A modified Hoek - Brown Failure Criterion for the Anisotropic Intact Rock

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Abstract: The Hoek - Brown criterion parameters (σ_{ci} , m_i and s) are significantly influenced by the strength anisotropy of intact rock. In the present study, the criterion was modified by incorporating a new parameter (k_β), to account for the effect of strength anisotropy, thus being able to determine the strength of intact anisotropic rock under loading in different orientations of the plane of anisotropy. The range of the parameter (k_β) for the rocks tested has been analytically investigated by carrying out triaxial tests, in different orientations of the foliation plane. The proposed modification was studied for metamorphic rocks (gneiss, schist, marble), but could also be applied to other rock types exhibiting “inherent” anisotropy, e.g. sedimentary as well as igneous rocks. The proposed modified criterion is intended for use for prediction of strength of intact rock, but can also be extended to rock masses.

Keywords: Intact rock, strength, anisotropy, failure criteria, metamorphic rocks, laboratory tests.

1. INTRODUCTION

The Hoek - Brown failure criterion [1] was initially proposed for the determination of the intact rock strength and the rock mass strength in isotropic conditions.

Nevertheless, in the last years, it is used for the determination of the strength of anisotropic rocks and rock masses.

In order to use this criterion for the prediction of strength in anisotropic intact rocks, careful selection of its parameters is necessary, namely the uniaxial compressive strength (σ_{ci}) and the material constants m_i and s .

Moreover, the determination of the minimum and maximum intact rock strength due to rock anisotropy, is very important in the selection of characteristic values of rock for the design and construction of civil engineering works e.g. tunnels, foundations and rock slopes.

2. ANISOTROPY OF INTACT ROCK

2.1. Nature of anisotropy

Earlier work on anisotropic rocks has revealed that: a) anisotropy stems mainly from schistosity, foliation and cleavage, for metamorphic rocks, and bedding planes and lamination, for sedimentary rocks, and b) anisotropy curve of the uniaxial and triaxial compressive strength is usually U-shaped e.g. [2]; [3]; [4]; [5]; [6]; [7]; [8]; [9].

The rocks that behave in an anisotropic way are mainly metamorphic rocks e.g. phyllites, shales, schists and slates, as well as gneisses due to the presence of banding. Marbles may also exhibit a low degree of anisotropy, but usually their behaviour is practically isotropic. Anisotropy in sedimentary rocks is common in siltstones, claystones and mudstones [10]. Furthermore, sandstones and limestones (as rock material) exhibit low anisotropy, due to the cementation of their rock-forming minerals.

Little information about strength anisotropy is available for rocks other than metamorphic and sedimentary. Anisotropy in igneous rocks can be encountered due to the presence of flow structure, e.g. in rhyolites [11].

In order to measure the intact rock strength, it is essential that no micro-fractures or other discontinuities are present in the specimen, which is tested. This can be achieved by taking intact samples of the rock from the surrounding rock mass. The difficulty of preparing samples with no visible fracture or discontinuity depends on the nature of the rock, its tectonic history and its geological environment. On the other hand, the structural features that are responsible for “inherent” anisotropy, discussed in the present work, mainly the alignment of rock-forming crystals in intact rock cannot be excluded from the samples tested in the laboratory when determining the rock strength. The “inherent” anisotropy due to the existence of bedding, foliation and schistosity is obvious in the macro-scale (in the order of meters), as presented in Figure 1a but its genesis originates from the micro-scale (in the order of millimeters), as shown in Figure 1b.

2.2. Degree of anisotropy

The variation of strength in uniaxial and triaxial loading conditions of intact rock with respect to the loading direction, β is defined as strength anisotropy. Its magnitude is described by the degree of anisotropy, R_C , which is defined as:

$$R_C = \frac{\sigma_{ci(90)}}{\sigma_{ci(min)}}, \text{ where } \sigma_{ci(90)} \text{ is the uniaxial compressive strength perpendicular to the}$$

planes of anisotropy, and $\sigma_{ci(min)}$ is the minimum value of σ_{ci} obtained from oriented samples under uniaxial compression, usually occurring when the principal loading axis (σ_1) forms an angle to the plane of foliation between $\beta=30^\circ$ and 45° . A

classification of strength anisotropy based on the ratio R_C , is given by Ramamurthy [12].

The plot of uniaxial compression tests on a graphitic phyllite [13], shown in Figure 2a, suggests that its strength is highly dependent upon the orientation of the schistosity relative to the direction of applied load. The maximum strength parallel to schistosity is of the order of 100 MPa while the minimum strength, with the schistosity inclined at approximately 30° to the direction of applied load, is approximately 15 MPa.

The decrease of the uniaxial compressive strength, σ_{ci} , for rocks exhibiting well developed foliation is in the order of two to five times. The effect of inclination of foliation on the intact rock strength remains significant also on weathered samples (Figure 2b), as it was shown by the tests carried out on phyllite samples by Sabatakakis & Tsiambaos [14].

Strength anisotropy remains significant when confined conditions exist, as it is seen by a large number of triaxial tests on Delabole slates [7] (Figure 3) and on Himalayan schists [15]. It has been found that the degree of strength anisotropy decreases with increasing confining pressure [2]. The ratio R_C , when calculated in terms of σ_1 , at the confined state (for σ_3 up to $\sigma_c/2$), describes the effect of confining pressure on strength anisotropy.

3. HISTORICAL REVIEW

3.1 Empirical Hoek - Brown failure criterion

The empirical failure criterion that was initially proposed by Hoek and Brown [1] for the intact rock is described by equation (1). Some modifications for its use in rock masses were recently done by Hoek et al. [16].

$$\sigma_1 = \sigma_3 + \sigma_{ci} \cdot \left(m \frac{\sigma_3}{\sigma_{ci}} + s \right)^\alpha \quad (1)$$

where, σ_3 is the minor principal stress

σ_1 is the major principal stress

σ_{ci} is the uniaxial compressive strength

m , s , α are material constants, m is equal to m_i , $\alpha=0.5$ and $s = 1$, when the rock is intact.

The behaviour of intact anisotropic rocks at failure depends upon the orientation of the planes of anisotropy in relation to the principal loading axis. The determination of uniaxial compressive strength, in these types of rocks, presents many difficulties.

The determined values of the Hoek - Brown parameters for intact rock (σ_{ci} , m_i , s) are defined based on the results of uniaxial and triaxial tests, when loading is applied perpendicular to the planes of anisotropy (foliation or bedding). The value of σ_{ci} and m_i will be significantly different in the case that failure will occur at the direction of such a plane [17].

Hoek and Brown [17] also propose that the maximum value of intact rock strength, σ_{ci} , should be used in rocks of high strength with good interlocking, while the minimum strength in rocks that have suffered intense tectonic fracturing or shearing and the resulting rock mass is of low quality.

The Hoek - Brown criterion is a failure criterion expressed in terms of minor and major principal stress (σ_3 and σ_1) and ignores the influence the intermediate principal

stress, as well as other criteria in rock mechanics. It is now known that, even in the case of an isotropic rock, the intermediate principal stress, σ_2 , will have an important influence on strength determination. The limitations of failure criteria expressed in terms of σ_1 and σ_3 are outlined in the work of Al-Ajmi & Zimmerman [18], and Colmenares & Zoback [19].

3.2 Initial treatment of anisotropy in the criterion

Hoek and Brown [1] state that the “single plane of weakness” theory, as proposed by Jaeger & Cook [20] and presented in Figure 4, is sufficient for the prediction of strength, when the rock behaves anisotropically due to the presence of a single plane of weakness (e.g. discontinuity plane) but doesn't describe adequately the strength behaviour of intact rock possessing inherent anisotropy, due to the presence of bedding or foliation, as in the case of siltstones, schists, gneisses etc. In developing the empirical failure criterion, the possibility of extension to deal with anisotropic failure and failure of jointed rock masses should be offered [21].

Consequently, in order to predict the strength of intact anisotropic rock, Hoek and Brown [1] suggested that the value of the constants m and s of their empirical criterion, should be altered accordingly based on the orientation of the foliation plane relative to the principal loading axis, β .

The variation of the constants m and s relative to the orientation is given by Equations (2) and (3).

$$\frac{m}{m_i} = 1 - Ae^{-\left(\frac{\beta - \xi_m}{A2 + A3\beta}\right)^4} \quad (2)$$

$$s = 1 - Pe^{-\left(\frac{\beta - \xi_s}{P2 + P3\beta}\right)^4} \quad (3)$$

where ξ_m : orientation angle, β , where m is minimum, ξ_s : orientation angle, β , where s is minimum, A is a constant given by equation (4).

$$A = m_i - \frac{m_{\min}}{m_i} \quad (4)$$

P constant equal to $P=(1-s_{\min})$ while A_2 , A_3 , P_2 and P_3 are constants. The variation of the ratio m/m_i and s relatively to orientation angle, β , for Martinsburg slate are presented in Figure 5 (reproduced from [1]).

3.3 Studies of anisotropic intact rock strength

Colak & Unlu [22] applied equation (2) to triaxial data from tests performed in sandstone (possessing low degree of anisotropy) and siltstone, claystone (medium degree of anisotropy) and calculated the constant m_i for orientations of loading equal to $\beta=0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° (orientated constant named $m_{i(\beta)}$), assuming that the parameter, s , of the Hoek - Brown criterion equal to unity. The parameter $m_{i(\beta)}$ is equal to 0.4 times of the m_i value for the claystone samples when the load is applied in the angle of least strength ($\beta=30^\circ$), while for sandstone it is equal to 0.6 of m_i and 0.3 for siltstone.

Based on their research, it was found that the ratio $m_{i(90)}/m_{i(\beta)}$ correlates well with the degree of anisotropy of uniaxial strength (which for claystone is $m_{i(90)}/m_{i(\beta)}=2.5$ and R_c equal to 3.0). Additionally, the decrease of m_i related to the degree of anisotropy, thus greater decrease was observed for the rocks with medium degree of anisotropy (siltstone and claystone) and less for the rock with low degree of anisotropy (sandstone).

3.4 Remarks

The estimation of the strength of anisotropic intact rock by calculating, indirectly, the variation of the constants m and s is relatively tedious and requests a large

number of triaxial tests performed on orientated samples. Additionally, the methodology of estimating the variation of these constants, without taking into account the variation of the uniaxial compressive strength (σ_{ci}) due to strength anisotropy, can give misleading results and is practically incorrect.

In the present study, the variation of the uniaxial compressive strength due to anisotropy is taken into account in the application of the failure criterion and it was found that the decrease of the constant m of the intact rock, given in equation (2), when loading is at the orientation of minimum strength, relates to the degree of strength anisotropy (R_c).

The constant m_i is characteristic for each rock type, thus it shouldn't be changed. Its value is determined with a set of triaxial tests performed perpendicular to the planes of "inherent" anisotropy.

4. MODIFIED FAILURE CRITERION

4.1. Incorporation of anisotropy in the failure criterion

It would be considered that the degree of anisotropy is indirectly incorporated in the failure criterion, when fitting of triaxial data in different loading directions, is performed. This was the initiative to include a parameter in the criterion, the range of which practically relates to the degree of anisotropy of the intact rock. Based on the above, it becomes clear that it is more appropriate to determine the variation of strength of intact rock, due to the presence of “inherent” anisotropy, directly from the failure criterion instead of indirectly by performing the variation of the material constants m and s . In order to achieve this, a new parameter, k_β , that represents the effect of strength anisotropy, is considered in the failure criterion. This parameter refers to the degree of strength anisotropy of the intact rock (R_C).

The proposed modification of the Hoek - Brown criterion, in order to account for the effect of anisotropy on strength, is based on the inclusion of:

1. The variation of the uniaxial strength of intact rock due to the presence of foliation planes, $(\sigma_{c\beta})$, and
2. The parameter (k_β) , which denotes the range between the minimum and maximum strength of intact anisotropic rock.

The constant s is considered equal to unity due to the intact state of the rock. A fitting procedure, that involved the variation of s in the modified Hoek – Brown criterion, proved that it has a negligible effect on the prediction of strength of intact rock possessing inherent anisotropy, compared to the effect of the variation of the proposed parameter k_β . The influence of the constant s is predominantly on the “tensile” strength of the rock and its influence is most important at very low confinement (σ_3).

The following equation is suggested:

$$\sigma_1 = \sigma_3 + \sigma_{c\beta} \cdot \left(k_\beta \cdot m_i \frac{\sigma_3}{\sigma_{c\beta}} + s \right)^{0.5} \quad (5)$$

where, $\sigma_{c\beta}$ is the uniaxial compressive strength at an angle of loading β and k_β is the parameter describing the anisotropy effect.

When loading is performed perpendicular to the planes of “inherent” anisotropy of the intact rock, the parameter k_β is equal to unity ($k_{90}=1$) and the strength ($\sigma_{c\beta}$) is equal to the uniaxial compressive strength σ_{ci} .

The minimum value of this parameter, $k_\beta = k_{30}$, occurs when loading is performed at the angle of minimum strength which usually is when the angle, β , between the major principal stress (σ_1) and the foliation planes is between 30° and 45° .

4.2. Strength anisotropy in uniaxial compression

For the use of the proposed failure criterion (equation 5), the uniaxial compressive strength at different orientations, $\sigma_{c\beta}$, should be estimated. This can be determined either through uniaxial compression testing on orientated samples or it can be predicted from equation (6) when tests are not available. This expression describes adequately the variation of uniaxial compressive strength of intact rocks possessing strength anisotropy, and was initially proposed by Jaeger [23] and later modified by Donath [24].

$$\sigma_{c\beta} = A - D[\cos 2(\beta_m - \beta)] \quad (6)$$

where β_m is the angle at which the uniaxial compressive strength is minimum (usually between 30° and 45°), A and D constants. The values of constants A and D are determined given that the uniaxial compressive strength is known at least at three different loading angles, that of $\beta=0^\circ$, $\beta=30^\circ$ and $\beta=90^\circ$.

The variation of uniaxial compressive strength in different loading direction, can be predicted by equation (6) when the strength at orientations $\beta=0^\circ$, 30° and 90° is known. The constant D relates to the degree of strength anisotropy and thus it is expected that smaller degree of anisotropy will lead to a lower value of the constant D.

In the present study the uniaxial compressive strength for different orientations, β , was determined directly by uniaxial compression tests.

4.3. Fitting procedure

The fitting procedure in order to determine the variation of the parameter k_β is the following:

1. Initially, the value of σ_{ci} should be determined by uniaxial compression tests perpendicular to the planes of anisotropy.
2. Secondly, the value of the constant m_i is determined considering $k_{90}=1$, using the fitting procedure suggested by Hoek et al. [16], from a series of triaxial tests perpendicular to the planes of anisotropy (4 specimens in different confining pressures, σ_3).
3. For all other orientations, β , using the above value of m_i , the value of k_β , that gives the best fit to the triaxial data according to equation (5), is also determined.

The range of confining pressure for the fitting procedure influences significantly the constant m_i and the parameter k_β . In order that the current procedure is comparable to that used by Hoek and Brown [1], the confining pressure, σ_3 , of the triaxial tests that were used for the fitting procedure was in the range of 0 to $0.5 \sigma_{ci}$.

5. APPLICATION OF THE PROPOSED FAILURE CRITERION

5.1. General

The triaxial data from tests performed on Penhryn slate [6] as well as on gneiss, schist and marble from the present study were fitted to the proposed failure criterion (equation 5). The uniaxial and triaxial compression tests were performed on specimens, in which the planes of anisotropy were oriented at angles, β , equal to 0° , 15° , 30° , 45° , 60° , 75° and 90° , taken by careful drilling of rock blocks. The specimen diameter was 54 mm (NX size) with a height/diameter ratio between 2.0 and 2.5. The range of confining pressure used for the triaxial tests was $0 < \sigma_3 < \sigma_{ci}/2$ although few were performed at higher confining pressures (up to σ_{ci}) in order to investigate the effect of confining pressure on the strength of anisotropic intact rock.

5.2. Characteristics of tested rocks

The rocks tested are gneisses, schists and marbles. The samples of gneiss were collected from Northern Greece, from an area that a number of tunnels of Egnatia Highway were constructed. Two types of gneiss [25] were distinguished:

- a) a muscovite gneiss (denoted as gneiss A) with fine-grained texture (due to the elongation of micas) which consists of very thin bands rich in micas alternating with bands rich in crystals of quartz and feldspars. The bands vary in thickness from 0.2 to 0.5 mm resulting to a very well developed gneiss banding (Figure 1a).
- b) a muscovite - biotite rich gneiss (denoted as gneiss B), which has well-developed gneiss banding, better than that of gneiss A and is medium-grained. The thickness of the micaceous bands varies between 0.3 mm and 0.5 mm while that of the quartz – feldspar between 0.6 mm and 1.8 mm.

The schist belongs to the group of the Athens Schist formation and is a fine-grained rock with very well developed and continuous schistosity planes [26].

The marble, which was used in the antiquity for the construction of monuments of the Acropolis of Athens, is microcrystalline and is characterized as calcitic with low percentage of dolomite (3%). Its texture is of irregular grain size and has a coherent, banded texture due to parallel discontinuous dolomite bands. It is characterized by a low degree of anisotropy due to the incomplete orientation of dolomite crystals.

5.3. Strength anisotropy in uniaxial compression of tested rocks

As stated earlier, the magnitude of strength anisotropy of intact rock can be described by testing intact rock at different orientations of “inherent” anisotropy relative to the direction of applied load.

The variation of uniaxial compressive strength, due to the presence of foliation planes for gneiss and marble, is shown in Figures 6 and 7. The uniaxial strength tests for the schist were not adequate in order to describe the influence of loading angle on uniaxial compressive strength. Equation (6) was fitted to the uniaxial strength data of gneiss and marble and the values of constants A and D were determined, as shown in Table 1.

The minimum uniaxial strength was encountered, as expected, when loading was applied at an inclination of $\beta=30^{\circ} - 45^{\circ}$ relative to the banding planes of the gneiss samples. The uniaxial strength when loading perpendicular to the banding planes was greater than that when loading was applied parallel to them, for both gneisses (Figure 6).

The uniaxial strength of the marble varied slightly, with respect to the orientation of loading, resulting in a rock of low strength anisotropy. The minimum uniaxial strength was encountered, when loading was applied at a loading angle of $\beta= 30^{\circ}$ (Figure 7),

relative to the bedding planes of the marble samples. The uniaxial strength when loading was applied perpendicular to the bedding planes was almost equal to that when loading was applied parallel to them.

5.4. Strength anisotropy in proposed failure criterion

5.4.1. Analysis of triaxial data of Penhryn slate

The failure envelopes of Penhryn slate for different loading orientations, β , resulting from fitting the proposed equation (5) to the triaxial data, are given in Figure 8. The variation of the parameter k_β is summarized in Table 2. It can be concluded that the minimum value of this parameter is encountered, as expected, when loading takes place at an orientation of $\beta=30^\circ$ relative to the planes of foliation ($k_{30}=0.25$), while its maximum value ($k_{90}=1$) when loading is performed normal to them.

The ratio k_{90}/k_{30} is equal to 4 and agrees well to degree of strength anisotropy ($R_c=3.3$) of Penhryn slate for the range of confining pressure considered in the analyses ($0 < \sigma_3 < 68.9$ MPa).

5.4.2. Analysis of data for tested rocks

The degree of anisotropy of gneiss A is $R_c=2.2$, characterizing it as a medium anisotropy rock. The fitting of the modified failure criterion (equation 5) to the triaxial data of gneiss A is given in Figure 9. The constant m_i is equal to 24.6, which is in good agreement to that proposed by Marinos and Hoek [27]. The variation of the parameter k_β with foliation orientation, β , for the rocks tested in the present study is given in Table 3.

Gneiss B is characterized by a medium to high degree of anisotropy ($R_c=3.8$). The constant m_i is equal to 23.2. The failure envelopes of the gneiss B for different loading directions are presented in Figure 10.

The fitting of the modified failure criterion to the triaxial data of schist is given in Figure 11. The constant m_i is equal to 9.5. The minimum strength of the schist was observed at orientation β between 30° and 45° , since in some cases, this changed within the same specimen. The scatter of triaxial data of the schist tested, is relatively small due to the relatively low degree of anisotropy ($R_c=1.25$). Additionally, the heterogeneity of this rock type resulted in inconsistencies. More precisely, it would be expected that the strength of rock increases with increasing confinement, but this was not always true since some specimens failed at lower strength than expected due to presence of micro-cracks and weaker layers. These data were not included in the fitting procedure for the triaxial data of schist.

The fitting of the modified failure criterion to the triaxial data of marble is given in Figure 12. The constant m_i is equal to 9.6 while the variation of the parameter k_β with banding orientation, β , is relatively low due to the low strength anisotropy of the rock ($R_c=1.1$). The uniaxial and triaxial data for the rocks tested in the present study are given in the Appendix in Tables 4, 5, 6 and 7.

5.4.3. Variation of parameter k_β – relation with R_c

The variation of the parameter k_β with foliation orientation for the rocks tested in the present study is shown in Figure 13. Based on the above analysis it is evident that, the ratio k_{90}/k_{30} is greater for the rocks with a high degree of anisotropy, R_c , hence Penhryn slate, gneiss and reduces significantly for the rocks with a low degree of anisotropy, schist and marble (where k_{90} is the value of the parameter k_β when

loading is perpendicular to the foliation, equal to unity, and k_{30} is its value at the orientation of minimum strength, at $\beta=30^\circ - 45^\circ$).

The ratio k_{90}/k_{30} for Penhryn slate is equal to 4.0 while R_C is 3.3. Additionally, for gneiss A the ratio k_{90}/k_{30} is 2.3 and the value of R_C is 2.2. Finally, the ratio for schist and marble is equal to 1.33 and 1.09 respectively, while their degree of strength anisotropy is equal to 1.25 and 1.1.

The estimation of strength reduction of intact rock due to anisotropy is achieved by the ratio k_{90}/k_{30} , which was found to relate quite well to the degree of strength anisotropy, R_C . This means that the degree of strength anisotropy can be used to predict the reduction of strength of intact anisotropic rock, with relative accuracy, at the orientation of minimum strength.

6. VERIFICATION OF PROPOSED CRITERION

A dimensionless plot, which was introduced by Londe [28], is utilized in order to check the quality of the fitting procedure. As it is shown in Figures 8, 9, 10, 11 and 12, the triaxial data of anisotropic rocks have considerable scatter in a σ_3 - σ_1 plot, which in turn represents the degree of their strength anisotropy.

In this plot, a single curve can be used to define the Hoek - Brown criterion for all rock types. Londe's transformation was adapted to the proposed failure criterion of anisotropic intact rock as follows:

$$S_1 = S_3 + \sqrt{S_3} \quad (7),$$

$$\text{where } S_1 = \frac{\sigma_1}{k_\beta \cdot m_i \cdot \sigma_{c\beta}} + \frac{1}{k_\beta^2 \cdot m_i^2} \text{ and } S_3 = \frac{\sigma_3}{k_\beta \cdot m_i \cdot \sigma_{c\beta}} + \frac{1}{k_\beta^2 \cdot m_i^2}$$

The determination of S_1 and S_3 for anisotropic intact rock incorporates the value of parameter k_β for different loading angles, β , as determined from fitting the proposed criterion to the triaxial data, presented previously. It also incorporates the uniaxial compressive strength for different orientations, $\sigma_{c\beta}$, as determined from testing.

The accuracy of fitting equation (7) to the triaxial data, for the rocks tested, is justified as the majority of (S_3, S_1) points are plotted on this line. These points have a really good plot when compared to the scatter of the σ_3 - σ_1 plots for each rock. The dimensionless plot $S_3 - S_1$ of the 150 triaxial data points, of the present study, is shown in Figure 14.

Additionally, plotting the uniaxial compressive strength, determined by tests for different loading directions, $\sigma_{c\beta\text{-lab}}$, against that predicted from the failure criterion for anisotropic intact rock (equation 5), can also assess the accuracy of the proposed criterion. The data sets that plot off the diagonal line (1:1) are indications of

unacceptable fitting. The prediction of uniaxial strength by the proposed criterion is quite good, as the majority of the data plot on the diagonal line, shown in Figure 15.

Based on the analysis of triaxial data from other researchers (Martinsburg slate [2], schists [9], slate [3], shales [4] and limestone, gneiss [5]) it was profound, that the proposed modification of the Hoek - Brown failure criterion can predict the strength of intact anisotropic rock reasonably well, given that a number of conditions are provided, such as: a) the maximum confining pressure, σ_{3max} , for each orientation of loading should be half the uniaxial compressive strength of the intact rock for each case, and b) uniaxial compressive strength should be directly determined in different orientations, in order to have the minimum and maximum values.

7. CONCLUSIONS

Strength anisotropy of intact rock stems mainly from the presence of foliation and is commonly encountered in metamorphic rocks e.g. phyllites, shales, schists, slates and gneisses. Marbles and sedimentary rocks (siltstones, claystones and mudstones) may also exhibit a low degree of anisotropy. Studying the behaviour of anisotropic rocks, it is most important to determine the variation of uniaxial compressive strength, σ_{ci} , thus the degree of strength anisotropy. The degree of strength anisotropy, R_c , is determined by the ratio $\sigma_{ci(90)}/\sigma_{ci(min)}$, where $\sigma_{ci(90)}$ is the uniaxial compressive strength perpendicular to the foliation planes and $\sigma_{ci(min)}$ is the compressive strength when the orientation of foliation planes relative to the loading axis is between 30° and 45° .

The Hoek - Brown criterion parameters (σ_{ci} , m_i and s) are significantly influenced by the strength anisotropy of intact rock. The criterion is proposed for isotropic rock and rock masses, but it is widely used for intact rocks with “inherent” anisotropy.

In order to use this criterion for the prediction of strength in anisotropic intact rocks, careful selection of its parameters is necessary, namely the uniaxial compressive strength (σ_{ci}) and the material constants m_i and s .

The application of the Hoek - Brown failure criterion to intact anisotropic rock is accomplished by the incorporation of a new proposed parameter, k_β , which is determined by fitting the modified criterion to the triaxial test data, obtained from tests on samples with different orientations of the foliation plane.

The variation of the uniaxial compressive strength, $\sigma_{c\beta}$, is also incorporated in the modified criterion, while the constant m_i is characteristic for each rock type and independent of loading orientation. Its value is determined with a set of triaxial tests performed perpendicular to the planes of “inherent” anisotropy, as proposed by Hoek and Brown [17].

The proposed modified failure criterion is expressed by the equation:

$$\sigma_1 = \sigma_3 + \sigma_{c\beta} \cdot \left(k_\beta \cdot m_i \frac{\sigma_3}{\sigma_{c\beta}} + s \right)^{0.5}$$

where, $\sigma_{c\beta}$ is the uniaxial compressive strength at an angle of loading, β , and k_β is the parameter describing the anisotropy effect.

When loading is performed perpendicular to the planes of “inherent” anisotropy of the intact rock, the parameter k_β is equal to unity ($k_{90}=1$) and the strength ($\sigma_{c\beta}$) is equal to the uniaxial compressive strength σ_{ci} . The minimum value of this parameter, $k_\beta = k_{30}$, occurs when loading is performed at the angle of minimum strength which usually is when the angle, β , between the major principal stress (σ_1) and the foliation planes is between 30° and 45° .

The estimation of strength reduction of intact rock due to anisotropy is described by the ratio k_{90}/k_{30} .

It was found that the rocks possessing a high degree of strength anisotropy, R_c , are also characterized by a high value of the ratio k_{90}/k_{30} (i.e. slate and gneiss).

Furthermore, rocks having a low degree of anisotropy give a proportionally low ratio k_{90}/k_{30} (schist and marble).

It is evident, that the proposed modified Hoek – Brown criterion fits well the triaxial data of anisotropic intact rock, based on the research findings. The proposed modified criterion is intended for use for prediction of strength of intact rock, but can also be extended to rock masses.

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LIST OF FIGURES

Figure 1 a) Banding in gneiss intact rock in a rock slope exposure, b) petrographic texture of gneiss showing alternating layers of mica (Ms), quartz (Qtz) and feldspar (Fs) (x 25).

Figure 2 a) Influence of loading direction on uniaxial strength of graphitic phyllite (tested by Salcedo [13]), b) variation of compressive strength with inclination of foliation of phyllite (tested by Sabatakakis and Tsiambaos [14])

Figure 3. Influence of loading direction on triaxial strength of Delabole slate (tested by Brown et al. [7]).

Figure 4. Single plane of weakness theory (reproduced from Hoek [21]).

Figure 5. Variation of m/m_i and s relative to orientation angle, β , for Martinsburg slate (reproduced from Hoek and Brown [1]).

Figure 6. Strength anisotropy of gneisses in uniaxial compression.

Figure 7. Strength anisotropy of marble in uniaxial compression.

Figure 8. Failure envelopes of Penhryn slate (triaxial testing data from Attewell and Sandford [6], from modified criterion).

Figure 9. Failure envelopes of gneiss A at different loading orientations, β (from modified criterion).

Figure 10. Failure envelopes of gneiss B at different loading orientations, β (from modified criterion).

Figure 11. Failure envelopes of schist at different loading orientations, β (from modified criterion).

Figure 12. Failure envelopes of marble at different loading orientations, β (from modified criterion).

Figure 13. Variation of parameter k_β for analyzed triaxial data.

Figure 14. Dimensionless plot of $S_3 - S_1$ of triaxial data of present study.

Figure 15. Plot of predicted uniaxial compressive strength, $\sigma_{c\beta-crit}$, against that determined in the laboratory, $\sigma_{c\beta-lab}$ for tested specimens.

LIST OF TABLES

Table 1. Parameters characterizing the anisotropy of tested intact rocks in uniaxial compression.

Table 2. Variation of parameter k_β for Penhryn slate (tested by Attewell and Sandford [6]).

Table 3. Variation of parameter k_β for tested rocks.

ANNEX

Table 4. Triaxial data for Gneiss A

Table 5. Triaxial data for Gneiss B

Table 6. Triaxial data for Athens Schist

Table 7. Triaxial data for Marble

Figure1a
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Figure1b

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Figure2a
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Figure2b
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Figure3
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Figure4
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Figure5
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Figure6
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Figure7
[Click here to download high resolution image](#)

Figure8
[Click here to download high resolution image](#)

Figure9
[Click here to download high resolution image](#)

Figure10
[Click here to download high resolution image](#)

Figure11
[Click here to download high resolution image](#)

Figure12
[Click here to download high resolution image](#)

Figure13

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Figure14
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Figure15
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Table1

Table 1.

Rock type	σ_{ci}	Strength	Constant A	Constant D
	(MPa)	anisotropy, Rc	(MPa)	(MPa)
Gneiss A	60.6	2.2	52.7	26.1
Gneiss B	85.3	3.8	61.5	43.8
Schist	67.2	1.25		
Marble	89.7	1.14	83.9	11.2

Table2

Table 2.

Rock type	β ($^{\circ}$)	0	15	30	45	60	75	90
Penhryn	k_{β}	0.89	0.47	0.25	0.35	0.31	0.39	1
slate	$\sigma_{c\beta}$ – criterion	159.7	108.8	47.9	61.3	92.4	129.4	191.9
	r^2	0.94	0.97	0.94	0.97	0.94	0.99	0.94

Table3

Table 3.

Rock type	β ($^{\circ}$)	0	30	45	60	75	90
Gneiss A	k_{β}	1.79	0.42	0.38			1
	$\sigma_{c\beta}$ criterion (MPa)	39.4	35.5	46.7			66.5
	r^2	0.98	0.79	0.97			0.97
Gneiss B	k_{β}	0.88	0.59	0.41			1
	$\sigma_{c\beta}$ criterion (MPa)	45.4	23.4	34.5			85.7
	r^2	0.97	0.96	0.96			0.96
Schist	k_{β}	0.96	0.75	0.76	1		1
	$\sigma_{c\beta}$ criterion (MPa)	64.3	63.1	58.2	64		66.4
	r^2	0.91	0.92	0.90	0.96		0.99
Marble	k_{β}	0.99	0.91	0.93		1	1
	$\sigma_{c\beta}$ criterion (MPa)	88.1	76.1	84.3		90.1	88.4
	r^2	0.98	0.96	0.96		0.86	0.98

APPENDIX

Laboratory data used for the analysis (Total number of data: 150)

Table 4. Triaxial data for Gneiss A

$\beta=0^0$		$\beta=30^0$		$\beta=45^0$		$\beta=90^0$	
σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1
(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)
0	42,5	0	21,1	0	37,5	0	58,30
0	39,4	0	35,3	3	62,5	0	62,9
0	30,9	1,8	60,1	3,0	75,5	3,6	126,3
0	36,5	1,8	37,7	7,8	84	7,8	133,0
3,6	112,5	3,0	55,8	15,6	108,8	15,6	198,5
4,2	111,6	3,0	62,5	27,0	147,2	31,2	257,2
7,8	132,3	6,0	66,8	31,2	156,2		
9,0	120,7	6,0	72,9				
15,6	181,0	12,0	81,7				
18,0	199,5	12,0	81,3				
31,2	269,6						

Table5

Table 5. Triaxial data for Gneiss B

$\beta=0^0$		$\beta=30^0$		$\beta=45^0$		$\beta=90^0$	
σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1
(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)
0	33,4	0	22,4	0	37,9	0	84,9
0	48,4	1,8	29,9	3,6	47,5	0	62
3,6	87,6	3,6	55,8	7,2	63,6	3,6	148,6
7,8	115,2	7,2	58,6	14,4	92,5	3,6	135,6
15,6	142,8	14,4	92,5	15,6	90,5	4,8	130,0
31,2	201,4	21,6	99,0	15,6	109,6	7,2	191,4
		28,8	131,9	31,2	147,7	10,2	165,8
				24,0	111,8	10,2	166,1
				31,2	133,3	15,6	208,9
						15,6	213,8
						21,0	219,1
						31,2	286,6
						38,4	344,3
						46,0	360,0

Table6

Table 6. Triaxial data for Athens Schist

$\beta=0^0$		$\beta=20^0$		$\beta=30^0 - 45^0$		$\beta=60^0$		$\beta=90^0$	
σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1
(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)
0	57,7	0	57,7	0	51,6			0	67,2
3,6	88,7	3,6	76,4	7,2	100,9	3,6	88,1	3,6	86,1
3,6	94,3	3,6	79,8	12,0	106,7	7,2	84,9	7,2	104,0
7,2	105,1	7,2	113,6	15,6	112,2	15,6	145,0	24,0	153,6
7,8	110,3	7,2	80,0	24,0	134,4	24,0	164,8	46,0	236,0
15,6	105,4	15,6	122,3	31,2	143,2	31,2	187,5		
15,6	130,2	15,6	125,3	31,2	178,7				
31,2	188,7	15,6	117,7						
31,2	228	24,0	158,4						
		24,0	132,3						
		24,0	149,2						
		24,0	143,8						
		31,2	160,4						

Table7

Table 7. Triaxial data for Marble

$\beta=0^0$		$\beta=30^0$		$\beta=45^0$		$\beta=75^0$		$\beta=90^0$	
σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1	σ_3	σ_1
(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)
0	80	0	71	0	84,7	0	69,2	0	79,7
4,8	125	3,6	76,8	3,6	107,5	0	97,9	0	89,7
9,6	140	15,6	137,2	3,6	94,9	4,8	121,3	7,2	130,7
19,2	168	31,2	192,6	7,2	123,8	4,8	132,9	15,6	165,6
40,0	242	46,0	230,2	15,6	157,2	9,6	140,9	24,0	200,5
		3,6	109,5	15,6	141,8	9,6	136,0	46,0	252,8
		15,6	145,4	24,0	188,1	19,2	170,2		
		31,2	203,4	27,0	181,2				
		7,2	121,4	31,2	227,6				
				31,2	201,0				
				46,0	243,7				