + P e = M \quad (52)$$

where F_{bbf} is the row compressive yield capacity referred to the beam bottom flange; F_{linkt} is the rigid link tensile capacity that joins the second bar to the supports; d and e are, respectively, the distances from the loading application centre to the rigid link and the beam bottom flange.

Assuming M to be equal to the yield bending moment of the first bolt-row $M_{br,l}^y$ given in Eq. 39, F_{bbf} , P and e are already known, the problem variables are F_{linkt} and d . Then, isolating F_{linkt} from Eq. 51, substituting it into Eq. 52, and then isolating d leads to the expression for the lever arm position:

$$d = \frac{P e - M_{br,l}^y}{P - F_{bbf}} - e \quad (53)$$

which also satisfies the condition where F_{bbf} and $M_{br,l}^y$ simultaneously reach the yield.

2.4.2. Lever arm evaluation for the complementary cases disregarding axial forces and/or considering compressive forces applied to the joint

The lever arm d , for these cases, is evaluated as the ratio between the sum of bending moments referred to the bolt-rows and the axial force at the beam bottom flange and the sum of forces referred also to the bolt-rows and the axial force minus the distance from the load application centre to the beam bottom flange:

$$d = \frac{\sum_{i=1}^{nbr} f_{br,i}^y b_i + P e}{\sum_{i=1}^{nbr} f_{br,i}^y + P} - e \quad (54)$$

where nbr is the number of joint bolt-rows, $f_{br,i}^y$ is the yield strength of bolt-row i and b_i is the distance from joint bolt-row i to the beam bottom flange centre.

The lever arm d evaluated in either Eq. 53 or Eq. 54 take into account the change of the joint compressive centre position according to the axial force levels and bending moment applied to the joint, before the yield of the first weakest component is reached.

3. Application of the proposed generalised mechanical model

Application of the generalised mechanical model, developed in section 2, to predict the joint behaviour requires the following steps:

- (a) Generation and adoption of a joint model in consonance with the generalised mechanical model presented in Fig. 12.
- (b) Joint design according to Eurocode 3 (2005).
- (c) Characterisation of the joint components: force-displacement relationship of each component according to the approach suggested in section 2.1.
- (d) Identification of all the possible situations (model for compression, tension, tension/compression) given that loading may vary from pure bending to pure compressive/tensile axial force with all intermediate combinations. These intermediate combinations are derived from the adopted model in step (a).
- (e) Evaluation of the limit bending moments for the adopted models in step (d), with the aid of Eq. 35, to define the application domains of each one.
- (f) Evaluation of the lever arm d according to the proposed procedure in section 2.4, Eq. 53 for tensile forces and Eq. 54 for either compressive forces or without axial forces, considering the change of the joint compressive centre position.
- (g) Prediction of bending moment versus rotation curves for each axial force level, according to the approach described in section 2.3.

It is worth highlighting that the incorporation of this approach into a nonlinear semi-rigid joint finite element formulation does not require steps (d) and (e), because the complete joint modelling already considers all the possible situations of loading through each component force-displacement characteristic curve. In order to explain how each step is evaluated, six extended endplate joints tested by Lima et al. (2004) were tested.

3.1. Extended endplate joints

Starting with the application of step (a) previously described and using the extended endplate joint properties, Fig. 17, the following mechanical model was adopted, Fig. 18. Next step (b), with the joint material (Table 3) and geometric (Fig. 17) properties, the theoretical values of the strength and initial stiffness for the extended endplate joint components are evaluated according to Eurocode 3 (2005) and are presented in Table 4.

With the evaluated properties of the joint components, the characterisation of the force-displacement relationship for each component can be calculated according to the proposed formulation. Table 5 presents the results of this step (c).

Based on the adopted mechanical model, step (a), Fig. 18, four derived models are identified and presented in Fig. 19. These four models, referred to step (d), are able to deal with the eight load situations presented in Table 6. For the experimental tests used in this section, only three load situations depicted in Table 6 were necessary:

- Number 3, where only bending moment is applied to the joint and the proposed model presented in Fig. 19(c) is sufficient to model the joint.

- Number 5: where a compressive axial force is applied to the joint followed by a bending moment increase. This situation uses the proposed models depicted in Figs. 19(a) and 19(c).

- Number 6: where a tensile axial force is firstly applied to the joint with a subsequent bending moment application. In this case, the proposed models in Figs. 19(b) and 19(c) are utilized.

Before analysing the adopted mechanical models in Fig. 19, it is necessary to identify each model applicability domain, which depends on whether the joint components are subjected to either compression or tension, for a given combination of bending moment and axial force. This is done by evaluating the limit bending moments (M_{lim}), step (e), for the adopted models in Fig. 19 with the aid of Eq. 35, relative to the experimental axial force levels. This step does not require the knowledge of the lever arm position d since yield of joint bolt-rows are not affected by this position. In this case, only the joint rotation and the joint row yield corresponding to the beam flanges are affected. The results of the limit bending moment evaluations are illustrated in Table 7. For the EE1 experiment (load situation number 3, Table 6) any bending moment applied to the joint model, Fig. 19(c), induces tension on the joint first bolt row and compression on the beam bottom flange. For the EE2, EE3 and EE4 experimental tests (load situation number 5, Table 6), the limit bending moment, which induces tension on the beam top flange is obtained by using the proposed mechanical model shown in Fig. 19(a). The EE6 and EE7 tests (load situation number 6, Table 6), the limit bending moment, which leads the third bolt row to compression, is calculated by the proposed mechanical model illustrated in Fig. 19(b).

Based on these limit bending moments, an appropriate mechanical model can then be adopted from those shown in Fig. 19. For instance, for EE4 test if the bending moment applied to the joint was smaller than 29.83 kNm, Table 7, the compressive model presented in Fig. 19(a) should be used. For larger values the tensile-compressive model should be utilised. On the other hand, if the proposed mechanical model, Fig. 18, was implemented into a nonlinear structural analysis programme, where each component was described by its force-displacement characteristic curve, these joint components would be automatically activated or deactivated according to its compressive/tensile characteristic (Fig. 14), without the need to previously define a model for each load situation as shown in Fig. 19 and Table 6.

The proposed mechanical models presented in Figs. 19(c) and 19(d) require the evaluation of the lever arm d , step (f). Table 7 presents the lever arm d positions evaluated for the mechanical model shown in Fig. 19(c), where Eq. 53 is used for tensile forces applied to the joint and Eq. 54 is utilised for all the other complimentary cases. Regarding the mechanical model in Fig. 19(d), the lever arm d positions were not calculated since they were not contemplated in the Lima et al. (2004) experiments.

Finally, with the steps (a) to (f) evaluated for the adopted model in Fig. 19, it is possible to predict the bending moment versus rotation curves for each axial force level, step (g), used in the experimental tests carried out by Lima et al. (2004). Table 8

presents the values evaluated for each moment-rotation curve, according to the approach described in section 2.3. Point 1 (θ^y, M^y), Table 8, defines the onset of the joint yield and is evaluated in Eq. 50, by using the yield strength (Table 4, f_i^y) and the elastic effective stiffness (Table 4, k_i^e) for rows i . Point 2 (θ^u, M^u) represents the joint ultimate capacity and is obtained by utilising Eq. 50 and the ultimate strength (Table 5, f_i^u) and the plastic effective stiffness (Table 5, k_i^p) for rows i . Point 3 (θ^f, M^f), Eq. 50, is obtained by adopting a 0.10-radian final rotation for the joint and the reduced strain hardening effective stiffness (Table 5, k_i^h) for rows i . With these results in hand, the results of each analysis compared to their equivalent experimental tests are illustrated in Figs. 20(a) to 20(f). Subsequently, Fig. 21 presents the whole set of numerical results.

4. Results and discussion

Six experimental moment-rotation curves, of Lima et al. (2004), were used to validate the proposed mechanical model in section 2 as well as demonstrate its application.

Fig. 20(a) illustrates the comparisons between the proposed model and the EE1 test moment-rotation curve that was only subjected to bending moments. For this case, the point that characterises the joint initial stiffness was defined by yielding of the endplate in bending. The initial stiffness is slightly underestimated in 22 % by the mechanical model whilst the flexural capacity is rather over predicted in 19 %, Table 9.

Figs. 20(b), 20(c) and 20(d) present comparisons between the proposed model and moment-rotation curves of EE2, EE3 and EE4 tests that respectively consider compressive forces of 10%, 20% and 27% of the beam axial plastic capacity. For these three compressive cases, the joint initial stiffness was defined by yielding of the beam bottom flange in compression. A very good correlation between the experimental tests and numerical results are obtained, Table 9.

Figs. 20(e) and 20(f) illustrate the results for EE6 and EE7 moment-rotation curves that respectively consider tensile forces of 10% and 20% of the beam axial plastic resistance. For these last two cases the joint plasticity was governed by yielding of the endplate in bending, followed by yielding of the beam bottom flange in compression. An accurate prediction of the initial stiffness and flexural capacity are observed, Table 9. However, as the tensile force increases to 20% of the beam axial plastic resistance, slight difference is exhibited underestimating the flexural capacity in 12%, Fig. 20(f).

Fig. 21 illustrates the set of numerical results where is possible to observe that the extended endplate joint subjected both to compressive and tensile forces has its initial stiffness and flexural capacity decreased as either compressive or tensile force increases. This reduction in the initial stiffness is more pronounced for tensile forces applied to the joint. Additionally it is worth highlighting that the joint initial stiffness is strongly influenced by the rigid link lever arm d . Joints presenting similar rigid link lever arms d exhibited a small variation of the initial stiffness as can be seen on the compressive force numerical results, Fig. 21: $P = +10\% N_{pl}$, $P = +20\% N_{pl}$ and $P = +27\% N_{pl}$.

Generally the global behaviour of the numerical moment-rotation curves, obtained by using the generalised mechanical model proposed in this work, are in agreement with the test curves, Lima et al. (2004), producing numerical results that closely approximate the initial stiffness and flexural resistance, Table 9. These small discrepancies might be attributed to the simplifications made in the generalised mechanical model as well as possible inaccuracies in material and geometrical properties.

5. Conclusions

Based on the general principles of the component method, a generalised mechanical model was proposed to estimate the endplate joint behaviour subjected to bending moments and axial forces. This mechanical model is able to deal with three basic requirements for the joint performance: strength, stiffness and deformation capacity. Application and validation of this mechanical model, using experimental tests executed by Lima et al. (2004) on six extended endplate joints, was performed and led to accurate prediction of the experiment's key variables.

The utilization of this generalised mechanical model is simple and supplies an approach to estimate the bending moment versus rotation curve for any axial force level. The tri-linear characterisation of the joint components suggested in this work showed to be capable to supply reasonable approximations for the moment-rotation curve construction. However, it is still required further experimental examination and numerical analysis using different ranges of joints to check the validity and application of the proposed strain hardening coefficients, Table 2, beyond the scope of studied joints in this work.

The approach proposed for evaluation of lever arm d , section 2.4, take into account the change of the joint compressive centre position according to the axial force levels and bending moment applied to the joint. This strategy was responsible for a satisfactory estimation for the joint initial stiffness even before yielding of the first weakest component was reached.

First order approximations for the trigonometric expressions were done throughout the generalised mechanical model formulation. In this way, Fig. 22 presents the error due to these approximations versus joint rotations. According to Nethercot & Zandonini (1989), rotations beyond 0.05 radians have little practical significance. In this work a joint final rotation twice the value given by Nethercot & Zandonini (1989), i.e. equal to 0.10 radians, was assumed. For this value is possible to observe in Fig. 22 that the error is 0.017 % and for a 0.05-radian rotation the error is 0.002 %. This indicated that the developed equations in this work are accurate for the usual problems involving beam-to-column joints.

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Figures

Fig. 1. Connection and mechanical model for web cleat connections, Wales & Rossow (1983).

Fig. 2. Mechanical model for flange and web cleated connections, Chmielowiec & Richard (1987).

Fig. 3. Mechanical model for full welded joints, Tschemmerneegg (1988).

Fig. 4. Mechanical model for bolted joints, Tschemmerneegg & Humer (1988).

Fig. 5. Idealization of beam-to-column connection, Madas (1993).

Fig. 6. Mechanical model, Jaspart et al. (1999).

Fig. 7. Spring model for extended endplate joints, Lima (2003).

Fig. 8. Spring model for flush endplate joints, Lima (2003).

Fig. 9. Nonlinear spring connection model, Ramli-Sulong (2005).

Fig. 10. Moment-rotation curves for the extended endplate joints tested by Lima (2003) and obtained from numerical simulations, Lima (2003).

Fig. 11. Moment-rotation curves for the flush endplate joints tested by Lima (2003) and obtained from numerical simulations, Simões da Silva et al. (2004).

Fig. 12. Proposed generalised mechanical model for semi-rigid joints.

Fig. 13. Constitutive laws of the endplate joint components, Simões da Silva et al. (2002).

Fig. 14. Force-displacement curve for components in tension and compression.

Fig. 15. Proposed prediction of the bending moment versus rotation curve for any axial force level.

Fig. 16. Proposed generalised mechanical model for semi-rigid joints – lever arm d .

Fig. 17. Extended endplate joint, Lima et al (2004).

Fig. 18. Proposed mechanical model.

Fig. 19. Proposed mechanical model for each analysis stage.

(a) EE1 test versus proposed model.

(b) EE2 test versus proposed model.

(c) EE3 test versus proposed model.

(d) EE4 test versus proposed model.

(e) EE6 test versus proposed model.

(f) EE7 test versus proposed model.

Fig. 20. Comparisons between experimental moment-rotation curves, by Lima et al. (2004), and built curves by using the proposed mechanical model.

Fig. 21. Prediction of six moment-rotation curves for different axial force levels.

Fig. 22. First-order approximations error magnitudes versus joint rotation.

Tables

Table 1. Summary of the mechanical models to predict the joint behaviour.

<i>Authors (date)</i>	<i>Joint/Connection Type</i>	<i>Forces</i>
<i>Wales & Rossow (1983)</i>	<i>Double web cleat connections</i>	<i>Bending moment and axial force</i>
<i>Kennedy & Hafez (1984)</i>	<i>Header plate connections</i>	<i>T-stub: axial force</i>
<i>Chmielowiec & Richard (1987)</i>	<i>All types of cleated connections</i>	<i>Bending moment and shear</i>
<i>Tschemmerneegg (1988)</i>	<i>Welded connections</i>	<i>Bending moment</i>
<i>Tschemmerneegg & Humer (1988)</i>	<i>Endplate bolted connections</i>	<i>Bending moment</i>
<i>Madas (1993)</i>	<i>Flexible endplate, double web angle and top and seat angle connections</i>	<i>Bending moment and axial force</i>
<i>Jaspart et al. (1999) and Cerfontaine (2003)</i>	<i>Extended and flush endplate connections</i>	<i>Bending moment and axial force</i>
<i>Pucinotti (2001)</i>	<i>Top-and-seat and web angle connections</i>	<i>Bending moment</i>
<i>Simões da Silva & Coelho (2001)</i>	<i>Welded beam-to-column joints</i>	<i>Bending moment and axial force</i>
<i>Sokol et al. (2002)</i>	<i>Endplate joints</i>	<i>Bending moment and axial force</i>
<i>Lima (2003)</i>	<i>Extended endplate joints</i>	<i>Bending moment and axial force</i>
<i>Lima (2003) and Simoes da Siva et al. (2004)</i>	<i>Flush endplate joints</i>	<i>Bending moment and axial force</i>
<i>Ramli-Sulong (2005)</i>	<i>Flush and extended endplate, top-and-seat and/or web angles, and fin-plate connections</i>	<i>Bending moment and axial force</i>
<i>Urbonas & Daniunas (2006)</i>	<i>Endplate bolted beam-to-beam joints</i>	<i>Bending moment and axial force</i>

Table 2. Values adopted for the strain hardening coefficients, μ .

<i>Designation - Component</i>	<i>Plastic μ^p</i>	<i>Ultimate μ^u</i>
<i>1 - Column web in shear</i>	<i>0.5</i>	<i>0.10</i>
<i>2 - Column web in compression</i>	<i>0.3</i>	<i>0.06</i>
<i>3 - Column web in tension</i>	<i>0.3</i>	<i>0.06</i>
<i>4 - Column flange in bending</i>	<i>0.2</i>	<i>0.04</i>
<i>5 - Endplate in bending</i>	<i>0.1</i>	<i>0.02</i>
<i>7 - Beam or column flange and web in compression</i>	<i>∞</i>	<i>∞</i>
<i>8 - Beam web in tension</i>	<i>∞</i>	<i>∞</i>
<i>10 - Bolt in tension</i>	<i>0.6</i>	<i>0.12</i>

Table 3. Steel mechanical properties, Lima et al. (2004).

<i>Specimen</i>		<i>Yield Strength (MPa)</i>	<i>Ultimate Stength (MPa)</i>	<i>Young's Modulus (MPa)</i>
<i>Beam IPE240</i>	<i>Web</i>	363.4	454.3	203713
	<i>Flange</i>	340.14	448.23	215222
<i>Column HEB240</i>	<i>Web</i>	372.02	477.29	206936
	<i>Flange</i>	342.95	448.79	220792
<i>Endplate</i>	<i>Web</i>	369,44	503,45	200248

Table 4. Theoretical values of the resistance and initial stiffness of the extended endplate joint components, Fig. 17, evaluated according to Eurocode 3 (2005).

<i>Component</i>		f_{cp}^y (kN)	k_{cp}^e (kN/mm)	f_i^y (kN)	k_i^e (kN/mm)
Beam top and bottom flange (compression)	<i>cwc (2)</i>	680.1	2127.3	321.3	453.55
	<i>bfc (7)</i>	542.2	∞		k_{btb}/k_{bbf}
	<i>cws (1)</i>	321.3	576.5		
Beam bottom flange	<i>cwc (2)</i>	680.1	2127.3	542.2	747.69
	<i>bfc (7)</i>	542.2	∞		k_{bbf}
	<i>cws (1)</i>	642.5	1152.9		
First bolt row (h=267.1mm)	<i>cwt (3)</i>	533.2	1205.4	289.8	548.20
	<i>cfb (4)</i>	408.3	6938.4		k_{br1}
	<i>epb (5)</i>	289.8	4223.1		
	<i>bt (10)</i>	441.00	1629.6		
Considered individually					
Second bolt row (h=193.1mm)	<i>cwt (3)</i>	533.2	1031.1	252.4	451.53
	<i>cfb (4)</i>	408.3	5936.7		k_{br2}
	<i>epb (5)</i>	341.8	2160.9		
	<i>bwt (8)</i>	492.3	∞		
	<i>bt (10)</i>	441.0	1629.6		
	<i>cwc (2)</i>	390.4	-		
	<i>bfc (7)</i>	252.4	-		
	<i>cws (1)</i>	352.7	-		
	Bolt row belonging to the bolt group formed by bolt rows				
Third bolt row (h=37.1mm)	<i>cwt (3)</i>	735.1	-		
	<i>cfb (4)</i>	713.8	-		
	<i>bfc (7)</i>	0.0	-	0.0	554.77
	<i>Note: As the resistance of the second bolt row was limited by the collapse of the beam flange in compression, the third bolt row does not contribute to the joint overall resistance.</i>				k_{br3}

Table 5. Characterisation of the extended endplate joint components, Fig. 17, according to the approach given in section 2.1.

<i>Component</i>		f_{cp}^u (kN)	k_{cp}^p (kN/mm)	k_{cp}^u (kN/mm)	f_i^u (kN)	k_i^p (kN/mm)	k_i^u (kN/mm)
Beam top and bottom flange (compression)	<i>cwc</i> (2)	872.6	638.19	127.64	412.2	198.55	39.71
	<i>bfc</i> (7)	714.5	∞	∞			k_{btb}/k_{bbf}
	<i>cws</i> (1)	412.2	288.23	57.65			
Beam bottom flange	<i>cwc</i> (2)	872.6	638.19	127.64	714.5	302.88	60.58
	<i>bfc</i> (7)	714.5	∞	∞			k_{bbf}
	<i>cws</i> (1)	824.4	576.45	115.29			
First bolt row (h=267.1mm)	<i>cwt</i> (3)	684.1	361.62	72.32	394.9	145.42	29.08
	<i>cfb</i> (4)	534.4	1387.68	277.54			k_{br1}
	<i>epb</i> (5)	394.9	422.31	84.46			
	<i>bt</i> (10)	490.0	977.76	195.55			
Second bolt row (h=193.1mm)	Conside						
	<i>cwt</i> (3)	684.1	309.33	61.87	332.6	102.82	20.56
	<i>cfb</i> (4)	534.4	1187.34	237.47			k_{br2}
	<i>epb</i> (5)	465.8	216.09	43.22			
	<i>bwt</i> (8)	615.4	∞	∞			
	<i>bt</i> (10)	490.0	977.76	195.55			
	<i>cwc</i> (2)	500.9	-	-			
	<i>bfc</i> (7)	332.6	-	-			
	<i>cws</i> (1)	452.5	-	-			
	Bolt						
Third bolt row (h=37.1mm)	<i>cwt</i> (3)	943.2	-	-			
	<i>cfb</i> (4)	934.2	-	-			
	<i>bfc</i> (7)	0.0	-	-	0.0	128.08	25.62
	Note: As the resistance of the second bolt row was limited by the collapse of the beam flange in compression, the third bolt row does not contribute to the joint overall resistance.						k_{br3}

Note: f_{cp}^u is given in section 2.1, k_{cp}^p and k_{cp}^u are given in Eqs. 1 and 2, respectively.

Table 6. Load situations applied to the joint and theirs respective mechanical models.

<i>Nº</i>	<i>Load situations</i>		<i>Mechanical model(s)</i>
	<i>Bending moment</i>	<i>Axial Force</i>	
1	-	+P	Compressive, Fig. 19(a)
2	-	-P	Tensile, Fig. 19(b)
3	+M	-	Tensile-Compressive, Fig. 19(c)
4	-M	-	Compressive-Tensile, Fig. 19(d)
5	+M	+P	Fig. 19(a) and Fig. 19(c)
6	+M	-P	Fig. 19(b) and Fig. 19(c)
7	-M	+P	Fig. 19(a) and Fig. 19(d)
8	-M	-P	Fig. 19(b) and Fig. 19(d)

Note: +P and -P are compressive and tensile axial forces applied to the joint, respectively. +M is the bending moment that compresses the beam bottom flange and tensions the beam top flange, whilst -M is the bending moment that tensions the beam bottom flange and compresses the beam top flange.

Table 7. Applicability of each model, M_{lim} , and evaluation of lever arm d according to the experimental axial force levels.

<i>Experimental data</i>		<i>M_{lim} (kNm), Eq. 35</i>		<i>Lever arm d (mm)</i>	
<i>Test</i>	<i>N (kN)</i>	<i>Compressive Fig. 19(a)</i>	<i>Tensile Fig. 19(b)</i>	<i>Tensile-Compressive Fig. 19(c)</i>	
EE1 (only M)	0.00	*NA	*NA	0.00 to M^f	Eq. 54: 117.55
EE2 (+10% N_{pl})	135.94	0.0 to 15.65	*NA	15.65 to M^f	Eq. 54: 93.99
EE3 (+20% N_{pl})	193.30	0.0 to 22.25	*NA	22.25 to M^f	Eq. 54: 86.66
EE4 (+27% N_{pl})	259.20	0.0 to 29.83	*NA	29.83 to M^f	Eq. 54: 79.53
EE6 (-10% N_{pl})	-127.20	*NA	0.0 to	15.96 to M^f	Eq. 53: 42.85
EE7 (-20% N_{pl})	-257.90	*NA	0.0 to	32.36 to M^f	Eq. 53: 19.48

Note: "+" indicates compressive axial forces and "-" tensile axial forces. M^f is given in Eq. 50.

*NA = not applicable.

Table 8. Values evaluated for the prediction of the moment-rotation curves for different axial force level.

<i>Point</i>	<i>EE1 (only M)</i>		<i>EE2 (+10% N_{pl})</i>		<i>EE3 (+20% N_{pl})</i>		<i>EE4 (+27% N_{pl})</i>		<i>EE6 (-10% N_{pl})</i>		<i>EE7 (-20% N_{pl})</i>	
	θ	M	θ	M	θ	M	θ	M	θ	M	θ	M
	mrad	kNm	mrad	kNm	mrad	kNm	mrad	kNm	mrad	kNm	mrad	kNm
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	6.9	103.8	7.6	100.6	7.4	92.6	7.0	84.9	10.2	91.1	11.8	78.0
2	22.1	142.3	23.4	136.6	22.6	127.4	21.6	118.5	29.0	118.3	32.4	101.2
3	100.0	146.9	100.0	147.8	100.0	148.2	100.0	148.6	100.0	146.1	100.0	145.2

Note: Points 1, 2 and 3 defined in Eq. 50.

Table 9. Comparisons between the experimental and the proposed model initial stiffness and the experimental and the proposed model design moment.

	<i>Initial Stiffness (kNm/rad)</i>							
	<i>Model</i>	<i>Exp</i>	<i>Lima</i>					
EE1 (only M)	15071	19438	20290					
EE2 (+10% N_{pl})	13183	13554	15685					
EE3 (+20% N_{pl})	12604	15411	16228					
EE4 (+27% N_{pl})	12078	12647	16668					
EE6 (-10% N_{pl})	8948	9087	13060					
EE7 (-20% N_{pl})	6598	6750	14905					

	<i>Initial Stiffness (kNm/rad)</i>				<i>Design Moment (kNm)</i>			
	<i>Model</i>	<i>Exp</i>	<i>Mod/Exp</i>	<i>%</i>	<i>Model</i>	<i>Exp</i>	<i>Mod/Exp</i>	<i>%</i>
EE1 (only M)	15071	19438	0.78	22	141	119	1.19	-19
EE2 (+10% N_{pl})	13183	13554	0.97	3	133	125	1.06	-6
EE3 (+20% N_{pl})	12604	15411	0.82	18	123	118	1.04	-4
EE4 (+27% N_{pl})	12078	12647	0.96	4	113	113	1.00	0
EE6 (-10% N_{pl})	8948	9087	0.98	2	110	112	0.99	1
EE7 (-20% N_{pl})	6598	6750	0.98	2	89	101	0.88	12

Note: Negative percentage means overestimated value in X % whilst positive percentage indicates underestimated value in X %. Joint design moment is determined according to Eurocode 3 (2005), through the intersection between two straight lines, one parallel to the initial stiffness and another parallel to the moment-rotation curve post-limit stiffness.

Nomenclature

b_i	distance from joint spring/row i to the beam bottom flange centre
bfc (7)	beam flange in compression
bt (10)	bolts in tension
bwt (8)	beam web in tension
cfb (4)	column flange in bending
cwc (2)	column web in compression
cws (1)	column web in shear
cwt (3)	column web in tension
d	lever arm: distance from the loading application centre to the rigid link
d_i	system displacements, $i=1..4$: u_{b1} , θ_{b1} , u_{b2} , θ_{b2}
e	distance from the loading application centre to the beam bottom flange
epb (5)	endplate in bending
$f_{br,i}^y$	yield strength of the joint bolt-row i
f_{cp}^y	joint component yield capacity
f_{cp}^u	joint component ultimate capacity
f_i	force in spring/row i
f_i^y	yield capacity of spring/row i
f_i^u	ultimate capacity of spring/row i
k_i	tangent effective stiffness of spring/row i
k_{cp}^e	joint component elastic stiffness
k_{cp}^p	joint component plastic stiffness
k_{cp}^u	joint component reduced strain hardening stiffness
k_i^e	elastic effective stiffness of spring/row i
k_i^p	plastic effective stiffness of spring/row i
k_i^u	reduced strain hardening effective stiffness of spring/row i
nbr	number of bolt-rows
nc	row/spring component number
ns	system spring/row number
u_{b1}	first bar displacement
u_{b2}	second bar displacement
u_i	absolute displacement of spring/row i (first bar)
ul_i	absolute displacement of spring/row i (second bar)

Capital letter

C_i	spring/row i vertical coordinates
F	internal loading vector
F_{bbf}	row compressive yield capacity (beam bottom flange)
F_{linkt}	rigid link tensile capacity, which joins the second bar to the supports
K_{ij}	terms of the system stiffness matrix, $i=1..4$ and $j=1..4$
M	bending moment applied to the connection
M^f	bending moment referred to a 0.10-radian joint final rotation
M^u	bending moment that leads the joint to the failure
M^y	bending moment that leads the joint to the yield
$M_{br,i}^u$	bending moment that leads to the failure of the joint spring/row i , located between the first and second bars

$M_{br,i}^y$	bending moment that leads to the yield of the joint spring/row i , located between the first and second bars
$M_{fr,i}^u$	bending moment that leads to the failure of the joint spring/row i , located between the second bar and supports
$M_{fr,i}^y$	bending moment that leads to the yield of the joint spring/row i , located between the second bar and supports
$M_{j,lim}$	limit bending moment of spring/row j , located between the first and second bars
N_{pl}	beam's axial plastic capacity
P	axial load applied to the connection
U	system strain energy
W	load total potential

Greek Letters

$\alpha_{1,2,3,4}$	coefficients of Eq. 41
$\eta_{1,2,3,4}$	coefficients of Eq. 41
θ	joint rotation
θ^u	joint rotation capacity necessary to develop the joint plastic bending moment
θ^y	joint rotation capacity necessary to develop the joint yield bending moment
θ^f	joint final rotation (assumed to be equal to 0.10 radians)
θ_{b1}	first bar rotation
θ_{b2}	second bar rotation
κ	stiffness coefficient (Eq. 18)
λ	stiffness coefficient (Eq. 18)
μ^p	plastic stiffness strain hardening coefficient
μ^u	ultimate stiffness strain hardening coefficient
ξ	stiffness coefficient (Eq. 23)
ρ	stiffness coefficient (Eq. 18)
v	stiffness coefficient (Eq. 26)
φ	stiffness coefficient (Eq. 27)
χ_1	stiffness coefficient (Eq. 23)
χ_2	stiffness coefficient (Eq. 20)
ψ	stiffness coefficient (Eq. 20)
ω_1	stiffness coefficient (Eq. 23)
ω_2	stiffness coefficient (Eq. 20)

Capital letter

Δ_i	spring/row i relative displacement
$\Delta_{br,i}$	spring/row i relative displacement located between the first and second bars
$\Delta_{fr,i}$	spring/row i relative displacement located between the second bar and the supports
Δ_i^y	relative displacement that leads to the yield of the model spring/row i
Δ_i^u	relative displacement that leads to the failure of the model spring/row i
Z	stiffness coefficient (Eq. 25)
Π	total potential energy functional
X	stiffness coefficient (Eq. 22)
Ω	stiffness coefficient (Eq. 22)