Windage Sources in Smooth Walled Rotating Disc Systems

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Abstract

This paper presents experimental data and an associated correlation for the windage resulting from a disc rotating in air, characteristic of gas turbine engines and relevant to some electrical machine applications. A test rig has been developed which uses an electric motor to drive a smooth bladeless rotor inside an enclosed pressurised housing. The rig has the capability of reaching rotational and throughflow Reynolds numbers representative of a modern gas turbine. A moment coefficient has been used to allow a non-dimensional windage torque parameter to be calculated and agreement with the relevant data in the literature has been found within 10 %. Infrared measurements have been performed which allow direct surface temperatures of the rotating disc to be obtained. Laser Doppler Anemometry measurements have been made which allow velocities in the flow field of the rotor-stator cavity to be examined and tangential velocities corresponding to rotationally and to radially dominated flow conditions are shown. The importance of the flow regime in relation to the resulting windage has been identified and in particular it is noted that windage is a function not only of the ratio of rotational and radial flow dominance as defined by the turbulence parameter, but also, for a given value of the turbulence parameter, the magnitude of the rotationally induced and superimposed flows. The

experiments extend the range of data available for windage in rotor-stator systems and has been used to produce a correlation suitable for applications operating up to the range $Re_\phi = 10^7$.

Keywords: Windage heating; rotating flows; Internal air system

1 Introduction

Typical turbine inlet air temperatures have risen from approximately 800 °C during the Whittle era, to around 1700 °C in a current engine. If left unchecked, these temperatures would cause rapid degradation of the metal components, and contribute to accelerated fatigue and creep, reducing the service life of the engine components. A commonly used method of controlling metal temperatures is to use some of the compression stage mainstream flow and feed it to the critically hot components, in particular the nozzle guide vanes, turbine blades and turbine discs, in order to cool them from within. The ability to more accurately quantify windage in terms of non-dimensional parameters appropriate for the rotating flows found inside gas turbines is a well recognised requirement for the successful design of modern engines. Designing for effective cooling presents a significant challenge; cooling air is subject to heating by viscous friction as it passes over rotating and static surfaces inside the engine. As the cooling air passes through the cooling circuit and absorbs heat, its effectiveness is continuously reduced, requiring that more mainstream air must be used. The success with which cooling is managed has direct impact on cycle efficiency and service life.

The work reported here presents correlations between the magnitude of windage over a range of real engine representative dimensionless conditions. A test rig has been built which allows experiments to be performed where direct torque and enthalpy rise measurements may be made. The physical mechanisms responsible for windage heating have also been studied with

the aid of Laser Doppler Anemometry (LDA) to measure velocities within the rotor-stator cavity and infrared (IR) to directly measure the disc surface temperature. The improved understanding of flow fields and thus local disc heating add to the accuracy of disc life prediction.

The term windage may be defined as the viscous friction heating which results from the relative velocities across the boundary layers between the fluid and the rotating disc, and between the fluid and the stationary casing surfaces. This contributes to losses which may be measured as either a shaft torque or heat rise of the fluid passing through the system.

Nomenclature

a	Radial displacement, inner	[m]
b	Radial displacement, outer	[m]
C	Constant	
G	Stator gap to disc radius ratio	
K_0	Core rotation factor, no superimposed flow	
K_r	Core rotation factor, superimposed flow present	
$\dot{m}$	Mass flow rate	[kg / s]
M	Moment	[N m]
Q	Volumetric flow rate	[m ³ / s]
r	Radial displacement, local	[m]
s	Disc to stator axial spacing	[m]
T_F	Throughflow factor	
u	Velocity, stationary frame of reference	[m / s]
x	Linear displacement	[m]

X_c	Non-dimensional source region radius	
X_{SEAL}	Developed labyrinth seal length	[m]

Subscripts

bal	Pressure balance half of test rig housing
r	Pertaining to radial direction
ϕ	Pertaining to tangential direction
x	Axial displacement
z	Pertaining to axial direction

Greek Symbols

β	Core flow swirl rate variable, or diameter ratio	
ε_M	Core rotation rate factor	
μ	Dynamic viscosity, or micro	[kg / m s]
ω	Angular velocity	[rad / s]
ρ	Density	[kg / m ³]
τ_ϕ	Tangential shear stress in fluid	[N / m ²]

Dimensionless Groups

C_M	$= M / \frac{1}{2} \rho \omega^2 b^5$	Moment coefficient (both sides of disc)
C_W	$= \dot{m} / \mu b$	Throughflow Reynolds number
λ_T	$= C_W / Re_\phi^{0.8}$	Turbulent flow parameter
Re_ϕ	$= \rho \omega b^2 / \mu$	Rotational Reynolds number

2 Description of Test Rig

The major measurement section of the rig comprises a rotating disc housed in a pressurised casing. See Figure 1. The disc is driven by means of a 55 kW electric motor which has a

maximum speed of 3000 rpm. In order to achieve the high speeds required for these experiments a 5:1 ratio step-up gearbox is used to transmit drive to the disc. The main casing of the test rig is formed from two steel castings. Rim seals of an 'L' shape cross section provide a cylindrical wall around the periphery of the cavity, the axial overlap between the rim seals and disc is 1 mm. The maximum axial clearance between the rotor and the stator, s , is 22.0 mm, giving a typical gap ratio $G = s / b = 0.1$. A central sealing ring is finished with an 'Apticote 800 / 38' abradable coating that is designed to be worn away in the event of contact when the disc is fully expanded by rotational and thermal loads. This has been designed to have a cold radial clearance of 0.4 mm and allows a safe running clearance to be maintained without risk of damage to the disc extremity. The 0.45 m diameter disc has a tapered cross section and is manufactured from titanium alloy IMI 318. It is mounted on a central driveshaft via a flange and may be driven at up to 13 000 rpm. Labyrinth sealing fins machined into its outer surface provide a controlled route for the air passing through the cavity. Compressed air is supplied to each side of the main casing at up to 0.4 kg / s, and is exhausted through ports equally spaced around its circumference from a plenum chamber located radially outward of the disc. An equal pressure is maintained both sides of the disc in order to ensure that no significant net pressure is exerted on the drive bearings.

Figure 1: Main Casing Geometry

Pressure measurements are required at inlet and outlet orifice plates for mass flow measurements, to record the steady state pressure in the rotor-stator cavity, and to ensure no net pressure is exerted on the drive bearings. Pressure lines for each of the measurement locations are connected to a 'DSS48C Mk 4 Scanivalve' pressure measuring instrument. This device employs a single pressure transducer in conjunction with a rotary valve which allows up to 48 pressure channels to be measured. This was capable of measuring up to the maximum test rig pressure condition of 7 bar absolute.

K-type thermocouples have been used at the inlet and outlet orifice plates to obtain temperature measurements for mass flow calculations. In order to measure the absolute temperature of the bead, it is necessary to independently measure the temperature of the data logger connection, which is referred to as a cold junction, and sum this with the thermocouple signal. For this purpose an 'LM 35CZ' precision integrated circuit temperature sensor is installed in the same insulated box used to house the thermocouple to data logger connections.

Infrared (IR) sensors with a calibrated measurement range of 10 to 140 °C were used to directly measure the surface temperature distribution on the rotating disc. These were installed using a cartridge system incorporating optical Zinc Selenide windows of diameter 20 mm, in order to protect the cells from the potentially damaging temperature and pressure of the gas stream. The windows were finished with a non-reflective coating which improves their transmissivity from 80 %, to 99.2 %. Laser Doppler Anemometry (LDA) was used to measure radial and tangential components of the air velocities in the rotor-stator cavity, using a 2D LDA probe and an air supply seeded with oil particles. The laser system is based around a ‘Spectra Physics Stabilite 2017’ Argon - Ion tube laser. The system is capable of measuring two dimensional speed and direction measurements of the velocity vector of a fluid particle in a flow field. A three dimensional measuring capability was achieved by using a traversable mounting chassis. Tracer particles are introduced to the main flow using an air pressurised jet atomiser. It has adjustable jets which flow oil particles that the laser beams can detect. A particle with a diameter of 1 μm has been shown to be the approximate maximum size which an oil particle will flow within an air flow field in a manner representative of the air itself, and without influencing the air motion by Ainsworth Thorpe and Manners [1]. Optical grade (BK7) windows are used to allow the beams to pass into the pressurised casing.

A Vibro-Meter in-line torque meter detects the load due to windage sources in the disc-casing arrangement. The device comprises a primary transducer which works on the principal of a variable inductance transformer, where the amount of screening between the inner and outer coils, mounted on the input and output shaft ends respectively, varies proportionally with the shaft twisting that results from an applied torque.

The operating ranges over which the test rig was operated were chosen with the chief criterion of representing the kind of conditions found in modern gas turbine engines. In order to achieve this, dimensionless parameters appropriate for the experiments proposed were used to ensure matching with real engine conditions would be realised. Rotating flow phenomena are commonly characterised by means of the rotational Reynolds number, $Re_\phi = \rho \omega b^2 / \mu$, where the characteristic dimension is the disc outer radius b , the fluid density and dynamic viscosity are ρ and μ respectively, and the disc rotates with a speed of ω . A disc rotating in a fluid will induce a bulk radial outflow of that fluid. This mechanism is commonly called entrainment. The mass flow, whether it is induced by rotation or whether it is deliberately superimposed can be characterised by the throughflow Reynolds number, $C_w = \dot{m} / \mu b$, where $\dot{m}$ is the mass flow rate through the system. The turbulent flow parameter, $\lambda_T = C_w / Re_\phi^{0.8}$, is particularly important to this work, as it provides a means of characterising flow regimes in enclosed rotating disc systems by providing an indication of whether the flow is rotationally or radially dominated. As the value of λ_T increases above 0.219, the flow transitions from rotationally to radially dominated. The non-dimensional operating ranges used are as follows:

Rotational Reynolds Number	$2.5 \times 10^6 \leq Re_\phi \leq 2.5 \times 10^7$
Flow Reynolds Number	$3.0 \times 10^4 \leq C_w \leq 1 \times 10^5$
Turbulent Flow Parameter	$0.05 \leq \lambda_T \leq 0.5$

This corresponds to the following dimensional operating conditions:

Rotational speed	0 - 13 000 rev/min
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Throughflow mass flow rate 0 - 0.4 kg/s (each side)

Throughflow temperature 290 - 350 K

Throughflow pressure 1 - 7 bar (abs.)

3 Windage Measurement Methodology

In order to broaden the relevance of this work, a non-dimensional torque parameter was used to quote the windage associated with a particular test. A perfectly smooth disc rotating in a fluid experiences a torque due to viscous friction at the surface as a result of the tangential shear stress, $\tau_{\phi,0}$. For a disc of inner and outer radii a , and b , respectively, an elemental ring on the disc located at radius r and of radial width dr will experience a torque given by:

$$M(r) = \tau_{\phi,0} 2\pi r^2 dr \quad \text{Equation 1}$$

This net torque, whether calculated or as with these experiments directly measured using a torque meter, may be used to find a non-dimensional moment coefficient which provides a useful means of comparing the windage resulting from a variety of flow and disc rotation conditions. The moment coefficient, C_M is defined as:

$$C_{M \text{ BOTH SIDES}} = \frac{M_{\text{BOTH SIDES}}}{\frac{1}{2} \rho \omega^2 b^5} \quad \text{Equation 2}$$

The presence of stationary walls close to the disc affects the core fluid rotational rate and also net torque. However, as the torque meter used in these experiments is a direct measurement of what the disc experiences, it accounts for these effects.

4 Comparison Methodology

The experiments reported here have been conducted with as part of continued investigations at the Thermo-Fluid Mechanics Research Centre into the windage associated with rotor-stator systems with the aim of improving the understanding and quantification of the prevalent flow structures. As part of obtaining good quality plain disc test data, a review of existing plain disc literature was performed and comparison with the existing data was made. The chief parameter used for comparison is the moment coefficient, C_M , as defined in section 3, against a range of dimensionless operating conditions also described in section 3. The data obtained using non-invasive measurements are described subsequently.

Because the torque meter used for these experiments registers all the sources of drag associated with the rotating disc and drive mechanism, the drag due to the driveshaft bearing friction is measured. Removing the driveline drag allows more accurate measurement of the windage drag, and allows more direct comparison with data in the literature. The driveline drag was measured by performing tests using a disc installed on the rig with the same mass as the plain disc, but with negligible windage due to a reduced diameter of 0.05 m. The un-pressurised rig was rotated up to 12 000 rpm and the torque meter readings were used to generate a correction that could be applied to subsequent raw torque meter data, by removing the torque due to bearing friction.

As with the driveline correction, removal of the torque absorbed by the stepped circumferential balancing seal at the exit of the pressurised cavity allows more direct comparison with data in the literature, where the presence of a seal is sometimes neglected. A seal model by Millward

[2] was used to modify the torque meter measurement for a given test, before calculating the net moment coefficient in the manner described in section 3. See Equation 3.

$$M_{SEAL} = C_{M SEAL} \pi \rho x_{SEAL} \omega^3 b^4 \quad \text{Equation 3}$$

$$C_{M SEAL} = 0.0382 \left(\frac{C_w}{Re_\phi} \right)^{0.55} \quad \text{Equation 4}$$

Where x_{SEAL} = Fully developed seal length

A review of the literature reveals the existence of many numerical models and several correlations of experimental data for the windage resulting from a disc rotating in fluid; See Childs [3]. This section is separated into categories of geometric configuration and by the corresponding fluid regimes represented by the data sourced from the literature. By virtue of the data preparation described previously, the Sussex rig data may be taken to represent simple rotor-stator geometry with a rotating disc, a stationary disc and a circumferential shroud. Because the new data was obtained with the test rig operating in the turbulent regime, comparison is made with turbulent rather than laminar flow cases.

5 Comparison with Free Disc Data

The free disc can be defined as a disc which rotates adjacent to an infinite and initially quiescent fluid. A review of free disc literature shows quite a variation in the moments coefficients predicted, diverging particularly as Re_ϕ is increased towards 10^7 . A representative range of established relationships is shown in Figure 2.

Figure 2: Example of Free Disc Moment Coefficients

The differences between these data may be explained by the following. Goldstein [4], Dorfman [5], and Bayley and Owen [6] used logarithmic boundary layer velocity profiles while von Kármán [7] used a 1/7th power law model. See Equations 5, 6, 7 and 8 respectively, for turbulent flow on both sides of a disc.

$$C_M = \left(1.97 \log(\text{Re}_\phi \sqrt{C_M}) + 0.03\right)^{-2} \quad \text{Equation 5}$$

$$C_M = 0.982 (\log_{10} \text{Re}_\phi)^{-2.58} \quad \text{Equation 6}$$

$$C_M = 0.131 \text{Re}_\phi^{-0.186} \quad \text{Equation 7}$$

$$C_M = 0.146 \text{Re}_\phi^{-0.2} \quad \text{Equation 8}$$

These velocity profile power laws are a development of the resistance formula of Blasius [8]; they are based on empirical data and ‘tailored’ to a particular range of fluid conditions, particularly the relative velocities. Approximating the boundary layer velocity profile using a

logarithmic rather than a power law distribution is considered to provide a better representation of a real fluid over a wider range of flow conditions as defined by the rotational Reynolds number. Goldstein's solution is differentiated by his use of numerical terms to match his result to a particular set of experimental data. Using the more physically realistic solutions of Dorfman, and Bayley and Owen as reference, comparison with the new data could be made. Plotting the data with respect to Re_ϕ reveals differences of characteristic. Error bars show the bounds of uncertainty associated with the new moment coefficients. See Figure 3.

Figure 3: Free Disc Correlations and New Data

The observable differences between the data may be understood by plotting the new data for cases only where $\lambda_T \approx 0.2$; good agreement between the data is then found. This is because when $\lambda_T \approx 0.2$, the maximum entrainment rate for the disc system is reached, and the fluid regime in the test rig may be considered to be similar to that of a free disc. See Figure 4.

Figure 4: Comparison of Data, $\lambda_T \approx 0.2$

6 Comparison with Data for Rotor-Stator Geometry

Comparison of the new data with rotor-stator arrangements is of interest because they more closely represent the geometry of the Sussex rig and geometry commonly found in gas turbines, where a rotating disc is in close proximity to a stationary disc, often with a circumferential shroud surrounding the axial gap separating the discs. The introduction of a stator adjacent to a rotating disc causes fluid structures quite different to that of the free disc to be formed. For small values of the gap ratio G , of around 0.05, there may be space enough only for a single boundary layer. With values of around $G \geq 0.1$, as used in the Sussex rig, separate boundary layers typically exist. The presence of a shroud allows fluid to be recirculated from the radial outflow from the rotating disc, radially inward along the stator. Although an increased shroud width introduces friction due to the increased surface area, the change in the cavity width alters the flow structure significantly, which itself alters the resulting moment coefficient. Experimental data from rotor-stator systems provides indication of the moment

coefficient with respect to gap ratio rather than a direct measurement of the shroud surface friction. Calculations such as shown by Gartner [9] indicate that for a gap ratio such as found in the Sussex rig, the shroud may contribute to around 10 % of the total friction.

The solutions of Schultz-Grunnow [10], Ippen [11], Soo and Princeton [12] and Daily and Nece [13] have been plotted from Equations 9, 10, 11, and 12 respectively, for turbulent flow on both sides of a disc.

$$C_M = 0.0622 \text{Re}_\phi^{-0.2} \quad \text{Equation 9}$$

$$C_M = 0.0836 \text{Re}_\phi^{-0.2} \quad \text{Equation 10}$$

$$C_M = 0.0412 \text{Re}_\phi^{-0.25} \left(\frac{s}{b} \right)^{-0.25} \quad \text{Equation 11}$$

$$C_M = 0.102 \left(\frac{s}{b} \right)^{0.1} \text{Re}_\phi^{-0.2} \quad \text{Equation 12}$$

In the case of Daily and Nece [13], and Soo and Princeton [12], where the disc radius and rotor-stator gap dimensions are required, the values for the Sussex rig have been used. See Figure 5.

Figure 5: Rotor-Stator Moment Coefficients

The differences between the data may be accounted for as follows. The correlation of Shultz-Grunnow [10] and the model of Ippen [11] do not account for cylindrical wall friction. Ippen used a numerical operator to match his result to a set of experimental data. Daily and Nece [13], and Soo and Princeton [12], account for axial spacing and its effect on core rotation rate, but the solution of Soo and Princeton is more appropriate for gap ratios smaller than used on the Sussex rig. The data of Daily and Nece was obtained by performing experiments using a test rig incorporating a rotor-stator arrangement. Their test rig used a completely enclosed rotor-stator system with no throughflow, where the fluid pumped by the disc was recirculated in the cavities either side of the disc. Rotationally dominated conditions therefore prevailed.

Although the Daily and Nece data represents distinctly different fluid conditions to those of the Sussex test rig, comparison is of interest because their data represents an enclosed system with similar geometry to that of the Sussex test rig, where the effects of a circumferential shroud,

stator walls, and axial spacing on rotor drag are all accounted for. Referring to Figure 6, although similarity in the trends may be observed between the Daily and Nece and the new data, the agreement is poor. This is due to the dissimilar fluid conditions, the significant difference in moment coefficient between the free disc and rotor-stator data may be explained as follows. When the rotating disc has a stator brought into proximity with it, the quiescent environment of the free disc no longer exists, and a core of fluid rotating at some fraction, typically 0.4, of the disc is generated, with separate boundary layers between the fluid and the disc and the fluid and the stator. This results in a reduction in the relative tangential velocity between the fluid and the rotating disc. The relationship between tangential velocity and shear stress across a boundary layer is given by the expression referred to as Newton's law of viscosity:

$$\tau_{\phi} = \mu \left[\frac{\partial u_{\phi}}{\partial z} \right] \quad \text{Equation 13}$$

It is important to note that it is the velocity gradient rather than the magnitude of the velocity difference that influences the shear stress. This explains why the sum of the stresses generated from the separate fluid to rotor and fluid to stator boundary layers of the rotor-stator arrangement, is less than the stresses developed in the single boundary layer of the free disc case. This corresponds to reduced moment coefficients for rotor-stator geometries.

7 Comparison with Data where Superimposed Throughflow Exists

The introduction of superimposed throughflow has a profound effect on the flow structure and rate of core rotation in a rotor-stator system. Superimposed flows exist in rotor-stator systems when the rate of remotely supplied flow exceeds the pumping, or entrainment, rate of the disc. The purpose of such flows is usually to cool internal components of the turbomachinery. The

flow may enter at the disc periphery and exit axially, along the axis of rotation. More commonly however, the flow enters axially, towards the disc, and exits radially at the disc periphery, as is the case with the Sussex rig. The rates of throughflow used for the experiments reported here are often greater than the entrainment rate of the disc. This level of superimposed throughflow alters the flow structure within a rotor-stator system by encouraging radially dominated conditions, more similar to that of the free disc. Given sufficient rate, the superimposed flow attenuates the core rotation towards zero. At high rates of throughflow the boundary layers become compressed, and although the relative tangential velocities in the stator boundary are reduced, the change in tangential velocity in the disc boundary layer is close to ωr . Correspondingly, the moment coefficients are increased. Experimental and numerical data from the literature shows that there is a significant effect of superimposed throughflow on the moment coefficient. A numerical study by Dorfman [5] showed that for the axial inlet case, doubling the ratio of axial to tangential velocity would increase the moment coefficient by 50 %, for laminar flow. Chew and Vaughan [14] predicted that although for the radial inlet case the free disc value of moment coefficient could not be exceeded, for the axial inlet case, the moment coefficient was found to increase with the value of λ_T . An attempt to model the rate of core rotation is found in the generalised solution of Owen [15]; see Equation 14. This approach models a system where fluid is being pumped through, which is similar to the arrangements commonly found in turbomachinery, and is representative of the conditions in the Sussex rig

$$C_M = \varepsilon_M \text{Re}_\phi^{-0.2} \quad \text{Equation 14}$$

Where ε_M depends on core rotation rate.

The result of Owen's solution is plotted using data from the Sussex experiments as set points, allowing a direct comparison with the new data. See Figure 6.

Figure 6: Comparison of the Moment Coefficient Data Obtained with Owen's Core Rotation Solution and Dorfman's Free Disc Correlation

A similarity in trend is evident between the new data and Owen's solution. The matching is closer than between the Sussex data and the correlation of Daily and Nece. A divergence between the data is however evident at the higher values of Re_ϕ shown. A characteristic of Owen's solution is that as the value of λ_T exceeds 0.25, the value of the moment coefficient decreases sharply. For cases where λ_T is increased by holding rotation constant while throughflow is increased, the relative tangential velocities between the rotating disc and fluid will increase to a limit, thus frictional drag and the moment coefficient will increase to a limit. The new data is therefore most appropriately compared to Owen's model where $0.15 < \lambda_T < 0.25$; away from the high rotation rates which are outside the range suitable for the $1/7^{\text{th}}$ power law used in Owen's model, and outside the throughflow dominated region where Owen's

model breaks down. Within this region the agreement is within approximately 20 %. See

Figure 7.

Figure 7: Comparison of Moment Coefficient Data Obtained with Owen's Core Ration Model

Daily Ernst and Asbedian [16] used a modified version of the test rig used by Daily and Nece [13] to perform experiments incorporating superimposed flows. Two methods of predicting the increases in moment coefficient were developed; a correlation of experimental data and a numerical model. See Equations 15 and 17 respectively.

$$\% C_M \text{ increase} = 1390 K_0 \frac{T_F}{\left[\frac{s}{b} \right]^{0.125}} \quad \text{Equation 15}$$

$$\text{Where:} \quad T_F = \frac{Q}{\omega b^3} \text{Re}_\phi^{0.2} \quad \text{Equation 16}$$

$$C_M = \left[\frac{0.663}{b^{23/5} \text{Re}_\phi^{0.2}} \int_0^b r^{18/5} (1 - K_r)^{5/4} \left[1 + \left(\frac{0.162}{1 - K_r} \right)^2 \right]^{3/8} dr \right] \quad \text{Equation 17}$$

Where:
$$K_r = \frac{K_0}{C T_F \left(\frac{b}{r} \right)^{13/5} + 1}$$
 Equation 18

Referring to Figure 8, the numerical model can be seen to predict moment coefficients lower than that found with the new experiments, particularly when the throughflow rates are high. The model is not particularly sensitive to the differing rates of throughflow. The Daily and Nece no throughflow solution is also shown, to highlight the step increase in moment coefficient predicted due to throughflow.

Figure 8: Comparison of the Moment Coefficient Data with the Numerical Throughflow Compensating Model of Daily Ernst and Asbedian

Plotting the correlation based model data against λ_T highlights a divergence between the data that occurs specifically as throughflow is increased beyond the entrainment rate of $\lambda_T \approx 0.2$, where the model predicts greater moment coefficients. Below this value agreement within 10 % is found. See Figure 9.

Figure 9: Comparison of the Moment Coefficient Data with the Correlation of Daily Ernst and Asbedian as a Function of λ_T

These divergences are explained by the characteristics of the compensating models developed by Daily Ernst and Asbedian [16], which were not performed at throughflow rates as high as the new experiments; a maximum of $C_w = 0.55 \times 10^4$ and $\lambda_T \approx 0.1$ was used, the moment coefficient limiting case of radially dominated flow with free disc like flow structures and compressed boundary layers, was not reached. Consequently, the correlation based model suggests that the moment coefficient should continue to increase indefinitely with increasing throughflow. The numerical model does observe the physical limit, but both models are considered to estimate the moment coefficient poorly once they are used, as in this case, outside the range of operation of the original experiments.

A solution by Owen [17] accounts for the core rotation rate using a term which is dependant upon the λ_T value of the fluid. Furthermore, the equation for $\lambda_T < 0.2$ models the core rotation at different regions of the disc. See Equation 19.

$$C_M = \left\{ \text{Re}_\phi^{-0.2} \left[0.0729 X_c^{\frac{23}{5}} + 0.0389 \left(\begin{aligned} &\left(1 - X_c^{\frac{23}{5}} \right) \\ &+ 14.7 \lambda_T \left(1 - X_c^2 \right) \\ &+ 90.4 \lambda_T \left(1 - X_c^{\frac{3}{5}} \right) \end{aligned} \right) \right] \right\} \times 2$$

Equation 19

$$\text{Where: } X_c = 1.79 \lambda_T^{\frac{5}{13}} \quad \text{Equation 20}$$

A source region around the hub of a disc was distinguished from the higher radial locations towards the disc periphery. This allows for the effect of a superimposed axial throughflow and the impinging jet effect to be modelled more accurately. For the case of $\lambda_T < 0.2$, and for low throughflow rates, agreement within the bounds of uncertainty can be seen between Owen's model and the new data. At the higher rates of throughflow agreement remains typically within 10 %. Plotting this data against λ_T highlights the similarity in the characteristics of the data. See Figure 10.

Figure 10 Comparison of the Moment Coefficient Data with Owen's Regional Model for $\lambda_T \leq 0.2$

Comparison with the relationship of Owen [17] given in Equation 23, is shown in Figure 11.

This is appropriate for the fluid regime where $\lambda_T > 0.2$.

$$C_M = 0.666 \lambda_T \text{Re}_\phi^{-0.2} \quad \text{Equation 21}$$

Agreement is found only where λ_T tends to 0.2. For all other conditions the predicted moment coefficient is significantly higher than the equivalent Sussex data. This is likely to be due to a lack of contemporary experimental data at high throughflow rates. The equation for this regime appears to be more sensitive to λ_T than the rate of throughflow specifically.

Figure 11: Comparison of the Moment Coefficient Data with Owen's Regional Model for $\lambda_T \geq 0.2$

Gartner [9] developed solutions for each of the cases $\lambda_T < 0.2$ and $\lambda_T > 0.2$. For $\lambda_T < 0.2$, Owen's multi region model [17] was modified, incorporating a new third term, which included an operator that was adjusted to give good agreement with experimental data collected as part of the original work. The data agrees with the new data less well than the original expression of Owen. See Figure 12.

Figure 12: Comparison of the Moment Coefficient Data with the Model Developed by Gartner for $\lambda_T \leq 0.2$

The case of $\lambda_T > 0.2$, is satisfied by an expression modelling high throughflow conditions specifically, see Equation 22.

$$C_M = \left\{ 0.2827 \text{Re}_\phi^{-0.25} \left(\frac{s}{b} \right)^{-0.25} \int_0^1 x^{\frac{15}{4}} \left[1 + \frac{0.02533 \lambda_T^2 \text{Re}_\phi^{-0.08} \left(\frac{s}{b} \right)^{1.14}}{(0.0017 + 0.0162 \lambda_T)^2} x^{-4} \right]^{\frac{3}{8}} dx \right\} \times 2$$

Equation 22

Comparing this to the new data shows similarity in characteristic, and good agreement, within approximately 10 %. See Figure 13.

Figure 13: Comparison of the Moment Coefficient Data with the Model Developed by Gartner for $\lambda_T \geq 0.2$

8 Correlation of the New Data

The results obtained from the tests described have been used to develop a new windage correlation. The correlations are presented in terms of the moment coefficient, C_M . The correlation takes the general form of:

$$C_M = [C_w]^a [Re_\phi]^b + C \quad \text{Equation 23}$$

Tests conducted where $\lambda_T < 0.2$ may be considered to possess rotationally dominated flow. Tests performed where $\lambda_T > 0.2$ may be considered to have radially dominated flow. Since the correlations are in terms of C_w and Re_ϕ , the flow regime is effectively considered, due to the following relationship:

$$\lambda_T = \frac{C_w}{Re_\phi^{0.8}} \quad \text{Equation 24}$$

The expression correlating the moment coefficients for a plain disc rotor-stator scheme is given as:

$$C_M = 0.52 [C_w]^{0.37} [Re_\phi]^{-0.57} + 0.0028 \quad \text{Equation 25}$$

The result of the plain disc rotor-stator scheme correlation is shown in Figure 14, with error bars of $\pm 8\%$. These error guides represent the typical uncertainty, with a 95 % confidence interval, associated with the moment coefficient values calculated from the new experimental data.

Figure 14: Collapse of the Moment Coefficient Data

Limits of Correlation

$$3.0 \times 10^6 \leq Re_\phi \leq 2.5 \times 10^7$$

$$3.0 \times 10^4 \leq C_w \leq 1.0 \times 10^5$$

$$0.05 \leq \lambda_T \leq 0.5$$

8.1 Comparison with Existing Correlations

Having successfully collapsed the measured data it is useful to compare the new correlation with an existing correlation. The windage model of Owen [17] is considered to provide a valid comparison where $\lambda_T \leq 0.2$; the correlation of Gartner [9] where $\lambda_T \geq 0.2$. The new correlation has been plotted with the results of Owen and Gartner, calculated using data points from the new experiments. See Figure 15.

Figure 15: Comparison of New Correlation with Owen and Gartner

Referring to Figure 15 it can be seen that good agreement is found between the correlations, particularly at the lower flow conditions. The differences which can be seen may be largely attributed to the experimental data available for comparison when the work of Owen and Gartner was performed. Although the λ_T values may be comparable, the conditions used for the new data involve a significant jet effect, due to the high velocity of the air entering the cavity and hitting the disc at the high throughflow conditions. This is known to increase the moment coefficient by increasing the relative tangential velocities at the low radius, and increasing the

boundary layer stresses. The new correlation thus provides an improvement in the prediction of the moment coefficient where $Re_\phi \rightarrow 10^7$ and $C_W \rightarrow 10^5$.

9 Non-Invasive Measurements

The flow velocity and associated disc surface temperature measurements obtained provide useful insight into the mechanisms driving windage and corroborate the existence of differing flow regimes as represented by the new correlation.

9.1 Laser Doppler Anemometry Measurements

Laser Doppler Anemometry has been used to measure flow velocities in the ‘test’ side cavity, allowing quantification of the flow structure at regimes defined by the parameter λ_T . The parameter β , the ratio of the tangential velocities of the fluid and the disc is plotted against the non dimensional distance from the disc, x / s . A qualitative curve has been fitted between the data points. The velocities were measured using a standard X, Y coordinate system, where the Y axis is true vertical. In order to obtain tangential velocities normal to the disc at the X, Y coordinates of the measurement locations, the raw data was transposed according to the angle displaced from the measurement location to true vertical using a matrix transformation function within the software provided by the laser equipment supplier, Dantec. The measurement field is shown in Figure 16.

Figure 16: LDA Flow Measurement Field

Radially Dominated Flow

The results are consistent with the flow fields found in simple wide gap ratio rotor-stator systems, where the core rotation velocity slows with increasing axial distance from the rotating disc. The rate of rotation near the disc is as may be expected for the radially dominated flow; reaching a maximum of around 10 % of the disc. See Figure 17.

Figure 17: Fluid Velocity Data, $Re_\phi = 0.27 \times 10^7$, $\lambda_T = 0.22$, $C_W = 0.3 \times 10^5$

Rotationally Dominated Flow

A developed core of fluid rotating at approximately 40 m/s was found for the nominally rotationally dominated case, where $\beta = 0.25$, such that the relative tangential velocity across the boundary layer at the disc periphery, after the fluid has been accelerated, is approximately 120 m/s. See Figure 18. This may be compared to the throughflow dominated case, where the fluid core velocity is 8 m/s, $\beta = 0.11$ and the relative tangential velocity is approximately 60 m/s. See Figure 17. Although the difference between the disc and fluid rate as expressed a ratio is greater for the throughflow dominated case, the magnitude of the velocity differential occurring with the rotationally dominated case is twice as great. The increased boundary layer stress and viscous friction drives the significant disc heating as measured with the infrared sensors. This data highlights the importance of considering the magnitude of the terms used to derive a value for the parameter λ_T , which may be achieved by altering the rate of disc rotation or the rate of throughflow.

Figure 18: Fluid Velocity Data, $Re_\phi = 0.8 \times 10^7$, $\lambda_T = 0.09$, $C_W = 0.3 \times 10^5$

9.2 Infra Red Measurements

The infrared measurements provide a means of measuring directly the temperature of the disc surface due to windage heating. Data is shown for cases representing nominally rotationally and radially dominated flow as defined by the parameter λ_T . The instrumentation was concentrated to the ‘test’ side of the rig casing, but a measurement on the ‘balance’ side was also taken in order to confirm flow symmetry and ensure that no significant heat transfer across the disc would occur. Conditions were considered to be stable enough for data to be collected once the changes in the temperature rise between the inlet and outlet air remained within 0.5 K over a 300 second period; small and simultaneous increases in the temperatures are due to changes in the temperature of the air delivered by the compressor. By examining the disc temperature with respect to radius, the windage corresponding to the regions within the cavity can be better understood. Referring to Figure 19, it can be seen that a temperature rise of approximately 1 K occurs between the radial locations of $r / b = 0.44$ and 0.68. The temperature rise between $r / b = 0.68$ and 0.93 is 1.5 K.

Figure 19: Disc Surface Temperature Map, $Re_\phi = 0.27 \times 10^7$, $\lambda_T = 0.22$, $C_W = 0.3 \times 10^5$

It can be seen perhaps most clearly by comparing the nominally rotationally and throughflow dominated cases, that the radial temperature gradient across the disc is influenced by the tangential velocity gradients in the boundary layers. For the case where $\lambda_T = 0.22$, nominally radially dominated flow, the temperature rise with respect to radius is approximately 2 K; for the equivalent rotationally dominated test, where $\lambda_T = 0.09$; achieved in this particular case by increasing the disc rotational rate while maintaining a similar throughflow rate, the temperature rise is increased to approximately 20 K; see Figure 20. This increase in the temperature rise is consistent with the relative tangential velocity data obtained using LDA.

Figure 20: Disc Surface Temperature Map, $Re_\phi = 0.8 \times 10^7$, $\lambda_T = 0.09$, $C_W = 0.3 \times 10^5$

10 Concluding Remarks

New experimental data has been presented for the windage associated with a rotating disc in a rotor-stator cavity with a superimposed throughflow of air. The data is relevant to gas turbine engine and some electrical machine applications. The data has been obtained at high rotational

Reynolds numbers and throughflow Reynolds numbers extending significantly the range of application and flow regime over previous studies. The best match between the new data and the moment coefficients available in the literature was found, as might be expected, when comparing to models which were designed to replicate the friction of a rotor-stator system with geometry and fluid conditions similar to that of the new test rig. For cases where $\lambda_T < 0.2$, the model of Owen [17] (Equation 19) gives the best agreement, within 10 %. For cases where $\lambda_T > 0.2$, the model of Gartner [9] (Equation 22) gives the best agreement, within 10 %. The IR disc temperature and LDA flow velocity data obtained as part of these experiments are useful in highlighting the parameters important to the moment coefficient and also disc heating. The new data has been used to develop a correlation for the moment coefficient as a function of both the throughflow and rotational Reynolds numbers. The correlation is valid for $3 \times 10^6 < Re_\phi < 2.5 \times 10^7$ and $3 \times 10^4 < C_w < 1 \times 10^5$ and matches the data within +/- 8 % with a 95 % confidence interval.

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