COMPARISON OF DEFLECTION CALCULATIONS AND SPAN TO DEPTH RATIOS IN BS 8110 AND EC2

by

Dr R L Vollum

November 2008

Senior Lecturer, Department of Civil and Environmental Engineering, Imperial College

Contact details:

Dr R. L. Vollum

Department of Civil and Environmental Engineering,

Imperial College Road

Imperial College

London SW7 2AZ

phone 0207 594 5992, fax 0207 594 5989, email r.vollum@imperial.ac.uk

Abstract

The current article proposes a procedure for calculating deflections in buildings with EC2 and evaluates the span-to-effective-depth rules in EC2 and BS8110. Guidance is given on the calculation of deflection with EC2 and updated recommendations are made for including the effect of construction loading in long-term deflection calculations. It is shown that deflections in slabs are very sensitive to the assumed loading, boundary conditions and concrete tensile strength if the maximum span moment is close to the cracking moment. The theoretical background to span to depth rules is discussed along with the assumptions made in the derivation of the EC2 span to depth rules. It is shown that the EC2 span-to-depth rules can lead to significant reductions in slab thickness compared with BS8110 since the rules account for the beneficial influence of the concrete tensile strength on deflection. It is also shown that the span to depth rules in EC2 overestimate the reduction in slab thickness which can be obtained by providing surplus reinforcement to equate the actual and permissible span to depth ratios. An amendment is proposed to improve the accuracy of the EC2 span to depth rules.

Calculation of deflection in EC2

EC2¹ calculates the mean curvature in cracked reinforced concrete members by interpolating between the curvatures in idealised uncracked and fully cracked sections as follows:

$$\Psi_m = \zeta \Psi_2 + (1-\zeta) \Psi_1 \quad (1)$$

where

$$\zeta = 1 - \beta (M_r/M)^2 \quad (2)$$

Ψ_1 and Ψ_2 are the curvatures in idealised uncracked and fully sections respectively including shrinkage and M_r is the cracking moment. The coefficient β in equation (2) accounts for the loss of tension stiffening with time due to additional internal- and macro-cracking under sustained load. EC2¹ states that β should be taken as 1 for short term loading and 0.5 for long-term loading but does not define the variation in β with time.

Back analysis of deflection data from laboratory tests and field measurements²⁻⁶ show that equation (1) gives good estimates of curvature and hence deflection if the material properties and loading are known. Difficulties arise in practice, since neither the concrete material properties nor the actual loading are known prior to construction or indeed subsequently. Deflections in slabs are particularly difficult to estimate reliably since they vary significantly dependent on whether or not the slab is cracked. Research²⁻⁶

suggests that deflections in slabs are largely governed by the most severe cracking which can arise during construction or subsequently in service. Cracking can arise during construction either at striking, or subsequently under loading from casting slabs above or stock piling construction materials.

Construction loading from casting slabs above: In multi-storey buildings, the self-weight of the most recently cast slab is transferred into the slabs below through props and backprops until the slab carries its self-weight. The magnitude of the load induced in slabs from casting slabs above depends on the type of formwork system, the number of levels of backprops and their spatial arrangement. If the most recently cast slab carries its self-weight after striking, the peak construction load is given by:

$$W_{\text{peak}} = W_{\text{self}} + C(W_{\text{self}} + W_{\text{con}}) \quad (10)$$

where c is a carry through factor of at least $1/(\text{number of supporting floors})$ and w_{con} is a construction load comprising formwork etc which is typically around 0.75kN/m^2 . Peak construction loads from casting slabs above depend on whether slabs carry their self weight before the slab above is cast. Beeby⁷ showed through analysis and measurements of prop forces at Cardington that the peak construction load occurs in the top slab of the supporting assembly when slabs carry their self-weight before the slab above is cast. The lower slab in the supporting assembly is most heavily loaded if slabs are not struck (i.e. carry their own weight) before the slab above is cast. Beeby's⁷ work showed that when backprops are installed finger tight, it is

reasonable in the absence of detailed calculation to take c as 0.7 in equation (10) if there is one level of backprops and 0.65 with two levels of backprops. In practice, backprops are usually preloaded during installation rather than being installed finger tight as at Cardington. The author measured significant preloads in the backprops at St George Wharf⁵ which corresponded to a uniformly distributed load of $\sim 1 \text{ kN/m}^2$. The effect of the preload was to induce a more even distribution of the construction load between the supporting slabs than measured at Cardington. Parametric studies indicate that it is reasonable to take the peak construction load w_{peak} as $0.004h \text{ kN/m}^2$ (where h is the slab thickness in mm) in deflection calculations for slabs up to 500mm thick where two levels of backprops are used and the backprops are preloaded during installation as at St George Wharf⁵.

Construction loads from casting slabs above can only be neglected if i) the formwork is supported by the columns or ii) sufficient backprops are provided to transfer the self-weight of the most recently cast slab into the ground. Caution should be exercised in neglecting construction loads since measurements of prop forces at Cardington³ at St George Wharf⁵ suggest that slabs can experience significant construction loads from casting slabs above even if the backprops continue to the ground due to the combined effects of prop shortening and ground settlement.

Pallet⁸ gives detailed guidance on designing slabs for peak construction loads at the ultimate limit state. He suggests that the load factor can be taken as 1.2 for dead and imposed loads during construction and that reduced material

factors of safety can be used at the ultimate limit state if the worst credible rather than characteristic material strengths are used. Shear failure is likely to be most critical during construction since the flexural strength in lightly reinforced members is almost independent of the concrete compressive strength.

Influence of peak loads on long-term deflection

The author⁶ recently tested a series of 6 simply supported one-way spanning slabs in the Concrete Structures Laboratory at Imperial College to determine the influence of short-term construction and in-service loads on long-term slab deflections. The slabs measured 500mm wide by 3600mm long by 150mm thick and were reinforced with 3T10 bars with 20mm cover. The slabs were simply supported over a span of 3300mm and loaded at their third points. Full details of the tests are given elsewhere⁶. The mid-span moment and curvature are plotted against time in Figs. 1 and 2 respectively for slabs S4 to S6. Slab S6 was a control specimen, which was not loaded with a peak construction load. Fig. 2 shows that curvatures in reinforced concrete slabs can be significantly influenced by the previous loading history which is usually unknowable in practice.

Rigorous method for deflection prediction

EC2¹ recommends that deflections should be calculated under the quasi permanent load which is given by:

$$W_{\text{perm}} = W_d + \phi W_i \quad (3)$$

where ϕ is taken as 0.3 for offices but depends on usage.

EC2¹ gives no guidance on how to account for the increase in deflection due to previous loading illustrated in Fig. 2. The author recently proposed⁶ a simple method for taking account of previous loading in deflection calculations which is a refinement of his earlier work which formed the basis of the rigorous method in Concrete Society Technical Report TR58⁹. The method was validated⁶ with deflection data from slabs S1 to S6 described above. It is proposed that mean curvatures are calculated with equation (1) in conjunction with equation (2) using equivalent values for the concrete effective elastic modulus and the interpolation coefficient ζ to account for the effects of previous loading. The equivalent elastic modulus of the concrete can be calculated for the concrete using the following equation from TR58⁹:

$$E_{LT} = \sum w_i / (w_1/E_{ceff1} + w_2/E_{ceff2} + w_3/E_{ceff3} + \dots) \quad (4)$$

where w_i is the load increment at time t_i and

$E_{ceff} = E_{ct} / (1 + \phi(t_i, t))$ where t_i is the age at application of w_i and t is the age at which deflections are required.

The interpolation coefficient ζ used in equation (1) needs to be modified to take into account the loss of tension stiffening induced by short-term peak loads greater than w_{perm} . Tension stiffening is reduced following the removal of short term peak loads since i) additional internal and macro cracks form under w_{peak} and ii) the slope of the unloading line is steeper than the

instantaneous loading line. The author⁶ has previously shown that the influence of peak construction loads can be included in deflection calculations by replacing ζ in equation (1) with a modified interpolation coefficient ζ^* given by:

$$\zeta^* = 0.5\zeta_{\text{peak}}(1+M_{\text{peak}}/M) \quad (5)$$

where M is calculated with the load w under which deflections are required and

$$\zeta_{\text{peak}} = 1 - \beta \left(M_{\text{rpeak}}/M_{\text{peak}} \right)^2 \quad (6)$$

where M_{rpeak} is the cracking moment when the peak construction load is applied and M_{peak} is calculated under the peak construction load. The ratio M_{peak}/M can be taken as w_{peak}/w for uniformly loaded slabs. Back analysis of deflection data from the author's slab tests⁶, Cardington²⁻⁴ and St George Wharf⁵ suggests it will usually be conservative to take β as 0.7 in equation (6) at the removal of the peak construction load if curvatures are calculated with ζ^* . The term $0.5(1+M_{\text{peak}}/M)$ in equation (5) for ζ^* accounts for the increment in deflection due to inelastic unloading from M_{peak} .

The maximum in service load, which is uncertain, may give rise to more severe cracking than the peak construction load if i) the slab is heavily loaded or ii) the backprops are continued down to the ground as is typical in low rise buildings. In the absence of better information, PD 6687¹⁰ recommends that

the peak load in deflection calculations should be taken as the frequent load combination which is defined in EC0 as:

$$W_{\text{freq}} = W_d + \psi W_i \quad (7)$$

where ψ is 0.5 for offices.

In practice, designers are normally interested in calculating a) maximum deflections and b) incremental deflections following fitting out. EC2¹ differs from BS8110¹¹ in that it requires deflections to be calculated under the quasi permanent rather than peak load. The change in loading proposed in EC2¹ seems questionable since damage to finishes and partitions is likely to be governed by peak deflections. Furthermore, it is difficult if not impossible to realistically predict deflections in building structures under quasi permanent loads since the deflections depend on the previous peak loading, its duration and the ratio of w_{perm} to w_{peak} all of which are usually unknown. Therefore, it is proposed that deflections should be calculated under the frequent rather than quasi permanent load to minimise the influence of prior loading on deflection.

Recommended procedure for calculating deflections with EC2

Difficulties arise in practice, since the loading and concrete material properties can only be estimated at the design stage. It is suggested that in the absence of better information, the following assumptions are made in deflection calculations:

1. The slab is struck at 7 days, the superimposed dead load is applied at 60 days and the permanent component of the imposed load at one year.
2. Creep and shrinkage strains are calculated with a relative humidity of 50%.
3. Two levels of backprops are used
4. The floor above is cast after 10 days.
5. When slabs are supported by slabs below during construction, the peak construction load w_{peak} should be taken as $0.04h \text{ kN/m}^2$ where h is the slab thickness in mm.
6. The permanent load should be taken as the quasi permanent load and be applied at 1 year.
7. Peak deflections are calculated under the frequent load case. The increment in load $w_{\text{freq}} - w_{\text{perm}}$ should be treated as an instantaneous load in equation (4) for E_{LT} .

Choice of concrete tensile strength: It is difficult to assess the effective tensile strength of concrete in slabs due to its inherent variability and uncertainties in the tensile stress induced by internal and external restraint of shrinkage. Back analysis of deflection data shows that the effective flexural strength of concrete in reinforced concrete slabs typically lies somewhere between the indirect and flexural strengths. Analysis of test data shows that the EC2 formula for the mean concrete tensile strength is rather conservative since it calculates the tensile strength in terms of the characteristic rather than mean concrete compressive strength. Back analysis of deflection data from

Cardington²⁻⁴ and St George Wharf⁵ suggests that it is reasonable to take the concrete tensile strength in deflection calculations as the mean tensile strength calculated in terms of the mean rather than characteristic concrete compressive strength. The corresponding concrete tensile strength is given by:

$$f_{ctm28eff}=0.3f_{cm}^{2/3} \quad (8)$$

An alternative approach⁷ is to take the concrete tensile strength as the mean of the tensile and flexural strengths calculated as follows in terms of the characteristic cylinder strength:

$$f_{ctm28eff}=0.3f_{ck}^{2/3}(1.3-h/2000) \quad (9)$$

Parametric studies show that equations (8) and (9) give similar strengths for slabs up to 400mm thick. Equation (8) is arguably preferable since the proportional reduction in effective tensile strength due to axial restraint from columns and shear walls is likely to be greatest in thin slabs.

Calculation of incremental deflection following fitout

The incremental deflection experienced by the partitions and other finishes depends significantly on whether slabs crack before or subsequent to fitting out. Cracking during construction tends to increase the overall deflection but frequently reduces the subsequent increase in deflection as illustrated in Fig.

2. The increment in deflection Δa seen by partitions and other finishes can be conservatively calculated as follows:

$$\Delta a = a_{\infty} - a_1 \quad (11)$$

where a_{∞} is the long term deflection calculated under the frequent load and a_1 is the deflection under the slab self-weight immediately before the installation of the finishes. If it is uncertain whether the slab will crack during construction, it is suggested that the incremental deflection is taken as the greatest of the values given by M1) and M2) below.

- M1) Neglect the effect of construction loading and calculate a_{∞} with the greatest of ζ_{perm} and $\beta \zeta_{freq}$ and a_1 with ζ_1 . Use the 28 day concrete tensile strength in the calculation of ζ_1 , ζ_{perm} and ζ_{freq} . Take β as 0.5 in the calculation of ζ_1 and ζ_{perm} and 0.7 in ζ_{freq} .
- M2) Include the effect of construction loading and calculate a_1 with ζ_{peak}^* and a_{∞} with ζ equal to the greatest of ζ_{peak}^* , ζ_{perm} or ζ_{freq} . Assume $\beta=0.7$ in the calculation of ζ_{peak} and ζ_{freq} .

Parametric studies show that Method 1) tends to give the greatest incremental deflection but the least total deflection.

Rational of span to depth ratios

It is helpful to review the underlying rational of controlling deflections with span to depth rules before making a detailed evaluation of the EC2¹ span to depth rules. The maximum permissible span to effective depth ratio corresponding to a deflection limit $\delta/L = \alpha$ can be expressed as:

$$L/d = \alpha/(X d \Psi_m) \quad (12)$$

where Ψ_m = mean curvature which can be calculated with equations (1) and (2), d is the effective depth, L is the maximum permissible span and X is a coefficient which relates the deflection to the loading arrangement and boundary conditions.

The product $d\Psi_m$ is readily shown to be independent of the section depth for specified values of the reinforcement index $\rho = A_s/bd$, d/h , reinforcement stress σ_s and concrete material properties since the curvatures in the uncracked and fully cracked sections are inversely proportional to the effective depth.

BS8110: Part 2¹² gives values for X derived on the assumption that the flexural rigidity is uniform along the length of the member. The assumption of uniform rigidity is reasonable for uncracked members and cracked members in which cracking is distributed over a significant proportion of the span. However, analysis shows that X reduces significantly below the values given in BS8110: Part 2¹² if the cracking is localised near mid-span as is the case when the peak moment is only slightly greater than the cracking moment. This effect is illustrated in Fig. 3 in which X is plotted against the loading ratio w/w_u for a simply supported beam with various reinforcement indices. X varies with the length of the span which is cracked which depends on the shape of the bending moment diagram and the loading ratio $\lambda M_u/M_r$ where λM_u is the maximum span moment. $\lambda M_u/M_r$ is proportional to $\lambda(d/h)^2/(1.3-h/2000)$ for given values of ρ and f_{ck} if the effective concrete tensile strength is calculated with equation (9) which implies that X and, hence, the permissible span to

depth ratio depends on λ , d/h , h , ρ and f_{ck} . The influence of h is undesirable and can be excluded by calculating the effective concrete tensile strength with equation (8). It follows that the effect of variations in λ , d/h , ρ and f_{ck} should be included in the calculation of permissible span to depth ratios.

EC2 span to depth rules

EC2¹ states that it is generally unnecessary to calculate deflections explicitly since deflection problems can be avoided by dimensioning members to comply with the span-to-depth ratios in the code. The span to depth rules given in EC2¹ were derived¹³ by curve fitting the results of a parametric study of a series of simply supported uniformly loaded members measuring 1000mm wide by 300mm deep. The effective depth was assumed to be $0.9h$ where h is the total section depth. EC2 states that slabs dimensioned with the EC2 span to depth rules will satisfy both the commonly used overall deflection limit of $L/250$ and the active deflection limit of $L/500$. The code is inconsistent with the background document which states that the slenderness limits were calculated by limiting the total deflection to $L/250$ even though it was found that limiting the deflection after the construction of partitions to $L/500$ was more restrictive. The following additional assumptions were made in the parametric studies used in the derivation of the EC2 span to depth rules¹³:

1. The mean 28 day direct concrete tensile strength was used in deflection calculations
2. The relative humidity was taken as 70%

3. The total characteristic load ($q_{tot}=g+q$) was assumed to equal $0.71q_{uls}$ where q_{uls} is the design ultimate load calculated with load factors of 1.35 for dead loads and 1.5 for imposed loads. The total dead load g was made up of the self-weight g_1 and the finishes g_2 where $g_1 = 0.36q_{tot}$ and $g_2 = 0.24q_{tot}$. The self weight g_1 was applied at 10 days, the load of the finishes g_2 was applied at 60 days and the permanent component of the imposed load $0.3q$ was applied at 365 days.
4. A short-term construction load equal to the permanent load of $g+0.3q$ was applied at striking.
5. The characteristic yield strength of the reinforcement was taken as 500MPa.
6. The ratio between the permanent ($g+0.3q$) and design ultimate loads was assumed to be 0.5.

The span-to-depth ratios in EC2 are defined by equation 13 below (equation 7.16 in EC2):

$$L/d_{basic} = K[11+1.5\sqrt{f_{ck}}\rho_0/\rho+3.2\sqrt{f_{ck}}(\rho_0/\rho-1)^{1.5}] \text{ if } \rho \leq \rho_0 \quad (13a)$$

$$L/d_{basic} = K[11+1.5\sqrt{f_{ck}}\rho_0/(\rho-\rho'+1/12\sqrt{f_{ck}}\sqrt{(\rho'/\rho_0)})] \text{ if } \rho > \rho_0 \quad (13b)$$

where

K accounts for structural form and is taken as 1 for simply supported spans, 1.3 for end spans of continuous spans, 1.5 for internal spans of continuous spans and 1.2 for flat slabs, ρ_0 is the reference reinforcement index $\sqrt{f_{ck}} \times 10^{-3}$, ρ is the required tension steel ratio at mid-span A_s/bd and ρ' is the required compression steel ratio at mid-span. EC2 states that $(L/d)_{basic}$ should be

reduced by $7/L$ for spans greater than 7m which support partitions likely to be damaged.

EC2 states that when other steel stress levels are used the basic span to effective depth ratios given by equation 13 should be multiplied by $310/\sigma_s$ where σ_s is defined in the code as the tensile stress in the reinforcement at mid-span “under the design load at SLS”. The code does not define what is meant by the design load at the SLS but goes on to state that it is normally conservative to assume:

$$310/\sigma_s = 500/(f_{yk}A_{sreq}/A_{sprov}) \quad (14)$$

where A_{sreq} is the area of flexural steel required for strength at mid-span and A_{sprov} is the area provided. The stress of 310MPa in equation (14) appears to have been calculated under the full service load (i.e. $g+q=0.71w_u$) since $0.71 \times 500/1.15 \sim 310\text{MPa}$.

Alternatively the stress σ_s can be calculated as follows:

$$\sigma_s = M(d-x_2)/I_2 \quad (15)$$

where M is the maximum span moment under the design service load (g_k+q_k), x_2 is the depth to the neutral axis of a fully cracked section and I_2 is the corresponding second moment of area. The author considers that x_2 and I_2 should be calculated with E_c to be consistent with equation (14). EC2 does not define the SLS loading case which should be used to calculate M . The author

considers that M is intended to be calculated under the total characteristic load $q_{\text{tot}} = g + q$ since this is consistent with the stress of 310MPa in equation (14).

Discussion of assumptions made in the derivation of the EC2 span to depth rules

The most contentious aspects of the derivation of the EC2 span to depth rules relate to the choices of the concrete tensile strength, loading ratio and the modification factor $310/\sigma_s$. The effect of these choices is to prevent the EC2 span to depth rules from accounting for variations in the loading ratio λ or d/h . The consequences of the omissions and assumptions made in the derivation of the EC2 span to depth rules are explored in the remainder of the current article.

Control of deflection by increasing $A_{\text{sprov}}/A_{\text{sreq}}$

In the UK, it is common practice to minimise slab thicknesses by increasing the area of area of flexural reinforcement provided in the span A_{sprov} to as much as twice that required for strength A_{sreq} . This practice frequently leads to significant economies in whole building costs since it reduces the building height and hence the area of external cladding. Increasing $A_{\text{sprov}}/A_{\text{sreq}}$ reduces the service stress in the reinforcement and, hence, the deflection. Consideration of equation (12) shows that increasing the area of reinforcement provided over that required for strength increases the permissible span of a given member under a specified loading ratio $w_{\text{perm}}/w_{\text{du}}$ (where w_{du} is the design ultimate load) by a factor Ω equal to:

$$\Omega = X_{ref} \Psi_{mref} / (X \Psi_m) \quad (16)$$

where the subscript *ref* denotes the member with $A_{sprov} = A_{sreq}$. In this case, $X \sim X_{ref}$ since M_r/M is not changed significantly by the addition of surplus reinforcement to control deflection. Numerical analysis shows that creep and shrinkage significantly reduce the factor by which curvature is reduced when A_{sprov}/A_{sreq} is increased. Parametric studies show that in the long term Ω can be approximated by:

$$\Omega = (A_{sprov}/A_{sreq})^{0.5} \quad (17)$$

Equation (17) is inconsistent with EC2 which states that “*where other stress levels are used the permissible span to depth ratios given by equation (13) should be multiplied by $310/\sigma_s$* ”. The code goes on to say that $310/\sigma_s$ can be approximated with equation (14) which implies that permissible span-to-depth ratios can be increased by a factor equal to A_{sprov}/A_{sreq} for grade 500 reinforcement. The inaccuracies in equation (14) were recognised in the drafting of the UK National Annex to EC2 which limits $310/\sigma_s$ to 1.5. Equation (17) is reasonably accurate unless the area of compression reinforcement is greater than A_{sreq} in which case it can be conservative.

Modification of EC2 span to depth rules to account for loading history and loading ratio

The EC2 span to depth ratios were derived for a loading ratio $w_{perm}/w_{uls} = 0.5$ with $A_{sprov} = A_{sreq}$ and need to be adjusted to account for variations in the

reinforcement stress σ_s due to changes in the loading ratio w_{perm}/w_{uls} . The loading ratio $w_{perm}/w_{uls} = 0.5$ assumed in the derivation of the EC2 span to depth rules is particularly unrealistic for lightly loaded slabs where the self-weight can exceed 50% of the design ultimate load. Analysis shows that the loss of tension stiffening due to loading ratios w/w_u greater than 0.5 can be included in equation (13a) if f_{ck} is replaced by an equivalent concrete strength $f_{ckequiv}$ given by:

$$f_{ckequiv} = (f_{cteff}/0.3)^{1.5} \leq f_{ck} + 8 \quad (18)$$

where f_{cteff} is calculated as follows:

$$f_{cteff} = (0.9h/d)^2 (0.5w_u/w) f_{ctmod} \quad (19)$$

where w_u is the design ultimate load and

$$f_{ctmod} = \sqrt{2w} k_{min} \quad (20)$$

where k_{min} is the least of $\sqrt{0.5f_{ctstrike}/w_{strike}}$, $\sqrt{0.7f_{ctpeak}/w_{peak}}$, $\sqrt{0.5f_{ct28}/w_{perm}}$ and $\sqrt{0.7f_{ct28}/w_{freq}}$. The term $(0.9h/d)^2$ in equation (19) accounts for the influence on the cracking moment of variations in h/d between the section under consideration and the reference section used in the derivation of the EC2 span to depth rules. Equation (19) modifies the concrete tensile strength to give the same interpolation coefficient ζ under the actual load w and $0.5w_u$ used in the calibration of the EC2 span to depth rules. Equation (19) is only

applicable to cracked slabs. Analysis shows that deflections under w_{freq} are relatively insensitive to the concrete tensile strength when the reinforcement index is greater than ρ_o since the curvature tends towards that in a fully cracked section. Therefore, it is proposed that the actual concrete strength is used in equation (13b) and that the permissible span to depth ratio is taken as the greatest of the values given by equations (13a) with f_{ckequiv} and (13b) with f_{ck} if $\rho < \rho_o$ calculated with f_{ckequiv} . The coefficient ρ_o should be calculated with f_{ckequiv} in equation (13a) and with f_{ck} in equation (13b). The proposed procedure is illustrated in Fig. 5. The critical span to depth ratio from equation (13a) (with f_{ckequiv} from equation (20)) or (13b) (with f_{ck}) as appropriate should be multiplied by $0.5w_u/w_{\text{freq}}$ to account for the difference between the actual loading ratio and that assumed in the derivation of the EC2 span to depth rules.

Influence of span

EC2 requires the permissible span to depth ratio to be reduced by $7/L$ for spans greater than 7m. The need for this reduction factor did not emerge in the calibration exercise described in the background document to the EC2 span to depth rules since i) the concrete tensile strength was taken as the splitting strength which is independent of the section depth, ii) the effective depth was assumed to be a constant proportion of the section depth and iii) the loading ratio M/M_u was assumed to be 0.5. The terms in equation (12) for $L/d_{\text{permissible}}$ are independent of the slab thickness and hence span, for given f_{ck} and reinforcement index ρ , when assumptions i) to iii) above apply. The term $7/L$ is needed in practice since assumptions i) to iii) do not apply to

practical slabs where i) the cover is independent of span and ii) the loading ratio w/w_u increases with span for constant superimposed loads. The reduction factor $7/L$ is required to compensate for a) the reduction in $M_r/(0.5M_u)$ with increasing span in members with constant cover and reinforcement index ρ (due to the increase in d/h and f_{ct}/f_{ft}) and b) the increase in loading ratio w/w_u with span (due to the increase in h) for slabs loaded with the same superimposed loads. The effect of a) and b) above is to reduce M_r/M which in turn increases the mid-span curvature above that implicit in the EC2 span to depth rules due to the increase in the interpolation coefficient ζ in equation (1). Reducing M_r/M also increases the length of the span which is cracked which affects X in equation (12) as shown in Fig. 3. The increase in d/h with span is particularly significant in lightly reinforced members since it causes a reduction in the reinforcement ratio ρ (due to the increase in lever arm) which in turn results in a significant increase in the permissible span to depth ratio given by equation (13).

Proposed modification to EC2 span to depth rules

It is recommended that permissible span to depth ratios are calculated as follows:

1. If $\rho < \rho_{\text{equiv}}$ take $L/d_{\text{permissible}}$ as the greatest value given by either equation (13a) (with f_{ckequiv} where f_{ckequiv} is given by equation (18) and $\rho_{\text{equiv}} = 10^{-3}\sqrt{f_{\text{ckequiv}}}$) or equation (13b) evaluated with f_{ck} and $\rho_o = 10^{-3}\sqrt{f_{\text{ck}}}$)
2. If $\rho \geq \rho_{\text{equiv}}$ calculate $L/d_{\text{permissible}}$ with equation (13b) in conjunction with f_{ck} and $\rho_o = 10^{-3}\sqrt{f_{\text{ck}}}$

The multiplier $310/\sigma_s$ in EC2 should be replaced by:

$$M = (0.5w_u/w)(A_{sprov}/A_{sreq})^{0.5} \quad (21)$$

where w is the load under which deflections are being calculated and w_u is the design ultimate load.

Parametric studies show that the load w should not be taken as less than $0.5w_u$ or greater than $0.65w_u$ in equation (19). The actual loading ratio w/w_u should be used in equation (21). When loads are not uniformly distributed, the ratio $0.5w_u/w$ in equations (19) and (21) should be replaced by $0.5M_u/M$ where M is the maximum span moment. It is unnecessary to reduce the resulting span to depth ratios by $7/L$ for spans greater than 7m as stated in EC2 since the influence of span is effectively eliminated by using $f_{ck \text{ equiv}}$ (from equation (18)) in equation (13) in conjunction with the multiplier M given by equation (21). It is recommended that A_{sreq} should be taken as the area of reinforcement required to resist the maximum design elastic bending moment in the span under all load combinations. Equation (21) is based on the assumption that the design ultimate load equals $1.35g_k+1.5q_k$ as assumed in the derivation of the EC2 span to depth rules. If the load factor for dead load is reduced to 1.25 in accordance with Equation 6.10 in EC0, the loading ratio $0.5w_u/w$ should be taken as $(1.35g_k+1.5q_k)/(1.25g_k+1.5q_k) \cdot 0.5w_u/w$.

Evaluation of proposed modification to EC2 span to depth rules.

The author has carried out a comprehensive series of parametric studies to compare total and incremental deflections in continuous slabs with depths equal to the minimum allowed by:

1. BS8110¹¹
2. EC2 with $M = (7/L)(A_{s,prov}/A_{s,req})$ where $7/L \leq 1$
3. EC2 as interpreted by the Concrete Centre¹⁴ with $M = (310/\sigma_s \leq 1.5)7/L$
where $\sigma_s \leq (f_{yk}/\gamma_s)[w/(\gamma_G G_K + \gamma_Q Q_K)][A_{s,req}/A_{s,prov}]/\delta$ where $7/L \leq 1$
4. EC2 as modified by the author

Deflections were calculated, with and without construction loading, in a 3 span slab that was continuous over simple supports. The span was varied between 6m and 10m in increments of 1.0m, the superimposed dead load was taken as 1.0kN/m² and the imposed load was taken as 1.5, 2.5 or 5 kN/m². The concrete strength was varied between 30 and 50MPa. When present, the peak construction load was taken as 0.004h (where h is the slab thickness in mm). Maximum and incremental deflections were calculated using the rigorous approach described in this paper in conjunction with the recommendations given in paragraphs M1 and M2 below equation (11). Peak deflections were calculated under the frequent load combination.

In practice, a restraining moment is typically provided at external supports by the supporting walls or columns. Therefore, deflections were calculated with and without restraining moments of $0.04w_u L^2$ at the external supports. The design moments M_u were calculated with the bending moment coefficients

given in Table 3.12 of BS8110¹¹ unless noted otherwise (i.e. the design moment in the external span M_u was taken as $0.086w_uL^2$ in slabs with $M=0$ at the external supports and $0.075w_uL^2$ in slabs with $M=0.04w_uL^2$ at the external supports). The accuracy of Method 4) was improved when the moment at the external supports was $0.04w_uL^2$ if k was increased from 1.3 to 1.5 in equation (13) and A_{sreq} was calculated with the maximum design elastic moment in the external span under pattern loading which was typically around $0.07w_uL^2$.

Fig. 6 shows the slab thicknesses given by methods 1) to 4) for various scenarios. The stress σ_s was calculated under the frequent load in method 3). The slab thickness is independent of the construction load for methods 1) to 3) above but not method 4). Fig. 7 shows that deflections can exceed code limits in slabs sized using the BS8110¹¹ span to depth rules if the concrete strength is low (i.e. below 30MPa). Fig. 7 also shows deflections calculated under the total service load using the partially cracked section of BS8110¹² in conjunction with creep and shrinkage coefficients calculated in accordance with EC2. The deflections given by BS8110¹² are significantly greater than those given by EC2. This is significant since it shows that predicted deflections can exceed span/250 when calculated with BS8110¹² in slabs dimensioned with the BS8110¹¹ span to depth rules. This was known¹⁴ at the time the BS8110 span to depth rules were derived and was justified by the observation that slabs are seldom truly simply supported in practice. Figs. 8 and 9 show deflections can significantly exceed code limits in slabs dimensioned with the EC2 span to depth rules particularly when the slab thickness is reduced by increasing A_{sprov}/A_{sreq} . In practice the deflections are

likely to be significantly less than shown in Figs. 8 and 9 since some moment restraint is usually present at both internal and external supports. The main reason for the relatively large deflections in Figs. 8 and 9 is that an unrealistically low loading ratio of $w/w_u=0.5$ was assumed in the derivation of the EC2 span to depth rules. Deflections can increase disproportionately in slabs dimensioned with the EC2 span to depth rules as a result of additional cracking induced by increases in the loading ratio w/w_u . Figs. 7 to 13 show that deflections in slabs designed with the proposed method are closest to the code limits of span/250 for total deflections and span/500 for incremental deflections. Figs. 8 to 11 show that construction loading can be beneficial since it reduces the incremental deflection seen by partitions and finishes. Fig. 13 shows that the proposed method gives good results when all the loads on the member in Fig. 10 including self weight are increased by a factor of 4. Figs. 10, 12 and 13 show the proposed method tends to slightly overestimate the reduction in permissible slab thickness that can be achieved by increasing A_{sprov}/A_{sreq} . This arises because the proposed method does a) not fully account for the increase in the coefficient X in equation (16) that occurs due increased cracking as the slab thickness is reduced and b) equation (21) slightly overestimates the benefit of adding extra reinforcement as illustrated in Fig. 4. Fig. 13 shows that the accuracy of the proposed method for dealing with increases in A_{sprov}/A_{sreq} improves with increasing reinforcement index ρ since X becomes less sensitive to changes in slab thickness as the interpolation coefficient ζ increases.

Fig. 6 shows that method 3) which is used by the Concrete Centre¹⁴ tends to give the thinnest slabs of all the methods considered. Figs. 9 to 15 suggest that deflections are likely to exceed the EC2 limits in slabs designed with method 3) unless moment restraint is present at the supports. Deflections were calculated in slabs designed with method 3) assuming that there was no moment at the external support. The stress σ_s in equation (14) was calculated under w_{perm} . The deflections were calculated with construction loading assuming that in reality there was a restraining moment of $0.04FL$ at the external support. Fig. 14 shows that the resulting deflections were close to code limits and suggests that the slab thicknesses given by method 3) are justifiable and economic when moment restraint is present in reality but is neglected in the design of the flexural reinforcement in the span. The area of top reinforcement provided at the external support should be at least 50% of that in the span if this approach is adopted.

Conclusions

EC2 permits significant reductions in slab thickness compared with BS 8110¹¹ since it takes into account the actual tensile strength of the concrete in deflection calculations. The downside of the EC2 deflection calculation method is that predicted deflections depend significantly on assumptions made about loading and concrete material properties. This article proposes a standard method for calculating deflections with EC2 that can be used in the absence of better information. It is shown that the EC2 span to depth rules do account for the effects of construction loading and variations in d/h and loading ratio on long-term deflection. A modification is proposed to the EC2

span to depth rules which is shown to rationally account for these factors. Fig. 7 shows that calculated deflections can exceed BS8110 and EC2 limits in slabs designed in accordance with BS8110 and raises the question of whether the deflection limits in EC2 are too onerous. Comparison with BS5950¹⁶ shows that the deflection limits in EC2 are very onerous compared with the limits used for steel construction in the UK. It is suggested that the incremental deflection limit should be increased to span/360 for brittle partitions, to bring it in line with current UK practice for structural steelwork¹⁶, but not more than 20mm. It is also suggested that the overall deflection should be limited to span/200 where the slab is hidden by raised floors and ceilings. The deflections calculated in the slabs sized with the author's modification to EC2 are generally within these limits.

References

1. British Standards Institution. Eurocode 2, Design of Concrete Structures- Part 1: General rules for buildings, 2004
2. Vollum R.L., Moss R.M., AND Hossain T.R., "Slab deflections in the Cardington in-situ concrete frame building", Magazine of Concrete Research, 54, (2002) 23-34.
3. Hossain T.R and Vollum R.L., "Prediction of slab deflections and validation against Cardington data", Proc. of the Inst. of Civil Eng., Structures and Building, 152, (2002), 235-248
4. Vollum R.L., "Influences of shrinkage and construction loading on loss of tension stiffening in slabs", Magazine of Concrete Research, 54, August 2002, 273-282

5. Vollum R.L Investigation into backprop forces and deflections at St George Wharf, Magazine of Concrete Research, 55, (2003), No. 5, 449-460.
6. Vollum R.L., Afshar N. Influence of construction loading on deflections in reinforced concrete slabs, Magazine of Concrete Research, accepted for publication
7. Beeby A.W Criteria for slab loading, Proceedings of the Institution of Civil Engineers, Structures & Buildings 146, May 2001 Issue 2, 195-202
8. Pallet P.F. Guide to flat slab formwork and falsework, CONSTRUCT, 2003
9. Concrete Society. Deflections in Concrete Beams and Slabs, Technical Report 58, Concrete Society 2005.
10. British Standards Institution. PD 6687-2005. Background paper to the UK National Annex to BS EN 1992-1-1 and BS EN. 1992-1-2. 2006.
11. British Standards Institution, Structural Use of Concrete, BS8110-1: 1997, 2005
12. British Standards Institution. Code of Practice for special circumstances, BS8110-2: 1985
13. Peiretti HC, Serviceability limit states, Supporting document for section 7 of EN 1992-1
14. Moss R.M Brooker O How to design concrete structures using Eurocode 2: Part 3 Slabs, CONCRETE CENTRE, 2006

15. Beeby, AW Modified proposals for controlling deflections by means of ratios of span to effective depth, Technical Report 456, Cement and Concrete Association, Wexham Springs 1971
16. British Standards Institution BS5950-1:2000, Structural use of steelwork in building – Part 1: Code of practice for design – Rolled and welded sections

List of Figures

Figure 1: Load history for slabs S4 to S6

Figure 2: Comparison of deflections in slabs S4 to S6

Figure 3: Influence of load ratio on X

Figure 4: Influence of A_{sprov}/A_{sreq} on curvature

Figure 5: Influence of $f_{ckequiv}$ on $L/d_{permissible}$

Figure 6: Comparison of slab thicknesses

Figure 7: BS8110 with construction load: Continuous slab ($M=0$ at external support)

Figure 8: EC2 without construction load: Continuous slab ($M=0$ at external support)

Figure 9: EC2 with construction load: Continuous slab ($M=0$ at external support)

Figure 10: Proposed method without construction load: Continuous slab ($M=0$ at external support)

Figure 11: Proposed method with construction load: Continuous slab ($M=0$ at external support)

Figure 12: Proposed method with construction load: Continuous slab ($M=0.04FL$ at external support)

Figure 13: Proposed method with all loads including self weight increased by a factor of 4: Continuous slab ($M=0.04FL$ at external support)

Figure 14: Concrete Centre method with construction load: Continuous slab (Deflections calculated with $M=0.04FL$ at external support)

Figure 1: Moment v time

Figure 2: Curvature v time

Figure 3: Influence of load ratio on X

Figure 4: Influence of A_{sprov}/A_{sreq} on curvature

Figure 5: Influence of $f_{ckequiv}$ on permissible span to depth ratio

Figure 6: Comparison of slab thicknesses

a) Total deflection

b) Total deflection

c) Incremental deflection

Figure 7: BS8110 with construction load: Continuous slab ($M=0$ at external support)

a) Total deflection

b) Incremental deflection

Figure 8: EC2 without construction load: Continuous slab ($M=0$ at external support)

a) Total deflection

b) Incremental deflection

c) Total deflection

Figure 9: EC2 with construction load: Continuous slab ($M=0$ at external support)

a) Total deflection

b) Incremental deflection

a) Total deflection

Figure 10: Proposed method without construction load: Continuous slab ($M=0$ at external support)

a) Total deflection

b) Incremental deflection

Figure 11: Proposed method with construction load: Continuous slab ($M=0$ at external support)

a) Total deflection

b) Total deflection

c) Total deflection

Figure 12: Proposed method with construction load: Continuous slab
 $(M=0.04FL \text{ at external support})$

Figure 13: Proposed method with construction load: Continuous slab
($M=0.04FL$ at external support)

a) Total deflection

b) Incremental deflection

c) Total deflection

Figure 14: Concrete Centre method with construction load: Continuous slab
(Deflections calculated with $M=0.04FL$ at external support)