A Rational Design Model for Web Stiffeners

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Abstract

Existing design methods for web stiffeners differ widely in concept and in the stiffener size which results. An investigation has been conducted to provide a rational, usable and validated design method for transverse stiffeners. The proposed design method takes account of the axial compression in stiffeners caused by tension field action and external forces, the transverse forces on the stiffeners required to enforce effective division of the web plate, and the tendency for the stiffened plate to buckle overall. Design criteria are established to ensure that the resulting stiffeners do not yield before plate failure, and that the plating is effectively divided into panels. The transverse stiffener dimensions required by the proposed design model have been validated against finite element results for plates subjected to pure shear, and to combined shear and direct in-plane loading. Finally, the stiffener sizes are compared with those required by the existing methods.

KEY WORDS: Stiffeners; Stiffener forces; Plate girders; Webs; In-plane Loading; Design; Non-linear analysis; Buckling

1. Introduction

Web stiffeners in plate girders have been designed successfully for more than 100 years. However, the design bases and assumptions in different codes and proposals differ widely, and this has an important effect on web thickness/panel size optimisation, and hence on the weight and cost of the stiffened web.

Some approaches are based on critical buckling, taking no account of the effects of initial imperfections in the plate and stiffener, or of post-buckling effects. For these reasons, some methods apply an empirical factor on the required stiffener rigidity obtained from the critical buckling criterion [1]. Other approaches,

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including British Standard BS5400 [2], EUROCODE 3 [3] and AASHTO [4], are based principally on the in-plane forces resulting from tension field action and do not give sufficient rational recognition to the out-of-plane forces on the stiffener exerted by panel bending. By designing stiffeners primarily to resist axial forces, the stiffeners required by these approaches tend to become progressively more conservative as the panel depth/length ratio increases [5, 6].

Rahal and Harding [7, 8] proposed that the stiffeners be designed as a simply supported beam subjected to a distributed lateral load. The design equation for the lateral load was developed by a curve-fitting operation on the lateral reactions acting on a non-deflecting nodal line, which was introduced to replace the stiffener. The effect of axial force caused by tension field action was found to be small for the cases considered, and the effect was therefore neglected. The tallest panel considered in the finite element (FE) analysis had an aspect ratio of 2:1.

For webs in pure shear, a study at Imperial College London [6, 9] also demonstrated that the axial forces on the stiffener are of secondary importance, and that out-of-plane forces provide a more rational basis for design. The local panel bending effects and the significance of the tendency for stiffened plates to buckle overall were identified. The studies led to the formulation of a design model that shows good agreement with elasto-plastic finite element analyses for concentric flat bar stiffeners on a two-panel web in pure shear.

In order to improve phenomenological insight and to reduce the complexity to a form suitable for design, the non-linear post-buckling behaviour of stiffened webs subjected to in-plane shear and direct loading (Fig.1) was investigated by experimental, analytical and numerical studies [5]. Tests were conducted on transversely stiffened plate girders with different geometrical configurations and loading arrangements, and the test results were used to validate the elasto-plastic post-buckling behaviour as represented by the FE analysis program. The design formulations proposed in the present paper take account of the axial compression in stiffeners caused by tension field action and external forces, the transverse forces on the stiffeners required to enforce effective division of the web plate into plate panels, and the tendency for the stiffened plate to buckle overall. The stiffener axial forces and the transverse forces due to panel bending effects were discussed in detail in previous papers [10, 11].

2. Determination of Stiffener Forces

The components of the stiffener forces to be included in the proposed design model have been studied separately in Ref. [5] and the previous papers [10, 11], and corresponding predictive formulae have been developed. This section proposes a method for predicting the total stiffener forces that take account of the combination of the different effects.

2.1. Axial Force In Stiffeners

Using the concept of the tension field theory, with modifications that take account of stiffener compressibility and panel deflection, a simple validated method for the estimation of the stiffener axial force has been developed [10] for stiffened plates subjected to combined shear, compression and bending. The modifications are based on FE non-linear elastic buckling analyses of plates with a straight non-deflecting compressible stiffener. FE parametric studies also demonstrated that the stiffener axial force due to the effects of Poisson's ratio and the out-of-plane deflection of the panel is relatively small, and that for an applied shear stress up to the ultimate shear strength, the effect of panel plasticity on the stiffener axial force may be neglected. Also, the magnitudes of initial panel and stiffener imperfections do not significantly affect the stiffener axial force.

It is proposed [10] that for plates subjected to general combined shear, compression and bending, the axial force P_s in the stiffener can be represented by the following equations:

$$\frac{P_s}{\tau_{cr} dt} = \begin{cases} C_1 \left(\frac{\tau}{\tau_{cr}} - 1 \right) \cdot (1 + C_2 \ln \Omega) & \text{for } \tau > \tau_{cr} \\ 0 & \text{for } \tau \leq \tau_{cr}, \text{ or } \tau < \sigma_c / 5 \text{ and } \tau < \sigma_b / 5 \end{cases} \quad (1)$$

where Ω denotes the stiffener extensional rigidity ratio ($\Omega = A_s / at$), and:

$$\begin{aligned} C_1 &= \frac{1}{2\sqrt{d/a}} \\ C_2 &= 0.27 + 0.05 \ln(d/a) \end{aligned} \quad (2)$$

τ_{cr}^0 , σ_{ccr}^0 and σ_{bcr}^0 are the critical stresses of the plate panel, subjected to shear, compression or bending in isolation, respectively. The stresses which produce buckling when acting in combination are denoted by τ_{cr} , σ_{ccr} and σ_{bcr} . The panel dimensions are width a , depth d and thickness t (see Fig.1).

The critical shear stress τ_{cr} in the presence of compression and bending can be calculated using Eqn. (3):

$$\frac{\sigma_{ccr}}{\sigma_{ccr}^0} + \left(\frac{\sigma_{bcr}}{\sigma_{bcr}^0} \right)^2 + \left(\frac{\tau_{cr}}{\tau_{cr}^0} \right)^2 = 1 \quad (3)$$

where τ_{cr}^0 , σ_{ccr}^0 and σ_{bcr}^0 can be determined by the equations (4) to (6):

$$\tau_{cr}^0 = k_{cr} \frac{\pi^2 D}{d^2 t} \quad k_{cr} = 5.35 + 4/\alpha^2 \quad \alpha = \min\left(\frac{d}{a}, \frac{a}{d}\right) \geq 1 \quad (4)$$

$$\sigma_{ccr}^0 = k_{cr} \frac{\pi^2 D}{d^2 t} \quad k_{cr} = \begin{cases} 4 & \text{for } d/a \leq 1 \\ \left(\frac{a}{d} + \frac{d}{a}\right)^2 & \text{for } d/a \geq 1 \end{cases} \quad (5)$$

$$\sigma_{bcr}^0 = k_{cr} \frac{\pi^2 D}{d^2 t} \quad k_{cr} = \begin{cases} 24 & \text{for } d/a \leq 1.5 \\ 24 + 73\left(\frac{2}{3} - \frac{a}{d}\right)^2 & \text{for } d/a \geq 1.5 \end{cases} \quad (6)$$

2.2. Bending Moment Due to Local Panel Bending Effect

Buckled panels cause a distributed lateral force on, and bending moment in, the stiffener, which is referred to as the “panel bending effect”. In the proposed design model, the bending moment in a straight rigid knife edge at the ultimate load of the plate, taking account of the effects of panel plasticity, is taken as the stiffener bending moment due to the panel bending effect, and is denoted as M_{pb} .

By comparison of the FE solutions obtained for the plate with a straight knife edge, with and without introducing plasticity of the panels, Xie ([5]) found that M_{pb} can be estimated using:

$$M_{pb} = F_{pl} M_{km} \quad (7)$$

where M_{km} is the maximum knife edge moment obtained from the non-linear elastic analysis, and F_{pl} represents the effects of panel plasticity on the knife edge moment. It was found that F_{pl} may be approximately represented by:

$$F_{pl} = 1.6 - 0.35 \sqrt{\sigma_{0p}/355} \quad \text{and} \quad F_{pl} \geq 1.0 \quad (8)$$

Eq. (8) represents an increase in the knife edge moment due to panel plasticity as the applied shear stress approaches the ultimate shear strength, the plastic deformation in the panels increases rapidly, which in turn results in a significant increase in the panel deflection, and hence in the knife edge moment.

The calculation of the elastic knife edge moment M_{km} is discussed in detail in Ref. [11] and summarised in the following subsections.

2.2.1. Plates subjected to pure shear

From the FE results of non-linear elastic buckling analyses of plates stiffened by a knife edge, it has been found [11] that for plates under pure shear the elastic knife edge moment M_{km} can be estimated using the following equations:

$$\begin{aligned} \frac{M_{km}b}{Dt} &= 29.71 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right) - 0.524 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right)^2 \\ &\quad - 6.96 \left(\frac{\tau}{\tau_{cr}} \right) + 4.962 \left(\frac{\tau}{\tau_{cr}} \right)^2 + 21.37 \left(\frac{\tau}{\tau_{cr}} \right)^3 - 2.865 \left(\frac{\tau}{\tau_{cr}} \right)^4 \quad \text{for } d/a \leq 1 \\ \frac{M_{km}b}{Dt} &= 83.62 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right) - 16.59 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right)^2 \\ &\quad - 14.03 \left(\frac{\tau}{\tau_{cr}} \right) + 19.70 \left(\frac{\tau}{\tau_{cr}} \right)^2 + 54.99 \left(\frac{\tau}{\tau_{cr}} \right)^3 - 7.676 \left(\frac{\tau}{\tau_{cr}} \right)^4 \quad \text{for } 2 \leq d/a \leq 2.2 \\ \frac{M_{km}b}{Dt} &= 69.47 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right) + 1.942 \left(\frac{w_{0p}}{t} \right) \left(\frac{\tau}{\tau_{cr}} \right)^2 \\ &\quad - 16.96 \left(\frac{\tau}{\tau_{cr}} \right) + 80.11 \left(\frac{\tau}{\tau_{cr}} \right)^2 - 7.281 \left(\frac{\tau}{\tau_{cr}} \right)^3 - 0.264 \left(\frac{\tau}{\tau_{cr}} \right)^4 \quad \text{for } d/a \geq 2.4 \end{aligned} \quad (9)$$

where w_{0p} is the magnitude of the initial panel imperfection (see Fig.3 of Ref. [10]). The values of M_{km} for $1 < d/a < 2$ and $2.2 < d/a < 2.4$ can be obtained by linear interpolation.

2.2.2. Plates subjected to combined loading

For plates subjected to combined shear, compression and bending, it is proposed [11] that the knife edge moment M_{km} can be estimated using Eq. (9), with τ/τ_{cr} being replaced by an equivalent shear stress ratio given by:

$$\frac{\tau_{eq}}{\tau_{cr}} = \frac{1}{2} \left(\beta_c \frac{\sigma_c}{\sigma_{ccr}} \right) + \sqrt{\left(\frac{\tau}{\tau_{cr}} \right)^2 + \frac{1}{4} \left(\beta_c \frac{\sigma_c}{\sigma_{ccr}} \right)^2 + \left(\beta_b \frac{\sigma_b}{\sigma_{bcr}} \right)^2} \quad (10)$$

By curve-fitting the FE solutions for combined loading against Eq. (9), it was found that the factor β_c may be taken as approximately:

$$\beta_c = \begin{cases} 0.45 & \text{for } d/a \leq 1 \text{ or } d/a \geq 3 \\ 0.2 + 0.25 \left| \frac{d}{a} - 2 \right| & \text{for } 1 < d/a < 3 \end{cases} \quad (11)$$

$$\beta_b = 0.25 \quad (12)$$

Fig. 2 shows a good agreement in the knife edge moments between the FE solutions and the values calculated using the proposed equations for $d/a=1$ plates subjected to various stress ratios ($\tau/\sigma_c/\sigma_b$).

2.3. Bending Moment Due to Overall Buckling Effect

Stiffened plates can buckle in two basic ways. In overall buckling, the stiffeners buckle along with the plating; in local panel buckling, the plate panels buckle between the stiffeners. For plates with elastic stiffeners subjected to shear loading, a combination of overall buckling and local panel buckling invariably occurs, as the stiffeners are unable to enforce nodal lines at the position of stiffener. For stiffened plates subjected to compression or pure bending, a stiffener rigid enough will force a nodal line forming at the position of stiffener, but the combination of overall buckling and local panel buckling can still be seen in some higher critical modes.

The tendency for the stiffened plate to buckle overall was first studied by Stanway et al [6, 9] using a rigid-link model to quantify the overall buckling effect for plates under pure shear. Xie [5] presents an improved and extended study of the overall buckling behaviour, introducing a beam-plate model composed of a stiffener and two plate panels. By applying the overall buckling model for plates with an imperfect stiffener, the stiffener bending moment due to the overall buckling effect is derived by solving the eigenvalue problem formed by minimising the total potential energy of the system.

The overall buckling analyses were conducted for a wide range of stiffened plates subjected to shear, compression, bending and external force applied to one or both ends of the stiffener, respectively. The deflection functions were chosen to enable the overall buckling modes for each loading component to be represented, as shown in Table 1. For pure shear, the deflection function takes account of the inclined buckling mode.

For stiffened plates subjected to coexistent shear stress τ , compressive stress σ_c and bending stress σ_b , it is proposed [5] that the stiffener moment due to the overall buckling effect, denoted as M_{ov} , can be determined by:

$$M_{ov} = \frac{\pi^2 E I_s w_{0s}}{d^2} \left(\frac{\tau/\tau_{ov} + \sigma_c/\sigma_{c,ov} + \sigma_b/\sigma_{b,ov}}{1 - \tau/\tau_{ov} - \sigma_c/\sigma_{c,ov} - \sigma_b/\sigma_{b,ov}} \right) \quad (13)$$

where w_{0s} denotes the maximum magnitude of the initial stiffener deflection, and the overall buckling stresses for shear, compression and bending in isolation are given by, respectively:

$$\tau_{ov} = \frac{\pi^2 D}{d^2 t} \sqrt{8.155 \left(\frac{d^2}{4a^2} + \frac{4a^2}{d^2} \right) + 13.99 \cdot \left[\left(\frac{d^2}{4a^2} + 1 \right)^2 + \frac{b}{2a} \Psi \right]} \left/ \left(\frac{d^2}{4a^2} + 1 \right) \right. \quad (14)$$

$$\sigma_{c,ov} = \frac{\pi^2 D}{d^2 t} \left[\left(\frac{d}{2a} + \frac{2a}{d} \right)^2 + \frac{4ab}{d^2} \Psi \right] \quad (15)$$

$$\sigma_{b,ov} = \frac{9a^2 \pi^4 D}{2d^4 t} \sqrt{\left[\left(\frac{d^2}{4a^2} + 1 \right)^2 + \frac{b}{a} \Psi \right] \left[\left(\frac{d^2}{16a^2} + 1 \right)^2 + \frac{b}{a} \Psi \right]} \quad (16)$$

and Ψ is the stiffener flexural rigidity ratio ($\Psi=EI_s/Db$).

To include the destabilising effect of the external force Q_1 applied to one end of the stiffener, it is proposed [5] that Eq. (13) should be modified as:

$$M_{ov} = \frac{\pi^2 E I_s w_{0s}}{d^2} \left(\frac{\tau/\tau_{ov} + \sigma_c/\sigma_{c,ov} + \sigma_b/\sigma_{b,ov} + Q_1/Q_{1,ov}}{1 - \tau/\tau_{ov} - \sigma_c/\sigma_{c,ov} - \sigma_b/\sigma_{b,ov} - Q_1/Q_{1,ov}} \right) \quad (17)$$

where the $Q_{1,ov}$ is overall critical load for this condition, and can be expressed as:

$$Q_{1,ov} = 1.121 \left[\frac{2b}{a} \Psi + \left(\frac{d^2}{4a^2} + 1 \right)^2 + \left(\frac{d^2}{16a^2} + 1 \right)^2 \right] \frac{\pi^2 D a}{d^2} \quad (18)$$

For cases where the external force Q_2 is applied to both ends, it is proposed that Eq. (13) should be modified as:

$$M_{ov} = \frac{\pi^2 E I_s w_{0s}}{d^2} \left(\frac{\tau/\tau_{ov} + \sigma_c/\sigma_{c,ov} + \sigma_b/\sigma_{b,ov} + Q_2/Q_{2,ov}}{1 - \tau/\tau_{ov} - \sigma_c/\sigma_{c,ov} - \sigma_b/\sigma_{b,ov} - Q_2/Q_{2,ov}} \right) \quad (19)$$

where the overall critical load $Q_{2,ov}$ can be expressed as:

$$Q_{2,ov} = \left[\frac{b}{a} \Psi + \left(\frac{d^2}{4a^2} + 1 \right)^2 \right] \frac{\pi^2 D a}{d^2} \quad (20)$$

2.4. Combination of Force Components

The interaction between the panel bending and overall buckling effects was studied by Stanway [6] for pure shear using the “rigid-link” model. The panel bending effect was incorporated in the model by applying a lateral distributed force to the stiffener, which was taken as the lateral force acting on a straight rigid knife edge. It was established that the total stiffener moment may be expressed as [9]:

$$M_s = M_{ov} + \frac{M_{pb}}{1 - \tau/\tau_{ov}} \quad (21)$$

It is noted that the second term on the right-hand side of Eq. (21) is a conservative approximation. Further, for an adequate stiffener, the ratio of τ/τ_{ov} is much less than 1.0. Therefore, it is reasonable to simplify Eq. (21) as:

$$M_s = M_{ov} + M_{pb} \quad (22)$$

According to Eq. (22), the total stiffener bending moment M_s can be approximately calculated simply by adding the components of stiffener moment due to the local panel bending effect and the tendency for the stiffened plate to buckle overall. Such a simplification is especially useful for the combined loading conditions, where the interaction is much more complicated.

Based on the above discussion, it is proposed that the stiffener should be designed to resist the coexistent bending moment M_s and axial force P_s , with the stiffener cross-section consisting of the actual stiffener area only. It is reiterated that the axial force P_s is determined using Eq. (1), while the stiffener bending moment M_s is calculated using Eq. (22), where M_{pb} and M_{ov} are given in Eqs (7) and (13), respectively.

In Fig. 3, the elastic stiffener moment (i.e., $M_s=M_{pk}+M_{ov}$), estimated using the proposed method but taking the panel plasticity factor $F_{pl}=1$, is compared with the finite element solutions.

It should also be pointed out that the proposed method for predicting the stiffener forces is developed for stiffeners whose sizes are within the practical range; that is, the stiffener should be large enough to effectively divide the plate. For a weak stiffener, the knife edge model is not valid, and neither is the equation for estimating the bending moment due to the overall buckling effect. This is because the bending moment due to the overall buckling effect is derived from the overall buckling model by assuming that the applied stress is much less than the overall buckling stress. However, for a weak stiffener, this may not be true.

3. Design Criteria for Stiffeners

Transverse stiffeners are introduced mainly to increase the plate strength, but they also help to achieve the intended geometry of the girder section. An adequate stiffener should satisfy the following requirements:

- (1) *The chosen stiffener should enable the ultimate strength of the stiffened plate to be at least equal to the strength of the individual panel, assuming it is simply supported. The minimum flexural rigidity which satisfies this criterion will be denoted as Ψ_{ssu} .*
- (2) *The chosen stiffener should effectively divide the plating; that is, an effectively divided panel deflection mode should be maintained until the ultimate load of the stiffened plate is reached.*
- (3) *The chosen stiffener should not be unnecessarily large. The stiffener size adopted should ideally be such that a further significant increase in the stiffener size results in only a very small relative increase in the ultimate strength, while a significant reduction in the stiffener size results in a significant reduction in the ultimate strength.*
- (4) *The chosen stiffener should prevent a rapid reduction in the load carrying capacity of the stiffened plate in the post-ultimate region. It is therefore desirable that stiffener failure does not occur before panel failure: it is assumed that this will be achieved if the stiffener does not yield at the maximum resistance of the stiffened plate.*

In this section, strength and stiffness requirements are introduced to size the stiffener. Attention is paid mainly to double-sided stiffeners, while special considerations required for single-sided stiffeners are presented in Subsection 3.3.

3.1. Strength Criterion

3.1.1. Calculation of Stiffener Stresses

As stated in Section 2.4, in the proposed design model, a stiffener is assumed to be subjected to the coexistent bending moment M_s and axial force P_s , with the stiffener cross-section consisting of the actual stiffener area A_s only. It is noted that the predictive formulae for the axial force in the stiffener are developed from the non-deflecting stiffened plate model, in which the stiffener is modelled using truss elements which lie in the mid-plane of the plate. Therefore, the axial force P_s should be taken as acting in the mid-plane of the plate, as illustrated in Fig. 4.

For double-sided flat bar stiffeners, the stiffener area A_s and the moment of inertia I_s are calculated as follows:

$$A_s = 2 h_s t_s \quad (23)$$

$$I_s = \frac{1}{12} t_s \left[(2h_s + t)^3 - t^3 \right] \quad (24)$$

The definitions of h , h_s and t_s are illustrated in Fig. 1. For single-sided flat bar stiffeners, the stiffener area A_s and the moment of inertia I_s are taken as:

$$A_s = h_s t_s \quad (25)$$

$$I_s = \frac{1}{12} t_s h_s^3 \quad (26)$$

It is indicated in Fig. 4 that for double-sided stiffeners, the stiffener can be designed as a simple beam loaded by an axial force equal to P_s acting at the centroid of the stiffener cross-section, and a bending moment M_s about the centroidal axis 1–1. For single-sided flat bar stiffeners, the stiffener can be designed as a simple beam loaded by a bending moment M_s and an eccentric axial load P_s acting at a distance $(h_s+t)/2$ from the centroidal axis 1–1 of the stiffener cross-section. It is apparent that for the single-sided stiffener, an additional bending moment resulting from the eccentricity of the axial load P_s should be included, and the two possible combinations of the stiffener forces should be considered for the design of the eccentric stiffener.

It is proposed that for double-sided (concentric) stiffeners, the maximum stress in the stiffener can be calculated as follows:

$$\sigma_{s,\max} = \frac{P_s}{A_s} + \frac{M_s}{Z_s} \quad (27)$$

For load bearing stiffeners, the stiffener axial force P_s in Eq. (27) should be replaced with the total axial force $P_{s,\text{total}}$, which is proposed to be as follows:

$$P_{s,\text{total}} = P_s + Q_1, \quad \text{or} \quad P_s + Q_2 \quad (28)$$

where Q_1 and Q_2 are the external stiffener forces acting at one end and at both ends, respectively. A more detailed discussion on load bearing stiffeners is presented in Section 4.2. As required in the design codes [2-4], a bearing stiffener should be double-sided and symmetric about the mid-plane of the plate.

3.1.2. Strength Requirement

The purpose of the strength requirement is to ensure that stiffener failure does not occur before panel failure. This is necessary because otherwise the stiffener deflection might be such that the panel strength would not be mobilized, and because stiffener failure causes a rapid reduction in resistance of the stiffened plate.

Stiffener failure may be prevented in design by providing a stiffener for which the yield stress is not exceeded, referred to as “the no yield criterion”. The no yield criterion has been adopted in the design methods proposed by Stanway et al [9], and by Rahal and Harding [8]. Another criterion, as adopted in the method by Rockey et al [12], is to provide a stiffener for which the plastic moment capacity is not exceeded, referred to as “plastic collapse criterion”. It is noted that after yielding, the stiffener stiffness will diminish, the deflection will increase, and the analytical assumptions may be violated.

For the strength design of the transverse stiffener, the no yield criterion is adopted here. It is proposed that the maximum stress in the stiffener, calculated using Eq. (27), should not exceed the yield stress of the stiffener; that is:

$$\sigma_{s,\max} \leq \sigma_{0s} \quad (29)$$

For design of the stiffener the ultimate load for a simply supported panel is taken as the load applied on the stiffened plate. However, the actual plate strength is often greater than the strength for the simply supported panel, and therefore it is desirable that the required stiffener size according to the strength criterion would ensure the stiffener remains elastic at the actual ultimate load of the stiffened plate, and this condition is checked against the FE results in the section below.

3.1.3. Required Stiffeners according to the Strength Criterion

In order to show to what extent the forces given by the proposed method result in a stiffener which satisfies the strength criterion, the flexural rigidity of the smallest stiffener (double-sided, $h/t_s=10$) which satisfies the strength criterion of Eq. (29), denoted as Ψ_{dz} , is compared with the ultimate strength results from the non-linear elasto-plastic FE analyses. The applied load was taken as the ultimate load of the simply supported panel obtained from FE analysis, as given in Ref. [5]. Stiffener sizes for plates under different loading conditions were determined.

- *Plates subjected to pure shear*

In Ref [5], the FE τ_u vs. Ψ results given by Fortune [13] are used to validate the required stiffener size Ψ_{dZ} calculated according to the proposed method for a range of cases with $d/a=0.5$ to 7 and $b/t=80$ to 180. As an illustration, Fig. 5 presents the comparisons made for $b/t=180$ and $d/a=1$ and 2, with the yield stress of stiffener and panel material being 245N/mm^2 . Also included in Fig.5 is the flexural rigidity of the smallest stiffener (double-sided, $h/t_s=10$) which satisfies the stiffness criterion (Section 3.2), denoted as Ψ_{dl} .

According to the strength criterion, the stiffener with a rigidity of Ψ_{dZ} should remain elastic at the ultimate load of the plate. This is indicated by the position of Ψ_{dZ} in relation to Ψ_p (the largest value of Ψ , of those considered, for which yielding of the stiffener occurs at the ultimate load) and Ψ_e (the smallest value of Ψ , of those considered, for which yielding of the stiffener does not occur at the ultimate load). The value of Ψ for which the stiffener just remains elastic at the ultimate load lies between Ψ_p and Ψ_e .

It was found that for all the cases examined by Xie [5], the required stiffener rigidity Ψ_{dZ} is in close proximity to the region bounded by Ψ_p and Ψ_e , and mostly greater than Ψ_p . The implication of this observation is that the stiffener forces determined in accordance with the proposed method are satisfactory.

For a stiffener design model, the basic requirement is that it should provide a stiffener which will ensure that the ultimate load for the stiffened plate is at least as great as the ultimate load for the simply supported panel. The minimum flexural rigidity which satisfies this criterion is denoted as Ψ_{ssu} . In Fig. 5, Ψ_{ssu} is also indicated on the τ_u vs. Ψ results curves. For the calculation of Ψ_{dZ} to be satisfactory in sizing the required stiffener, it should be equal to or greater than Ψ_{ssu} ; that is:

$$\Psi_{dZ} \geq \Psi_{ssu} \quad (30)$$

It has been observed in Ref. [5] that the required stiffener rigidity Ψ_{dZ} is in close proximity to Ψ_{ssu} , and in most of the cases, is equal to or greater than Ψ_{ssu} . For $d/a=0.5$, $b/t=180$, it is found that the ultimate strength of the stiffened plates is negligibly less than the panel strength (about 0.5%), even for a stiffener

of great rigidity Ψ . However, the design stiffener Ψ_{dz} for this case is considered to be adequate, because any increase in Ψ from Ψ_{dz} will not result in appreciable increase in τ_u' for the stiffened plate. It was noted that for $d/a=0.5$ and 1, the region bounded by Ψ_p and Ψ_e is at a significant distance along the plateau of the τ_u' vs. Ψ curve, and this means that a stiffener which just remains elastic at the ultimate load will represent the required stiffener size.

However, for $d/a \geq 2$, although the stiffener rigidity Ψ_{dz} is satisfactory according to the proposed strength criterion, it was observed that it is not on the plateau of the τ_u' vs. Ψ curve, and therefore cannot be taken as an adequate stiffener. It is noted that for these cases, the stiffener which remains fully elastic at the ultimate load is relatively small compared to the required stiffener sizes, and deflects to a degree which would not provide adequate subdivision of the plate, and would result in a reduction in the plate resistance. For this reason, a stiffness criterion is also necessary to ensure that the design stiffener is adequate, and the stiffness criterion is presented in Section 3.2.

- ***Plates subjected to combined loading***

The proposed design method is based on theoretical solutions and extensive numerical results, and has been verified at each stage of model development. Therefore, only a group of typical cases were analysed to provide a basis for the validation of the proposed design model when used for the combined loading conditions. The plates of $d/a=2$ and $b/t=180$ were considered, and the ratios of individual loading components examined are: $\sigma_c:\tau=1:0, 1:2, 1:5$ for shear and compression, and $\sigma_b:\tau=1:0, 2:1, 1:1$ for shear and bending. These cases were chosen to cover the limiting loading conditions (i.e., longitudinal compression or bending alone) and the conditions with which the possible maximum errors in estimating the knife edge moment according to the proposed method may be associated.

In Fig. 6, the required stiffener size Ψ_{dz} according to the proposed strength criterion is compared with the ultimate strength results from FE analyses (more comparisons for other stress ratios were given in Ref. [5]). It was found that for the shear and compression cases considered, the ultimate load corresponding to the stiffener rigidity Ψ_{dz} is well above the ultimate load of the simply supported panel, that is, the requirement given in Eq. (30) is satisfied. For the combined shear and bending cases examined, it was

found that the ultimate load for the stiffened plate is slightly lower (about 2%) than the ultimate load for the simply supported individual panel.

It was observed that the required stiffener rigidity Ψ_{dz} is within or beyond the region bounded by Ψ_p and Ψ_e , that is, the stiffener determined in accordance with the proposed strength criterion will remain elastic at the ultimate load. However, it was observed [5] that the stiffener having a rigidity of Ψ_{dz} does not always effectively divide the plate into panels, which further demonstrates the necessity of introducing a stiffness criterion.

3.2. Stiffness Criterion

3.2.1. Calculation of Stiffener Deflection

For stiffened plates subjected to pure shear, it has been found that the stiffener deflection may be closely represented by a half sine wave [6, 9], and this shape has been assumed for stiffness design of the stiffener. This assumption is also consistent with that made in the overall buckling model (Section 2.3), from which the stiffener moment due to the overall buckling effect is derived.

As implied in the “no yield” strength criterion, the stiffener is considered to be elastic in the proposed design model. Therefore, using the half sine wave assumption, the stiffener deflection w_s can be written as:

$$w_s = \frac{M_s d^2}{\pi^2 E I_s} \quad (31)$$

where w_s and M_s are the maximum values along the stiffener length, and the stiffener moment M_s is given by Eq. (22).

In Fig. 7, the stiffener deflection given by Eq. (31) is compared with the FE solutions. The point corresponding to the ultimate load is indicated on the elasto-plastic w_s/t vs. M_s/M_{ps} curve. It can be observed that the variation in w_s with M_s is basically linear, which would be expected if the half sine wave prevails, especially prior to the ultimate load, and the agreement between Eq. (31) and the FE results is considered to be satisfactory. Similar comparisons were made by Stanway [6] for different

values of Ψ . In the design model by Stanway et al [9], the stiffener deflection is also estimated by means of Eq. (22).

3.2.2. Design Limit on Stiffener Deflection

For plates subjected to pure shear, it was found by Stanway et al [9] that if the stiffener effectively divides the plate, then a plate strength within the plateau of the τ_u ' vs. Ψ curve, and greater than that for simply supported panels, will be attained. To ensure that the plate is effectively divided, the deflection of the stiffener at the ultimate load, denoted as w_{su} , must be sufficiently small. The approach adopted in the proposed design model is therefore to specify a maximum allowable stiffener deflection which will ensure that the plate is effectively divided, and hence that the plate strength is within the plateau of the strength against Ψ curve.

The stiffness criterion originally developed by Stanway et al [9] is that the stiffener deflection measured over a panel buckle half wavelength should be small in relation to the amplitude of the panel buckle for a rigid stiffener, and the panel deflection is assumed to be that given by non-linear elastic analyses for a simply supported panel. This approach is logical in that it introduces a quantitative means to define the subdivision of the plate. However, it involves the estimation of the panel deflection, which has been found to be quite complicated even for pure shear, due to the variation of the deflection mode with the applied load. For plates subjected to combined shear and longitudinal loading, the estimation of the panel deflection will be much more difficult.

An alternative approach to ensure the subdivision of the plate is to limit the out-of-straightness of the stiffener. Assuming that the deflected shape of the stiffener is a half sine wave, the out-of-straightness of the stiffener may be represented by the ratio of w_s/d .

The stiffener deflection ratio w_s/d must be small enough to ensure that the effectively divided panel mode and the required plate strength are achieved. For plates subjected to pure shear, this means that a plate strength within the plateau of the τ_u ' vs. Ψ curve should be attained. In order to define consistently a point of reference for the start of the plateau, the following parameter is introduced:

$$\mu_u = \frac{\tau_u(\Psi) - \tau_u(\Psi = 0)}{\tau_u(\Psi = \Psi_{\max}) - \tau_u(\Psi = 0)} \quad (32)$$

where $\Psi_{\max}$ denotes the maximum finite stiffener rigidity considered in the parametric analyses.

Fig. 8 presents the μ_u vs. d/w_{su} results derived from the FE solutions given by Fortune [13] for plates subjected to pure shear. Marked on the μ_u vs. d/w_{su} curves are the positions for $\mu_u=0.95$. It can be seen that d/w_{su} for $\mu_u=0.95$ remains at the start of the plateau of the μ_u vs. d/w_{su} curve. Similar results are also found for other d/a and b/t ratios.

The values of d/w_{su} corresponding to $\mu_u=0.95$, interpolated from the FE results, are shown against the panel aspect ratio d/a in Fig. 9, where it is noted that a single value of the limit of d/w_{su} may be specified for a given d/a and different values of b/t .

For design of transverse stiffeners, it is proposed that the stiffener deflection at the ultimate load, w_{su} , calculated using Eq. (31), should be less than the design limit of the stiffener deflection, denoted as w_{sd} , that is:

$$w_{su} \leq w_{sd} \quad (33)$$

where w_{sd} for double-sided (concentric) stiffeners is given by:

$$\frac{w_{sd}}{d} = \begin{cases} \frac{1}{500} \left(\frac{b}{d} \right), & \text{for } \frac{d}{a} \leq 2 \\ \frac{1}{4000} \left(7 - 6 \frac{a}{d} \right), & \text{for } 2 < \frac{d}{a} < 3 \\ \frac{1}{800}, & \text{for } \frac{d}{a} \geq 3 \end{cases} \quad (34)$$

The proposed design limit of w_{su}/d given by Eq. (34) is also shown in Fig. 9. It is noted that for $d/a=2$, $b/t=80$, the proposed value ($w_{su}/d=1/1000$) is less than that for $\mu_u=0.95$. Nevertheless, it was found in Ref. [5] that a plate strength within the plateau of the μ_u vs. d/w_{su} curve can still be achieved for $w_{su}/d=1/1000$.

In Fig 5, the stiffener rigidity for which the stiffener deflection at the ultimate load is equal to the deflection limit specified in Eq. (34), denoted as Ψ_w , is also indicated. The value of Ψ_w is estimated by the interpolation of the finite element solutions. It can be seen in Fig. 5 that Ψ_w lies within the plateau of the τ_u' vs. Ψ curve. It should be noted that Ψ_w is from finite element solutions, not from the design model.

It is also proposed that the design limit of w_{su}/d given by Eq. (34) is valid for plates subjected to combined shear and longitudinal loading, because the ratio of w_{su}/d specified in Eq. (34) represents the requirement of the stiffener straightness for the effective subdivision of the plate for double-sided stiffeners.

3.2.3. Required Stiffener Size According to the Stiffness Criterion

In this section, the flexural rigidity of the smallest stiffener (double-sided, $h/t_s=10$) which satisfies the proposed stiffness criterion in Eqs. (33) and (34), denoted as Ψ_{dl} , is compared with the ultimate strength results from the non-linear elasto-plastic FE analyses. The stiffener moment M_s was calculated using Eq. (22). The applied load was taken as the ultimate load of the simply supported panel, and stiffener sizes for plates under different loading conditions were determined.

- ***Plates subjected to pure shear***

The comparisons of the required stiffener flexural rigidity Ψ_{dl} with the FE results of the ultimate shear stress ratio τ_u' vs. Ψ are presented in Fig. 5 for plates subjected to pure shear for $b/t=180$ and $d/a=1$ and 2, where the smallest stiffener which satisfies the strength criterion, Ψ_{dz} , is also included. A more extensive comparison is given in Ref. [5]. It was found that for the cases considered, the stiffener with a rigidity of Ψ_{dl} , determined in accordance with the proposed stiffness criterion, can ensure that the plate is effectively divided, and consequently a plate strength within the plateau of the τ_u' vs. Ψ curve, and greater than the strength for simply supported panels, is attained.

By comparison of the required stiffener rigidities Ψ_{dz} and Ψ_{dl} , it was found in Ref. [5] that for the cases examined for $d/a=0.5$ and 1, the adequate design size of the stiffener, denoted as Ψ_d ($\Psi_d=\max\{\Psi_{dz}, \Psi_{dl}\}$),

is governed by the strength criterion, whereas for all the other cases, the stiffness criterion governs. It was also observed that the design stiffener rigidity Ψ_d is in general at a satisfactory distance along the plateau of the τ_u vs. Ψ curve.

The above observation suggests that in order to provide an adequate stiffener design, both the strength and stiffness criteria are required. The two criteria are necessary when considering the possible combinations of stiffener properties. For example, if the ratio Z_s/I_s is increased, or if the yield stress of stiffener is increased, the strength criterion is more likely to produce an unsatisfactory stiffener. On the other hand, if the ratio Z_s/I_s is reduced, for example, by replacing a flat bar stiffener with a Tee stiffener, the stiffness criterion is more likely to result in an unsatisfactory stiffener. In the proposed design model, both the strength and stiffness criteria are used to quantify the required stiffener.

- ***Plates subjected to combined loading***

In Fig. 6, the required stiffener size Ψ_{dl} according to the proposed stiffness criterion is compared with the ultimate strength results from FE analyses for the cases discussed in Section 3.1.3. It is apparent that for the cases considered, for the required stiffener Ψ_{dl} , the plate strength is within the plateau region of the strength curve, and is very close to or greater than the strength for simply supported panels.

It was found in Ref. [5] that for the cases examined for combined loading, the design size of the stiffener Ψ_d is governed by the stiffness criterion, and Ψ_d is at a reasonable distance along the plateau of the strength curve. By examining the contour plots of the out-of-plane deflection at the ultimate load, it was seen that for the stiffener rigidity Ψ_{dl} , an effectively divided panel mode is achieved ([5]).

In Table 2, the numerical values of Ψ_{dZ} and Ψ_{dl} required for the combined loading are compared with those for pure shear. It is observed that for the same stiffener type, the design stiffener rigidity Ψ_d for the combined loading conditions may be considerably less than that required for the same plate to achieve its pure shear strength. This suggests that it would be considerably conservative if the stiffener designed to develop the maximum shear capacity of the panel is used for other combinations of shear and compression.

3.3. Design of Single-Sided Stiffeners

For single-sided stiffeners, a variety of assumptions in respect of the neutral axis and the effective section of the stiffener are made in existing design methods. According to BS5400 [2] and Eurocode 3 [3], the effective cross-section of the stiffener consists of stiffener section itself and a width of plate. In some other methods, as that proposed by Horne and Grayson [14], only the actual stiffener section is considered, but the neutral axis of the stiffener section is taken to be at the surface of the plate. They suggested that with the moment of inertia I_s calculated about an axis at the surface of the plate, the single-sided stiffener can be designed to achieve the same flexural rigidity Ψ as required for double-sided stiffeners.

In this section, based on the numerical results for plates with a single-sided flat bar stiffener ($h_s/t_s=10$), a method for calculating the stresses in the single-sided stiffener is proposed. Special design requirements for single-sided stiffeners are discussed.

3.3.1. Calculation of Stiffener Stress

It has been pointed out in Section 3.1.1 that a single-sided stiffener is to be designed as a simple beam loaded by a bending moment M_s (from the lateral distributed force) and an eccentric axial load P_s acting at a distance e , $e=(h_s+t)/2$ for flat bar stiffeners, from the centroidal axis $I-I$ (Fig. 4) of the stiffener cross-section (Fig. 1). The eccentric load P_s is statically equivalent to a force P_s applied at the centroid plus a couple equal to $P_s \cdot e$. To calculate the maximum stress in the stiffener, the two possible combinations of the stiffener forces should be considered for the design of the eccentric stiffener. It is recalled that according to the proposed design model, the stiffener cross-section consists of the actual stiffener section only.

An adequate stiffener should be rigid enough to divide the plate, and the axial force in such a stiffener is always a compressive force as given in Eq. (1). In Fig. 10, two possible combinations of the axial force P_s and the bending moment M_s are illustrated. For the case 1 (Fig. 10a), where the tip of the stiffener is in compression, the stiffener forces with respect to the centroidal axis $I-I$ are given by:

$$P_{s,1} = P_s, \quad M_{s,1} = M_s - P_s \cdot e \quad (35)$$

and the maximum compressive and tensile stresses can be calculated by:

$$\sigma_{s,C} = \frac{P_s}{A_s} + \frac{(M_s - P_s \cdot e)}{Z_{s,C}} \quad (36)$$

$$\sigma_{s,T} = -\frac{P_s}{A_s} + \frac{(M_s - P_s \cdot e)}{Z_{s,T}} \quad (37)$$

where $Z_{s,C}$ and $Z_{s,T}$ are the section moduli for the compressive and tensile edges of the stiffener, respectively.

For the case 2 (Fig. 10b), where the tip of the stiffener may be in tension, the stiffener forces with respect to the centroidal axis $I-I$ can be written as:

$$P_{s,1} = P_s, \quad M_{s,1} = M_s + P_s \cdot e \quad (38)$$

and the maximum compressive and tensile stresses can be calculated by:

$$\sigma_{s,C} = \frac{P_s}{A_s} + \frac{(M_s + P_s \cdot e)}{Z_{s,C}} \quad (39)$$

$$\sigma_{s,T} = -\frac{P_s}{A_s} + \frac{(M_s + P_s \cdot e)}{Z_{s,T}} \quad (40)$$

The FE solutions presented by Xie [5] demonstrate that for plates of $d/a=2$, the stress at the tip of the stiffener is always more compressive than at the plate edge at the ultimate load, while for plates of $d/a=5$ the tip of the stiffener is in tension, and the edge in contact with the plate is in compression. It is also noted that the stiffener bending moment $M_{s,1}$ in case 2 is greater than that in case 1 (Fig. 10).

In view of the above discussion, it is suggested that for the calculation of the maximum stresses in the single-sided stiffener, Eqs. (35) to (37) should be used for plates of $d/a \leq 2$, while for plates of $d/a > 2$, Eqs. (38) to (40) should be used. It should be pointed out that this suggestion is based on a small number of cases, and it is necessary that further verification should be made over a wide range of d/a and b/t .

3.3.2. Stiffener Deflection and Its Design Limit

For single-sided stiffeners, the stiffener deflection can still be estimated using Eqn. (31), which is derived from simple beam theory by assuming that the stiffener deflects in a half sine wave. The moment of inertia of the stiffener section I_s should be taken about the centroidal axis of the stiffener section. For flat bar stiffeners, I_s is given by Eq. (26).

The bending moment M_s in Eq. (31) should be replaced with the moment about the centroidal axis $I-I$ of the stiffener section, $M_{s,1}$. It is proposed in Section 3.3.2 that for $d/a \leq 2$, $M_{s,1}$ is given by Eq. (35), while for $d/a > 2$, $M_{s,1}$ is given by Eq. (38).

Fig. 11 presents the τ_u vs. d/w_{su} curves obtained from the FE analyses carried out for plates of $d/a=2, 5$ and $b/t=180$, with a single-sided flat bar stiffener ($h_s/t_s=10$) of varying flexural rigidity Ψ , subjected to pure shear. It is observed that by taking $d/w_{su}=500$ for $d/a=2$ and 400 for $d/a=5$ respectively, the required plate strength can be achieved. It is noted that these two values of d/w_{su} are equal to the twice the limit specified in Eq. (34).

For design of single-sided stiffeners, it is thus proposed that the design limit of the stiffener deflection, w_{sd} , can be taken as the twice the value proposed for the double-sided stiffeners (Eq. 34). That is, the value of w_{sd} for the single-sided stiffener can be expressed as:

$$\frac{w_{sd}}{d} = \begin{cases} \frac{1}{250} \left(\frac{b}{d} \right), & \text{for } \frac{d}{a} \leq 2 \\ \frac{1}{2000} \left(7 - 6 \frac{a}{d} \right), & \text{for } 2 < \frac{d}{a} < 3 \\ \frac{1}{400}, & \text{for } \frac{d}{a} \geq 3 \end{cases} \quad (41)$$

As for the suggestions made in Section 3.3.1, it is recognised that Eq. (41) is also based on a small number of cases, and further verification over a wide range of d/a and b/t is needed.

3.3.3. Required Stiffener Size

Plates subjected to pure shear are considered in this section. Table 3 presents the smallest stiffeners (single-sided, $h_s/t_s=10$) which satisfy the proposed strength criterion (Eq. 29) and the stiffness criterion (Eq. 33), represented by Ψ_{dZ} and Ψ_{dI} respectively. In Fig. 12, Ψ_{dZ} and Ψ_{dI} are compared with the finite element results of τ_u' vs. Ψ . It is reiterated that for design of the stiffener, the applied load was taken as the ultimate load of the simply supported panel.

It can be observed in Fig. 12 that the required stiffener rigidity Ψ_{dZ} is greater than Ψ_c for both cases considered, which implies that the stiffener of rigidity Ψ_{dZ} does not yield at the ultimate load. It is also seen that the required stiffener Ψ_{dZ} can ensure the plate strength to be greater or nearly equal to the panel strength. It is noted that for $d/a=2$, Ψ_{dZ} is smaller than Ψ_{ssu} , but the difference in the plate strengths for the two stiffener rigidities is negligible. However, by reference to the contour diagrams of the deflection modes at the ultimate load, it was found that the required stiffener according to the strength criterion, Ψ_{dZ} , does not produce an effectively divided panel mode (Ref. [5]).

As for the required stiffener in accordance with the stiffness criterion, it is apparent in Fig. 12 that the stiffener with a rigidity of Ψ_{dI} can ensure that the plate strength lies within the plateau of the τ_u' vs. Ψ curve, and is greater than the strength for simply supported panels. By referring to the contour plots, it was observed that the stiffener with a rigidity of Ψ_{dI} results in an effectively divided panel mode [5].

The design size of the stiffener is again taken as $\Psi_d=\max\{\Psi_{dZ}, \Psi_{dI}\}$, which is considered to be satisfactory for the cases considered. Nevertheless, the design equations proposed for single-sided stiffeners should be further validated over a wider range of cases.

4. Stiffener Design Procedure

For designing a transverse stiffener for a plate subjected to coexistent shear and direct in-plane loading (Fig. 1), the following variables are considered:

- *Components of the ultimate strength for simply supported panels: τ_u , σ_{cu} , σ_{bu} ;*
- *External stiffener forces for load bearing stiffeners: Q_1 (at one end), Q_2 (at both ends);*
- *Panel dimensions: a , d , t ;*

- *Initial imperfections: w_{0p} , w_{0s} , measured as in Fig. 3 of Ref. [10];*
- *Yield stresses of panel and stiffener material: σ_{0p} , σ_{0s} ;*
- *Cross-sectional properties of the stiffener: A_s , I_s , Z_s (see Section 3.3) and e (see Fig. 10);*

The calculation of the ultimate strength of simply supported panels is not within the scope of the present study. Formulae for predicting the strength of individual panels subjected to shear and compressive loading have been given by Davidson [15]. For the panel shear strength τ_u^0 (τ_u for pure shear), the predicted values are about 3~4% greater than the FE solutions [13]. Therefore, it is recommended that in the proposed stiffener design method, the shear strength of panels may be calculated using the formulae given by Davidson [15]. For panels subjected to combined loading, the components of the ultimate panel strength (τ_u , σ_{cu} , σ_{bu}) may be evaluated by using the known ratio between the applied stresses and an available method in the design codes, such as the interaction equation in BS5400 [2].

4.1. Intermediate Stiffeners

The proposed design method for intermediate transverse stiffeners is summarized below.

- (1) Calculate the critical buckling stresses for a simply supported panel, subjected to shear, compression and bending alone, using Eqs. (4), (5) and (6), respectively.
- (2) Calculate the critical shear stress τ_{cr} in the presence of compression and bending using the known ratio between the applied stresses and the interaction equation (3).
- (3) Calculate the equivalent shear stress τ_{eq} using Eq. (10), with the reduction factors β_c and β_b given by Eqs.(11) and (12).
- (4) Calculate the elastic knife edge moment M_{km} using Eq. (9), with τ_{eq} obtained in Step (3).
- (5) Calculate the plastic factor F_{pl} using Eq. (8), and the stiffener bending moment due to the panel bending effect M_{pb} using Eq. (7).
- (6) Calculate the design limit of the stiffener deflection, w_{sd} , using Eq. (34) for double-sided stiffeners, and Eq. (41) for single-sided stiffeners.
- (7) Choose a trial stiffener section and calculate A_s , I_s , Z_s , and e (for single-sided stiffeners). The stiffener properties are defined in Eqs. (23) to (26). The trial stiffener size may be chosen in terms of experience. A suggested trial stiffener is the smallest stiffener which satisfies the following criteria:

$$Z_s \geq \frac{M_{km} + M_{ov}^{KE}}{0.8 \sigma_{0s}} \quad (42)$$

$$I_s \geq \left(M_{km} + M_{ov}^{KE} \right) \frac{d^2}{\pi^2 E w_{sd}} \quad (43)$$

$$\begin{aligned} M_{ov}^{KE} &= \lim_{\Psi \rightarrow \infty} M_{ov} \\ &= 2a \left(\frac{d^2}{4a^2} + 1 \right) \cdot \tau t w_{0s} / \sqrt{8.155 \left(\frac{d^2}{4a^2} + \frac{4a^2}{d^2} \right) + 13.99} \end{aligned} \quad (44)$$

with:

- In Eq. (42), the stiffener axial force P_s is not included, but allowance for P_s is made for the trial by reducing the stiffener yield stress σ_{0s} by 20%.
- (8) Calculate the axial force P_s in the stiffener using Eq. (1), with the applied shear stress τ and the buckling shear stress τ_{cr} obtained in Step (2).
 - (9) Calculate the stiffener bending moment due to the overall buckling effect M_{ov} , using Eq. (13), with the overall buckling stresses τ_{ov} , $\sigma_{c,ov}$ and $\sigma_{b,ov}$ given by Eqs. (14), (15) and (16), respectively.
 - (10) Calculate the total stiffener moment $M_s = M_{pb} + M_{ov}$ (Eq. 22).
 - (11) Calculate the maximum stress in the stiffener $\sigma_{s,max}$. For double-sided stiffeners, Eq. (27) is used. For single-sided stiffeners, Eqs. (35) to (37) should be used for $d/a \leq 2$, and Eqs. (38) to (40) for $d/a > 2$.
 - (12) Calculate the stiffener deflection at the ultimate load, w_{su} , using Eq. (31). For single-sided stiffeners, the moment M_s in Eq. (31) should be replaced with the moment about the centroid axis $I-I$ of the stiffener section, $M_{s,1}$, as given by Eq. (35) for $d/a \leq 2$, and Eq. (38) for $d/a > 2$.
 - (13) Check the strength and stiffness criteria, given in Eqs. (29) and (33), respectively.
 - (14) If either the strength criterion (Eq. 29) or the stiffness criterion (Eq. 33) is not satisfied, then the chosen stiffener size must be increased. Conversely, if both criteria are more than satisfied, then the chosen stiffener size may be reduced. Subsequent stiffener sizes may be checked by returning to step (8) in the design procedure.

The above procedure may be converted into a computer program, from which the optimum stiffener size can be obtained. An optimum stiffener is defined by the optimal model:

$$\begin{aligned}
 &\min A_s \\
 &\text{subjected to: } \sigma_{s,\max} \leq \sigma_{0s} \\
 &\quad w_{su} \leq w_{sd} \\
 &\quad A_s > 0
 \end{aligned} \tag{45}$$

4.2. Special Considerations for Load Bearing Stiffeners

As required in BS5400, the section of a load bearing stiffener should be chosen to be double-sided and symmetric about the mid-plane of the plate.

For a stiffener loaded with external forces, either at one end or both ends, the total stiffener axial force $P_{s,\text{total}}$ is given by Eq. (28), and the stiffener bending moment due to the overall buckling effect M_{ov} should be calculated using Eq. (17) or Eq. (19), which takes into account the destabilising effect caused by the external forces.

The applied external force is resisted by the stiffener itself and by part of the plate together. Nevertheless, since the panel is not designed to resist the external force applied at the end of the stiffener, it is consistent to assume that the external force is all taken by the stiffener itself. Therefore, it is suggested that when designing a load bearing stiffener, only the stiffener section should be considered.

5. Comparisons of the Proposed Model with Existing Methods

In this section, comparisons of the required stiffener sizes are made between the proposed model and several existing design methods, including three design codes (namely, BS5400 [2], Eurocode 3 [3] and AASHTO [4] and the methods proposed by Horne & Grayson [14], by Rahal & Harding [8], and by Stanway, Chapman & Dowling [9]. Comparisons are made mainly for plates with a double-sided stiffener, subjected to pure shear, but single-sided flat bar stiffeners are also considered. For all the results described in this section, a flat bar stiffener with a side ratio of $h_s/t_s=10$ (for both double-sided and single-sided, as shown in Fig. 1) is assumed.

5.1. Double-Sided Stiffeners

Table 4 lists the minimum values of the stiffener flexural rigidity, denoted as Ψ_d , required according to the proposed model and the existing design methods for stiffened plates with a double-sided flat bar stiffener. Plates having a panel aspect ratio $d/a=0.5, 1, 2, 3, 5$ & 7 , and a slenderness ratio $b/t=80, 120$ & 180 , respectively, were considered.

To demonstrate the adequacy of the stiffeners given by the various design methods considered, the Ψ_d values were compared with the FE τ_u' vs. Ψ curves for all the cases considered ([5]). As an illustration, Fig. 13 shows the comparisons made for the cases of $d/a=0.5$ to 3 with $b/t=180$. It has been pointed out that the basic requirement for an adequate stiffener is to ensure that the ultimate strength for the stiffened plate is at least as great as the strength for the associated simply supported panel, represented by:

$$\Psi_d \geq \Psi_{ssu} \quad (46)$$

Also, an adequate stiffener should ensure that the plate strength is within the plateau of the τ_u' vs. Ψ curve.

5.2. Single-Sided Stiffeners

In Table 5, the minimum values of the stiffener flexural rigidity, Ψ_d , required according to the various design methods are listed for stiffened plates of $d/a=2, 5$ and $b/t=180$ with a single-sided flat bar stiffener having $h_s/t_s=10$, subjected to pure shear. In Fig. 14, the values of Ψ_d for different design methods are compared with the finite element results of τ_u' vs. Ψ . Also shown in Fig. 14 are the values of Ψ_{ssu} .

5.3. Comments

The results presented above demonstrate that there is a wide difference in the stiffeners required by the various existing design methods for a given plate geometry. This in itself was one of the strong reasons for carrying out the present investigation.

From the validation against the FE results, it was found that the proposed design model gives consistently satisfactory stiffeners for all the cases examined ([5]), without being overly conservative. Of all the design methods considered, the proposed model is the only approach which gives adequate results for both double-sided and single-sided stiffeners. In comparison with BS5400 and Eurocode, the proposed model gives in general heavier stiffeners for $d/a \leq 2$ plates, and considerably lighter stiffeners for $d/a \geq 5$ plates.

For the double-sided stiffeners, it was found that the three design codes examined could give inadequate stiffeners in some cases. AASHTO gives satisfactory stiffeners for $b/t=80$, where the post-buckling behaviour of the plate is not important. For slender plates (e.g. $b/t \geq 180$), AASHTO severely underestimates the required stiffeners. BS5400 gives inadequate stiffeners for $d/a \leq 2$, but unduly conservative stiffeners for very tall plates (e.g. $d/a \geq 5$). Eurocode underestimates the stiffeners required for $d/a \leq 1$ and $b/t=180$. For $d/a \geq 5$ plates, the stiffener from Eurocode is unduly conservative. For tall plates with $b/t=180$, Eurocode gives stiffeners which are considerably greater than those from BS5400, due to the difference in the magnitude of the axial force caused by tension field action.

For the single-sided stiffener cases examined, the proposed model gives satisfactory results. However, it is recognised that more cases should be studied in the future.

6. Conclusions

Based on the non-linear buckling theory and FE solutions, a rational and usable design model for transverse stiffeners is proposed. The proposed design model requires that the transverse stiffener be designed to resist the coexistent bending moment and axial force in the stiffener. The bending moment is determined by adding the components of stiffener moment due to the local panel bending effect and the tendency for the stiffened plate to buckle overall. The axial force caused by the tension field action acts in the mid-plane of the plate, resulting in an additional bending moment for eccentric stiffeners due to the eccentricity of this axial force. For eccentric stiffeners the axial force may contribute up to about 70% of the total stress. This occurs when the plate bending effect acts in opposition to the eccentricity moment. The proposed method gives forces acting in the stiffener itself, which does not require the effective width

of the plate to be considered. The moment of inertia is calculated about the centroidal axis of the actual stiffener section for both concentric and eccentric stiffeners.

Both strength and stiffness criteria were developed to quantify the required stiffener. The strength criterion requires that the maximum stress in the stiffener must not exceed the yield stress of the stiffener. The stiffness criterion requires that the stiffener deflection is sufficiently small to ensure that the stiffened plate is effectively divided.

The proposed design model has been validated against experimental results and FE solutions for stiffened plates and girders ([5]). It is found that the proposed design model gives satisfactory stiffeners for concentric and eccentric stiffeners, and for pure shear, shear and compression, and shear and bending, respectively. When compared with existing design methods, it is found that the proposed model gives more consistent stiffener sizes in relation to those required to satisfy the design criteria. The extent of the variability of existing methods can be seen in Tables 41 and 5, and Figures 13 and 14.

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NOTATION

a	panel width
A_s	cross-sectional area of stiffener
b	smaller panel dimension, $b=\min\{a, d\}$
d	panel depth and length of stiffener
D	flexural rigidity of plate, $D=Et^3/12(1-\nu^2)$

e	distance between the central line of the plate section and the centroidal axis of the cross-section of a single-sided stiffener
E	modulus of elasticity ($E=2.05 \times 10^5 \text{ N/mm}^2$)
F_{pl}	empirical factor for the effect of panel plasticity on the knife edge moment
h	$h_s+t/2$ (Fig. 1)
h_s	depth of stiffener on one side of plate (Fig. 1)
I_s	moment of inertia of stiffener alone
k_{cr}	critical buckling stress coefficient
M_k	bending moment in straight knife edge, derived from the lateral distributed force q_k
M_{km}	maximum magnitude of bending moment M_k along the straight knife edge
M_{pb}	stiffener bending moment due to the effect of local panel bending, $M_{pb}=F_{pl} \cdot M_{km}$
M_{ov}	stiffener bending moment due to the overall buckling effect
M_s	stiffener bending moment
P_s	axial force in stiffener, $P_s > 0$ for compression
Q_1	external force acting at one end of the stiffener
$Q_{1,ov}$	overall buckling load for plate subjected to external force acting at one end of the stiffener
Q_2	external force acting at both ends of the stiffener
$Q_{2,ov}$	overall buckling load for plate subjected to external force acting at both ends of the stiffener
t	thickness of plate panel
t_s	thickness of stiffener
w_p	additional panel deflection
w_{0p}	magnitude of initial panel deflection
w_{0s}	magnitude of initial stiffener deflection
w_{sd}	the design limit of the additional stiffener deflection
w_{su}	additional stiffener deflection at the ultimate load of stiffened plates
α	panel side ratio defined by $\alpha=\max(d/a, a/d) \geq 1$
β_b	equivalent shear stress factor for bending
β_c	equivalent shear stress factor for compression
ν	Poisson's ratio ($\nu=0.3$)
σ_b	applied bending stress (Fig.1)

σ_{bcr}	critical buckling bending stress
σ_{bcr}^0	critical bending stress of plate panel, subjected to pure bending
$\sigma_{b,ov}$	overall buckling bending stress of stiffened plate
σ_c	applied compressive stress (Fig.1)
σ_{ccr}	critical buckling compressive stress
σ_{ccr}^0	critical compressive stress of plate panel, subjected to compression alone
$\sigma_{c,ov}$	overall buckling compressive stress of stiffened plate
σ_{0p}	yield stress of plate panel
σ_{0s}	yield stress of stiffener
$\sigma_{s,max}$	maximum stress in stiffener
τ	applied shear stress (Fig. 1)
τ_{cr}	critical buckling shear stress
τ_{cr}^0	critical buckling shear stress of plate panel, subjected to pure shear
τ_{eq}	equivalent shear stress for combined shear and longitudinal loading conditions
τ_{ov}	overall buckling shear stress of stiffened plate
τ_u	ultimate shear stress
τ_u'	ultimate shear stress ratio, τ_u/τ_{0p} , with $\tau_{0p} = \sigma_{0p}/\sqrt{3}$
Ω	stiffener extensional rigidity parameter, $\Omega=A_s/at$
Ψ	stiffener flexural rigidity parameter, $\Psi=EI_s/Db$
Ψ_d	Ψ of the smallest stiffener required, $\Psi_d=\max\{\Psi_{dZ}, \Psi_{dI}\}$
Ψ_{dZ}	Ψ of the smallest stiffener which satisfies the strength criterion (Eq. 29)
Ψ_{dI}	Ψ of the smallest stiffener which satisfies the stiffness criterion (Eq. 33)
Ψ_e	Ψ of the smallest stiffener, of those examined, which remains fully elastic at the ultimate load of stiffened plates
Ψ_p	Ψ of the largest stiffener, of those examined, which has yielded at the ultimate load of stiffened plates
Ψ_{ssu}	the stiffener rigidity required to develop the ultimate strength of simply supported panels

Table 1 Overall Buckling Model for Individual Loading Conditions

Loading Condition	Beam-Plate Model	Deflection Function w
Pure Shear		$w = A_1 \cos \frac{\pi x}{2a} \cos \frac{\pi y}{d} + A_2 \frac{x}{a} \frac{y}{d} \cos \frac{\pi x}{2a} \cos \frac{\pi y}{d}$
Compression		$w = A_1 \cos \frac{\pi x}{2a} \cos \frac{\pi y}{d}$
Pure Bending		$w = a_1 \sin \frac{\pi y}{d} \sin \frac{\pi x}{2a} + a_2 \sin \frac{2\pi y}{d} \sin \frac{\pi x}{2a}$
External Force at One End		$w = a_1 \sin \frac{\pi y}{d} \sin \frac{\pi x}{2a} + a_2 \sin \frac{2\pi y}{d} \sin \frac{\pi x}{2a}$
External Force at Two Ends		$w = a_1 \sin \frac{\pi x}{2a} \sin \frac{\pi y}{d}$

Table 2 Comparison of the required stiffener rigidities Ψ_{dz} and Ψ_{dl} for combined loading conditions. $d/a=2$, $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$

$\sigma_c:\tau$ or $\sigma_b:\tau$	Ψ_{dz}	Ψ_{dl}	Ψ_d	$\Psi_d/\Psi_{d, \text{shear}}$
(a) Combined shear and compression conditions, $a/t=80$				
1:0	23.9	81.2	81.2	0.306
1:2	88.9	157.9	157.9	0.594
1:5	42.4	138.8	138.8	0.522
0:1	98.3	265.7	265.7	1.0
(b) Combined shear and bending conditions, $a/t=180$				
1:0	19.15	143.6	143.6	0.135
2:1	97.93	328.9	328.9	0.309
1:1	146.95	505.8	505.8	0.476
0:1	375.1	1063.5	1063.5	1.0

Table 3 Required stiffener rigidities Ψ_{dz} and Ψ_{dl} for single-sided stiffeners. Pure shear. $b/t=180$, $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$. $\Psi=EI_s/Db$, I_s as in Eq. (26)

d/a	Ψ_{dz}	Ψ_{dl}
2	131.4	213.7
5	897.6	3488

Table 4 Required stiffener flexural rigidities Ψ_d . Double-sided flat bar stiffeners ($h_s/t_s=10$)
Pure shear. $\sigma_{op}=\sigma_{os}=245\text{N/mm}^2$. $\Psi=EI_s/Db$, I_s as in Eq. (24)

b/t	d/a	Proposed Model (Note 1)	BS5400 (Note 2)	Eurocode (Note 2)	AASHTO (Note 2)	Horne & Grayson	Rahal & Harding (Note 1)	Stanway et al (Note 1)
80	0.5	11.27	0.888	8.183	10.92	9.876	12.57	13.07
	1	6.037	2.422	16.33	5.460	9.876	7.793	11.17
	2	131.9	45.39	130.8	87.36	66.34	57.85	150.9
	3	240.7	233.2	442.5	223.9	119.5	194.4	431.0
	5	580.4	1838	2055	660.7	225.8	999.5	700.2
	7	1105	7000	5617	1316	332.2	2619	1774
120	0.5	24.04	5.757	8.187	10.92	29.81	13.94	27.62
	1	16.10	8.180	16.38	5.460	29.81	13.51	29.61
	2	294.7	96.84	130.9	87.36	119.5	71.06	309.4
	3	444.3	388.1	442.1	223.9	199.3	232.7	648.8
	5	1024	2308	2447	660.7	358.8	1001	1019
	7	2037	7911	6685	1316	518.3	2976	2622
180	0.5	53.09	10.12	8.645	10.92	59.72	13.23	59.36
	1	42.25	22.04	16.40	5.460	59.72	16.51	58.85
	2	587.9	192.5	494.6	87.36	199.3	72.84	609.2
	3	714.6	598.4	2149	230.0	318.9	221.0	1022
	5	1588	2796	9946	869.9	558.1	841.0	1554
	7	3090	8645	24677	1903	797.4	2088	3888

Note 1: Applied shear taken as the panel strength τ_u from FE analysis.

Note 2: Applied shear taken as the panel strength τ_u , calculated from the same code.

Table 5 Required stiffener flexural rigidities Ψ_d . Single-sided flat bar stiffeners ($h_s/t_s=10$)
Pure shear. $\sigma_{op}=\sigma_{os}=355\text{N/mm}^2$. $\Psi=EI_s/Db$, I_s as in Eq. (26)

b/t	d/a	Proposed Model (Note 1)	BS5400 (Note 2)	AASHTO (Note 2)	Horne & Grayson	Rahal & Harding (Note 1)	Stanway et al (Note 1)
180	2	213.7	346.2	407.3	62.0	25.78	217.5
	5	3488	3821	3429	170.0	813.4	453.2

Note 1: Applied shear taken as the panel strength τ_u from FE analysis [1].

Note 2: Applied shear taken as the panel strength τ_u , calculated from the same code.

Fig. 1 Two-panel stiffened plate subjected to combined shear, compression and bending

Fig. 2 Comparisons of the knife edge moments between the proposed equations and FE solutions. Coexistent shear, compression and bending. Plates with a straight knife edge. τ_{eq} as in Eq. (10).

Fig. 3 Comparison of the elastic stiffener moment from the proposed model with the FE solutions.

Plates with an imperfect stiffener. $b/t=180$. $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$. Pure shear

Fig. 4 Design forces acting in the stiffener

Fig. 5 Comparison of stiffener requirements from the proposed model, with the τ_u' vs. Ψ results from FE analyses, given by Fortune [13], with $\sigma_{0p}=\sigma_{0s}=245\text{N/mm}^2$. $\Psi=EI_s/Db$. Pure shear

Fig. 6 Comparison of stiffener requirements from the proposed model, with the τ_u' vs. Ψ results from FE analyses, with $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$. $d/a=2$, $b/t=180$. $\Psi=EI_s/Db$. Combined loading conditions.

Fig. 7 Comparison of w_s vs. M_s curves from finite element analyses with Eq. (31). Pure shear.

Initial imperfection w_0 as in Fig. 3 of Ref. [10]. $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$. $\Psi=EI_s/Db$

Fig. 8 Variation in the ultimate strength ratio μ_u with the stiffener deflection at the ultimate load w_{su} . Produced from the FE solutions by Fortune [13], with $\sigma_{op}=\sigma_{os}=245\text{N/mm}^2$. μ_u as in Eq. (32). Pure shear

Fig. 9 Values of the stiffener deflection at the ultimate load w_{su} for $\mu_u=0.95$. Pure shear. Proposed design limit of the stiffener deflection as given in Eq. (34). μ_u as in Eq. (32)

(a) Case 1, for $d/a \leq 2$

(b) Case 2, for $d/a > 2$
Fig. 10 Possible combinations of the axial force and the bending moment in a single-sided stiffener.

Fig. 11 Relationship of the ultimate shear strength and the stiffener deflection at the ultimate load. Plates with a single-sided stiffener. w_0 : type B for $d/a=2$; type A for $d/a=5$, as in Fig. 3 of Ref. [10]

Fig. 12 Comparison of stiffener requirements from the proposed model, with the τ_u' vs. Ψ results from FE analyses. $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$. Single-sided stiffeners, $\Psi=EI_s/Db$, I_s as in Eq. (26). Pure shear

Fig. 13 Comparison of the stiffener flexural rigidity Ψ_d required by the proposed model and the existing methods, with the FE τ_u' vs. Ψ results from [13]. Double-sided flat bar stiffeners ($h_s/t_s=10$)

$$\sigma_{0p}=\sigma_{0s}=245\text{N/mm}^2. \Psi=EI_s/Db, I_s \text{ as in Eqn. (24). Pure shear}$$

Fig. 14 Comparison of the stiffener flexural rigidity Ψ_d required by the proposed model and the existing methods, with the FE τ_u vs. Ψ results. Single-sided flat bar stiffeners ($h_s/t_s=10$) $\sigma_{0p}=\sigma_{0s}=355\text{N/mm}^2$.

$$\Psi=EI_s/Db, I_s \text{ as in Eq. (26). Pure shear}$$