~~THE~~ ELASTIC BEHAVIOUR OF ROCKS
AT LOW FREQUENCIES

A Thesis

submitted by

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for the

Degree of Doctor of Philosophy

in the

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May 1961

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ABSTRACT

An apparatus has been constructed to measure the variations with frequency of Young's modulus and the Internal Friction in rock specimens, over a frequency range of 2 to 40 cycles per sec. This follows work by Birch and Bancroft (4500 cycles per sec. to 140 cycles per sec.) and by Bruckshaw and Mahanta (120 cycles per sec. to 40 cycles per sec.). The apparatus uses beam-shaped samples of rock which are clamped at each end and forced into vibration below resonance by an electromagnetic transducer. The amplitude and phase of the vibrating rock beam are compared with those of a steel beam of known properties, using a Fizeau fringe optical technique.

Measurements have been made on several sedimentary and igneous rocks, and the effects of oil and water content have been studied. The results show a decrease in both Young's modulus and internal friction as the frequency is decreased below 10 cycles per sec. The two effects appear to be linked. The internal friction is considerably increased in sandstones when water or oil is present, and increases with frequency; Young's modulus is greatly decreased by water, but oil has little effect on it. It is suggested that a "viscous-type"

mechanism is operative in rocks at very low frequencies, and that it is associated with the various centres of inhomogeneity in the specimen.

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SECTION 1

INTRODUCTION

The seismic method has been widely used in the study of the structure of the rock strata below the surface of the earth. The method consists of the production of a shock wave near the earth's surface; some of the propagated elastic energy is reflected or refracted by the various **sub-surface** rock strata and is detected by suitable instruments when it returns to the surface. Information concerning the elastic properties of the rocks concerned can be deduced by measuring the time interval between the moment of production of the shock wave and the time of arrival of the reflected or refracted wave at the recording instrument. By a suitable analysis, it is possible to deduce the shape of the strata and to discover "structures" which are often associated with the accumulation of oil; this is the geophysical method of prospecting for oil. Similarly, the distribution of the elastic properties of the rocks concerned can also be deduced, and a general study of the elastic behaviour of rocks can be made. Such a study is clearly very important in the interpretation of seismic measurements.

The fundamental measured quantity in seismic work is the time interval mentioned above. This depends on the velocity of the stress wave which is a function of the elastic constants and the density of the rocks encountered. The amount of absorption or internal friction occurring is also important, since this governs the final amplitude of the stress wave at the recording instrument. Both the elastic constants and the internal friction are affected by a number of parameters such as the pressure, temperature and moisture content applying, and also by the frequency content of the stress wave. It is clear that these parameters are very important, and that a knowledge of their effects is very useful in forming a correct analysis of seismic results. Direct investigation of these effects by special seismic techniques is very difficult, and it is easier to study them in the laboratory where quantitative investigations of any parameter can be made. The effects of pressure and temperature have been very fully studied in this way. Laboratory experiments on the variation of elastic behaviour with frequency have been largely confined to frequencies greater than 1000 cycles per sec; since the frequencies normally associated with seismic work lie between about 0.5 cycles per sec. and 300 cycles per sec., it is clear that work in this lower region is of considerable importance.

The present work is thus concerned with the variation with frequency of Young's modulus and the internal friction in rocks over a low frequency range from 2 - 40 cycles per sec. The effects of moisture content and oil content have also been studied.

Section 2 gives an account of some of the most important methods of measurement of elastic moduli and internal friction and in Section 3 some of the mechanisms which have been put forward to explain the observed behaviour of rocks are discussed. This is followed by a review of previous relevant work. The results of earlier workers on elastic moduli and internal friction are stated and compared, and conclusions are drawn regarding the general trend of their results.

The apparatus used is described in Section 5. The need for the present work is emphasized and the necessary apparatus which must be sensitive at frequencies down to 2 cycles per sec. is discussed. In a full discussion the way in which the present equipment meets these requirements is indicated. Section 6 contains the theory of the method, and the corrections which must be applied. A detailed account of the method of operation is given. The various mechanical problems associated with the apparatus are considered in Section 7. The experiments performed to study these problems are described, and the results of experiments to

test that the apparatus was working properly are given. An experimental error is deduced from these experiments and is shown to be of the order expected from more theoretical considerations.

In Section 8 the results are tabulated. Results are given for the variation of Young's modulus with frequency, the variation of internal friction with frequency, for some experiments on materials other than rocks, and finally, for the effect of oil and water content on the behaviour of rocks. All experiments were over the frequency range of 2 to 40 cycles per sec. The results are then discussed and compared with those of previous workers. Reasonable agreement is found in general.

The final section contains the conclusions. It is found that a variation in Young's modulus and internal friction with frequency occurs and that the two variations are linked, suggesting a viscous type of mechanism. A qualitative explanation of the effects of water and oil is given, suggesting that the increased dissipation produced is due to movements through the pores and cracks in the rock. Finally, the basic loss process occurring is discussed in the light of both previous and present work. The suggested viscous type of mechanism is thought to be associated with the various centres of inhomogeneity in the rock samples.

SECTION 2

METHODS OF MEASUREMENT OF ELASTIC BEHAVIOUR OF ROCKS

The particular quantities investigated in this work are Young's modulus and the internal friction. A discussion of the various earlier methods of measuring these quantities is given below, and the results of actual measurements are discussed in a later section.

Methods of Measurement of Elastic Moduli

When a material is subjected to a stress, a strain is, in general, produced. For a homogeneous, isotropic, elastic material, the stress and strain may be linearly related by various constants, depending on the type of stress. These constants are called the elastic moduli of the material. For a unidirectional stress the constant is called Young's modulus (E); for a shear it is the rigidity modulus (n); for simultaneous stresses in three mutually perpendicular directions it is the bulk modulus (k). The three constants are related by the equations

$$E = 2n (1 + \sigma) = 3k (1 - 2\sigma),$$

where σ is Poisson's ratio which represents the ratio of lateral contraction to longitudinal extension in a unidirectionally stressed material. If any two of these quantities are known, the other two may be calculated. There are a number of methods of measuring them, some of which are mentioned below.

1. Static Methods

A specimen, normally in the form of a cylindrical rod, is loaded to produce a measurable deformation. The loading is usually axial and compressional, though torsional (shearing) stresses have been used. The relevant modulus is then found directly by measuring the displacement for a known stress. Considerable magnification is required and several methods are available. An optical lever arrangement giving a sensitivity up to 1×10^{-6} cm. has been used; interference fringes and resistance strain - gauges have also provided a high degree of accuracy. E and σ are the quantities usually determined in compressional experiments, and n and σ in torsional experiments. The method suffers from the fact that large stresses are required to produce a satisfactory deformation, and the dependence of E on the applied stress makes its value somewhat indefinite, as is shown later.

2. Dynamic Methods. (Laboratory)

The resonant frequency of a clamped bar of material is a function of the relevant elastic constant, and of the dimensions and mass distribution of the bar. The latter quantities being measurable, the modulus can be found if the resonant frequency is determined. In the case of a centre-clamped prismatic or cylindrical specimen in longitudinal vibration, the velocity V_B of longitudinal waves is

$$V_B = \sqrt{\frac{E}{\rho}}$$

where ρ = density.

$$\text{At resonance, } V_B = \frac{2}{2n + 1} \cdot f \cdot l.$$

where l is the length of bar,

n is an integer, and

f is the resonant frequency.

E is thus easily found. The specimen is forced into vibration by means of an electrostatic or electromagnetic transducer, and the amplitude of vibration is measured as a function of frequency by a similar transducer, thus enabling the resonant frequency to be found. Generally, clamping and loading corrections are necessary.

An alternative method is to force a specimen into vibration below resonance, and to measure the amplitude. E can

then be determined if the magnitude of the applied force, and the dimensions of the specimen, are known.

These methods are the ones most commonly used. Difficulties are introduced by clamping (or supporting) the specimens, and in calculating corrections for loading etc. These are discussed in a later section.

3. Pulse Methods

These methods provide the only way of finding the bulk modulus directly in the laboratory. The energy in pulses of several megacycles per sec. can be concentrated into a narrow beam by diffraction effects (if the radiating element is several wavelengths in aperture) and the time of transit of a pulse over a known distance of material can be measured.

In an extended elastic medium, only two types of body waves are possible - longitudinal and transverse (or shear). The respective velocities V_L and V_T of these are given by

$$\begin{aligned}
 V_L &= \sqrt{\frac{E}{\rho} \cdot \frac{(1 - \sigma)}{(1 + \sigma)(1 - 2\sigma)}} = \sqrt{\frac{k + 4/3 n}{\rho}} \\
 &\quad \left(= \sqrt{\frac{k}{\rho}} \quad \text{if the medium cannot support shear.} \right), \\
 V_T &= \sqrt{\frac{n}{\rho}} = \sqrt{\frac{E}{2\rho(1 + \sigma)}} \\
 \text{and } \frac{V_L}{V_T} &= \sqrt{\frac{2(1 - \sigma)}{(1 - 2\sigma)}}
 \end{aligned}$$

It can be shown that the condition applying to the transmission of pulses in a rod, of suitable dimensions, approximates to that in an extended (i.e. unbounded) elastic medium.

Piezo-electric crystals are used to produce and detect the vibrations, and electronic delay methods are used for the time measurement. The various moduli can be deduced from the measured velocities.

Difficulties occur on account of scattering and diffraction effects. However, the relevance of such high frequency measurements to the results for rocks examined by seismic means is doubtful, as explained later.

4. Seismic Methods

In these methods, artificial shock waves are excited in the ground by the detonation of an explosive charge, or by a heavy hammer, and the velocities of the various waves produced are measured. The waves may be reflected or refracted from the various strata of rock below the surface of the earth. In addition to the longitudinal and transverse waves mentioned above, surface waves (Rayleigh and Love) are produced.

The arrivals of the various types of wave are recorded

at known distances from the shot point by means of seismographs which produce a continuous record of the ground motion on a moving photographic film. By the use of three component seismograms, measuring the vertical component and the two horizontal components in, and perpendicular to the direction of the stress wave, the various waves may be identified and the required constants deduced.

5. Earthquake Methods

In a similar manner to that described above, the various waves associated with earthquakes may be studied, and the relevant elastic moduli deduced. The main difference between this method and that above, is that the frequencies involved in this case are much lower.

Methods of Measurement of Internal Friction

The amplitude of vibration of any freely vibrating material is found to decrease with time, even in the absence of all external loss factors. The energy of vibration is converted into heat, and the processes by which this occurs are collectively known as "internal friction".

Internal Friction may be defined in terms of the quantity $\frac{\Delta W}{W}$ where ΔW is the Energy dissipated in one stress cycle and W is the maximum elastic energy stored in one cycle.

$\frac{\Delta W}{W}$ is known as the specific damping capacity or specific loss factor.

Another method of defining internal friction is in terms of the logarithmic decrement δ . This is defined as the logarithm of the ratio of successive amplitudes of vibration.

$$\delta = \log \frac{(n^{\text{th}}) \text{ amplitude}}{(n+1^{\text{th}}) \text{ amplitude}} = \log \frac{A_n}{A_{n+1}}$$

For small damping, it can be shown that

$$\delta \doteq \frac{1}{2} \frac{\Delta W}{W}$$

A further measure of internal friction is obtained from the resonance curve for a specimen, and is known as the Q factor. This is defined by

$$\frac{\Delta W}{W} = \frac{2\pi}{Q} \quad \text{where} \quad Q = \frac{f_0}{\Delta f}$$

f_0 is the resonant frequency and Δf is the width of the resonance curve at $\frac{1}{\sqrt{2}}$ of the resonant amplitude; this corresponds to the half-energy position.

Internal friction may also be defined in terms of the attenuation of a plane wave. The amplitude A_x at a distance

x from the zero position is given by

$$A_x = A_0 e^{-\alpha x}$$

where α is the attenuation coefficient. An exact relation between $\frac{\Delta W}{W}$ and α is difficult to obtain, on account of the differences between laboratory and field specimens. One relation is

$$\delta = \frac{\alpha c}{f}$$

where c is the velocity of the wave
and f is the frequency.

Born suggests that $\frac{\Delta W}{W}_{\text{bar}} \doteq 2.1 \frac{\Delta W}{W}_{\text{bulk}}$ so that

$$\frac{\Delta W}{W}_{\text{bar}} = 4 \frac{\alpha c}{f}$$

The various methods of measuring internal friction use similar arrangements for measuring the elastic moduli.

1. Static Methods

The energy loss per cycle (ΔW) is equal to the area of the hysteresis loop obtained in a stress-strain diagram for the specimen by slow loading and unloading. However, the area is generally too small for accurate measurement.

2. Dynamic Methods

(a) Free vibrations

The logarithmic decrement δ can be measured by observing the rate of decay in amplitude of a freely vibrating specimen (in the shape of a bar or cylinder) of material. The natural resonant frequency of the system can be varied by changing the specimen length, or by loading it. Photographic techniques have been used up to an upper limit of 1000 cycles per sec; the lower limit is governed by the maximum specimen size or by the maximum loading possible.

The method fails for very high internal frictions, when few oscillations occur, and for very low internal friction where losses to air and supports are important.

(b) Resonance method

As indicated above, a resonance curve can be plotted and the Q value obtained from it. The method is again inaccurate for very low and very high internal frictions. The lower frequency limit is as above. However, it can be used for high frequencies (up to 10^5 cycles per sec.) and, on account of the ease with which measurements can be made, has been used in the majority of cases.

(c) Forced vibration method

If dissipation occurs in a specimen undergoing a periodic stress cycle, a phase difference ϕ will occur between the exciting force and the vibration it produces. When the frequency of vibration is very much less than the resonant frequency, ϕ is related to $\frac{\Delta W}{W}$ by

$$\frac{\Delta W}{W} = 2\pi \tan \phi$$

ϕ is very small in practice, so that considerable sensitivity is required. The upper usable frequency is limited by resonance, but the great advantage of the method is that there is no lower limit to the frequency range. Again, very low internal frictions cannot be accurately measured.

3. Pulse Methods

By observation of the initial and final amplitudes of a pulse, the attenuation coefficient α can be found and related to $\frac{\Delta W}{W}$. However, the very high frequencies used produce scattering (if the wavelength is of the same order as the grain size) so that the significance of a comparison of these results with results obtained by other methods, is doubtful.

4. Seismic Methods

The situation in seismic methods may be complicated by the production of a spherical wavefront; the equation then becomes

$$\frac{A_x}{x} = \frac{A_0}{x} e^{-\alpha x}$$

in an ideal case.

As explained above, many complications arise, but a comparison of the value for α can be made with laboratory results. The frequency range is controlled by the frequency content of the shock produced and generally lies in the range 0.5 to 300 cycles per sec.

5. Earthquake Methods

The decay of earthquake waves has been studied. Values of α and $\frac{\Delta W}{W}$ have been deduced (for waves of periods about 200 sec.), but a comparison with other values is difficult because of the assumptions necessary concerning the conditions at great depths below the surface.

SECTION 3.

MECHANISMS TO DESCRIBE THE BEHAVIOUR OF ROCKS

The work of Zisman (see later) demonstrated that rock specimens cannot be assumed to obey the ideal equations of elasticity. He subjected his specimens to a stress cycle and obtained hysteresis loops; he also noted that creep occurred leading to a permanent set, and that there was a certain amount of elastic afterworking. Later workers found similar effects. It is clear that a time dependent factor must be introduced into the elastic equations. Several models to describe the anelastic behaviour of materials have been suggested, by putting in the time dependent term in different ways. The most important models are described below, and their behaviour is summarised in Fig. 1.

1. Maxwell Solid (elastico-viscous material)

The equation of perfect elastic behaviour

$$\sigma = \epsilon E$$

where σ = the stress,

ϵ = the strain

E = Young's modulus,

is replaced by $\dot{\sigma} = \dot{\epsilon} E - \frac{\sigma}{\tau}$

where γ is a relaxation time. For $t \ll \gamma$, the equation becomes that for a perfectly elastic solid.

The model can be represented by a spring (elastic element) in series with a dashpot (viscous element). The strain-time diagram is shown for a sudden stress applied and removed. The dotted line, in Fig. 1, represents the behaviour of a perfectly elastic solid, and the continuous one that for the particular model concerned. A permanent set occurs on account of creep.

It can be seen that the measured value of E would be smaller at low frequencies than at high frequencies. It can be shown that for low frequencies or small γ , δ is constant, and that for high frequencies or high γ , δ is inversely proportional to the frequency. Thus, as will be seen later, the model does not agree with observed results.

2. Voigt Solid (firmo-viscous material)

Voigt assumed that the stress is proportional to the sum of two terms, one dependent on the strain and the other dependent on the rate of change of strain. His equation is

$$\sigma = E\epsilon + \eta \frac{d\epsilon}{dt}$$

where E and η are constants.

This can be represented by a spring and dashpot in

parallel, and the strain-time curve for a sudden stress is as shown. For high frequencies the elastic modulus would be greater than for low frequencies. It can be shown that for low frequencies f and with low damping, $\delta \propto f$, and for high frequencies or high damping, δ is constant. However, if practical values of the various quantities are used, for most rocks $\delta \propto f$ and little transmission of energy would occur at frequencies greater than about 100 cycles per sec.

The solution has been modified by Stokes, and also by Ricker. Ricker's solution involves a critical frequency below which a proportionality of δ with frequency holds and little dispersion occurs. This proportionality should hold up to 200 cycles per sec. in some cases, but this figure is somewhat doubtful on account of the assumptions introduced.

3. Solido-Fluid Medium

Stoke's and Ricker's solutions fail for high values of η , or for high values of f . Many fluids become very viscous at low temperature (i.e. $\eta \rightarrow \infty$) and it is possible to modify the Voigt solution to link the behaviour of fluids with solids. The spring and dashpot are of variable magnitude; the dashpot vanishes as η approaches ∞ (ideal solid) and the spring vanishes for η approaches zero, (ideal fluid).

The mathematics is somewhat complicated, and involves

tensors. It can be shown that very little dispersion should occur. For small f , $\delta \propto f$ and for high f , $\delta \rightarrow 0$. Similarly $\delta \propto f$ for low η and $\delta \rightarrow 0$ for high η . Numerical values of η suggest that in the seismic region $\delta \propto f$ for low frequencies but that a transition occurs about 50 cycles per sec. beyond which δ decreases with f .

4. Boltzmann Solid

Boltzmann suggested that the behaviour of a solid under an applied stress is a function of its entire previous stress history. This can be expressed by

$$\sigma'(T) = \sigma(T) - \int_{-\infty}^T \sigma(t) \chi(T-t) dt .$$

$\sigma'(T)$ is the stress at time T after applying the stress $\sigma(T)$. T is the time of observation and t is the time stretching from $-\infty$ to T , being zero at, say, the beginning of an experiment. $\chi(T-t)$ is a memory function. The first term on the right hand side is thus the present contribution to stress, and the second term the contribution due to the previous history.

The solid may be visualised as a combination of springs and dashpots, but the equations are not, in general, soluble. The general effect on δ is to produce a maximum at some frequency.

5. Sokoloff and Skriabin Model (relaxation mechanism)

The Boltzmann equation was simplified by Sokoloff and Skriabin, who showed experimentally that $\chi(T-t)$ may be chosen such that

$$\chi(T-t) = \beta e^{-\lambda(T-t)}$$

where β and λ are constants for a particular rock.

The model can be represented as a spring in series with a spring and dashpot in parallel; the introduction of a further dashpot in series with the rest would produce creep and a permanent set. The strain-time diagrams are as shown.

It can be shown that the static modulus is smaller than the dynamic modulus, and that $\bar{\epsilon}$ approaches zero for both very low and high frequencies. Calculations indicate that the maximum $\bar{\epsilon}$ occurs for very low frequencies (< 1 cycle per sec.).

6. Combination Mechanisms (viscous)

Early measurements on solids suggested that $\bar{\epsilon}$ may be independent of frequency over a wide range; however, there are no actual measurements on rocks at low frequencies. For this reason, several mechanisms involving a sum of several springs and dashpots have been suggested. This is equivalent to assuming a sum of relaxation times - this could also be done for the Sokoloff-Skriabin model by taking a sum for the

term involving the memory function.

Quantitative calculations become very difficult, but the general effect is to increase the range in frequency over which the value of $\bar{\epsilon}$ is constant.

7. Solid Friction Models

It has been pointed out by several workers (recently by Knopoff) that any combination of viscous elements will lead to some frequency dependence of $\bar{\epsilon}$. A model is required which will predict no variation at all, and a solid friction mechanism was suggested.

If a purely resistive frictional force was responsible for the dissipation, the area of the stress-strain hysteresis loop would be constant and independent of frequency. The values of ΔW and hence of $\frac{\Delta W}{W}$ would be constant. Also, no dispersion would occur. Thus, the experimental results found for many solids could be explained. However, a difficulty occurs in that for sufficiently low stresses, no frictional movement should occur and there should be no loss. Also, most experimental results have been done in the proportional stress-strain region, and slipping of grains does not occur till the yield point is reached. The actual mechanism is thus somewhat obscure.

8. Thermal Mechanisms

A temperature gradient is in general produced in a vibrating material, and this produces heat flow for sufficiently low frequencies. A frequency variation would produce a change between adiabatic and isothermal conditions, and this would produce a maximum loss in energy at some optimum frequency. Also, there would be a difference between the adiabatic and isothermal values of Young's modulus. Since the critical frequency is very low, a superposition of effects would be required to explain the behaviour of rocks. A thermal mechanism alone could not explain the observed behaviour, since no account of creep etc. could be given.

It can be seen that a large number of models have been suggested to give an account of the behaviour of solids. However, even if one of these models gave an accurate account of the observed facts, the physical picture of what is happening would be little improved (except possibly in the thermal case). It is convenient to "explain" solid behaviour in terms of springs and dashpots (or frictional resistances), but the physical meaning of a dashpot when considering a piece of rock is by no means clear.

FIG. 1. SOLID MECHANISMS

Model	Mechanism	Effect on E	Effect on $\frac{\sigma}{\omega}$	Strain-time graph
Maxwell	<p>Viscous. $\dot{\sigma} = E \dot{\epsilon} - \frac{\sigma}{\tau}$</p>	$E_0 > E_s$ $E \rightarrow v.$ small for $v.$ low f .	$\dot{\sigma} = \text{const.}$ (small f, τ) $\dot{\sigma} \propto \frac{1}{f}$ (high f, τ)
Voigt.	<p>Viscous $\sigma = E\epsilon + \eta \frac{d\epsilon}{dt}$</p>	$E_0 > E_s$ $E \rightarrow v.$ large for $v.$ high f .	$\dot{\sigma} \propto f$ (low f, η) $\dot{\sigma} = \text{const.}$ (high f, η)
Solido-fluid	<p>Viscous. Combination of springs & dashpots of variable relative size.</p>	<p>Little dispersion.</p>	$\dot{\sigma} \propto f$ (small for η) $\dot{\sigma} \rightarrow 0$ (large η)	$\eta \rightarrow \infty$ (ideal solid) $\eta \rightarrow 0$ (ideal fluid)
Boltzmann	<p>Memory function. Combination of springs & dashpots.</p>	<p>Not generally soluble.</p>	<p>Not generally soluble.</p>	$\sigma(t) = F(\epsilon) + \int_{-\infty}^t \dot{\epsilon}(\tau) \chi(t-\tau) d\tau$
Sokoloff and Skrabin.	<p>Simplification of Boltzmann. $\chi = \beta e^{-\alpha t}$</p>	$E_0 > E_s$ Most change at $v.$ low f .	$\dot{\sigma} \rightarrow 0$ ($v.$ low f) $\dot{\sigma} \rightarrow 0$ ($v.$ high f)
Solid Friction	<p>Frictional.</p>	$E_0 = E_s$	$\dot{\sigma} = \text{const.}$	<p>—</p>
Thermal Mechanisms.	<p>Thermal. Isothermal and adiabatic.</p>	$E_0 > E_s$	Max. $\dot{\sigma}$ at $v.$ low f .	<p>—</p>

SECTION 4

REVIEW OF PREVIOUS WORK

The various methods of measurement of elastic constants and internal friction have been mentioned in a previous section. A large number of measurements have been made, and some of the more important results are discussed below.

Elastic Constants

1. Static methods

The earliest measurements were made by Nagaka (1900)⁽¹⁾ using a horizontal beam loaded at its centre. His results were not very accurate. Adams and Coker (1906)⁽²⁾ did several measurements on cylindrical cores of rocks, which were loaded (i.e. in compression) along the axis. Some of their results are given in Table 1.

Zisman (1933)⁽³⁾ made a large number of measurements (on granites and norites in particular). He measured E and ν for cylindrical specimens held in the jaws of a testing machine (providing 10^3 lb/in²), using an optical lever method

of accuracy 1 in 10^7 . The specimens were put through cycles of 1000 lb/in^2 and elastic hysteresis was observed, a hysteresis loop (of small area) being formed. Creep was found to occur, leading to a permanent set, and a certain amount of elastic afterworking was also observed. He found that the mean value of E (corresponding to the centre of the loop) was dependent on the load used, which explained inconsistencies in the results of previous workers. This variation was less for high loads and he suggested that this was due to porosity.

Zisman also modified methods by Adams and Coker⁽²⁾, Adams and Williamson⁽⁴⁾, and also Bridgeman⁽⁵⁾. A hydrostatic pressure of 840 kg/cm^2 was applied to the specimens, the displacement being measured by means of a resistance-strain gauge. Samples were cut in three perpendicular directions, and anisotropy was noticed. Thin copper foil was used to cover the specimens to keep the pressure fluid from them, and it was found in all cases that $E_{\text{(covered)}}$ was less than $E_{\text{(uncovered)}}$. As above, E was proportional to the pressure applied; the covered and uncovered values approached one another at about 800 kg./cm^2 . Similar results were observed by Bridgeman, using very high pressures up to $10,000 \text{ kg./cm}^2$.

Zisman compared his results with the seismic measurements

of Leet and Ewing⁽⁶⁾. His values for E were smaller than theirs; and his value for β' (the compressibility) for uncovered rocks agreed well with their value, though his "covered" value for β' was much different. He suggested that the uncovered case, in which the pressure fluid is in contact with the rock, more nearly approaches the natural state than does the covered case; also, porosity effects must account for his low values for E . Porosity aids extension, but at high frequencies there would be insufficient time for this to occur. Similarly, $E_{\text{(covered)}}$ is less than $E_{\text{(uncovered)}}$ since the pressure medium fills the pores. Thus the dynamic state is equivalent to the uncovered state, and the static state is equivalent to the covered state.

He also compared his results with those of Richards⁽⁷⁾, on limestone cores in Persia. Richards used very compact rocks, and obtained similar results for E from both seismic and static measurements. Muzzey's results (using a magnetostriction method) were also quoted, and the static values were again smaller than the dynamic, the difference being greater for the less compact rocks.

Generally then, static measurements demonstrated the importance of pressure and porosity effects on the elastic constants. The difference between static and dynamic values for E was often noticed.

2. Dynamic methods

An early measurement by Kusakabe (1904)⁽⁸⁾, who clamped his specimen at its centre and excited vibrations by tapping one end, is of only historic interest.

Ide⁽⁹⁾ excited his specimens electrostatically and detected the vibrations by a piezo-electric method. He used some of Zisman's specimens, and compared his dynamic results with Zisman's static results, and also with Leet and Ewing's seismic results. He investigated Sudbury norites, Quincy granites and Rockport granites, and concluded that the dynamic values for E exceeded the static values by between 5 per cent and 20 per cent. Similar results were obtained for the rigidity modulus, using torsional vibrations. He further demonstrated that rocks cannot be considered to be perfectly elastic media, by calculating σ by several different formulae and pointing out the discrepancies.

Similar methods were used by Jacobs and Bancroft⁽¹⁰⁾ (electrostatic), Wegel and Walther⁽¹¹⁾ (electrodynanic) and Barone and Giacomini⁽¹²⁾, who used a metallised current path in a magnetic field to produce excitation.

A large amount of work was done by Birch and Bancroft⁽¹³⁾ (1938). They used an electromagnetic resonance method to measure the torsional modulus for small (20 cm. x 2 cm.) cylindrical specimens of several rocks at pressures of

0 - 4000 kg/cm² and at temperatures 30 - 100°C. The torsional method was chosen as it was unaffected by the pressure medium which was compressed Nitrogen in an apparatus due to Bridgman.⁽⁵⁾ Thin copper foil was used to cover the specimens.

Hysteresis effects were observed at the lower pressures (up to 1000 kg/cm²), but this effect diminished as the pressure was further increased. They pointed out that the behaviour of rocks approached that of a perfectly elastic medium for high pressures. For example, for granite E was 2.55×10^{-11} at 1 kg/cm², 3.50×10^{-11} at 500 kg/cm² and 3.58×10^{-11} at 4000 kg/cm². The pressure effects were thus attributed to porosity.

Small negative temperature coefficients were observed, normally of the order of 50×10^{-6} per °C. Later workers obtained similar values, though Ide found an irreversible effect for temperatures of 500°C and greater. This produced a decrease in velocity, which was further decreased by a further heating.

Birch and Bancroft's first measurements were made at frequencies between 4,500 and 60,000 cycles per sec. In order to lower the resonant frequency, they used a large cylindrical column of granite 244 cm. long and 23 cm. in diameter. This was suspended by wires at the nodes, and excited into longitudinal or torsional vibrations by means

of a metal ring electromagnetic system. An electromagnetic pick-up was also used. A lower frequency of 569 cycles per sec. was reached in the fundamental longitudinal mode, and of 139 cycles per sec. in the fundamental torsional mode. However, no significant frequency effect on either the elastic constants or the internal friction was observed.

Dynamic measurements by Weatherby, Born and Harding⁽¹⁴⁾ (1935), first demonstrated the effect of moisture on the elastic constants of rocks. They noted that a reduction of nearly 50 per cent occurred in the value of E, for small moisture contents (5 per cent) in Amherst sandstone. Born⁽¹⁵⁾ (1941) also observed this effect, though a more interesting effect occurred regarding internal friction (see later). Oliphant⁽¹⁶⁾, using a similar method, also noted decreases (up to 12 per cent for 0.6 per cent water content). He also noted that E increased as the pressure was increased, and that a small negative temperature effect occurred. However, he observed no frequency effect.

Obert⁽¹⁷⁾ (1939) investigated rocks up to breaking point, and again noted the approach to perfect elasticity for high pressures, and the greater pressure effect for less compact rocks.

Regula⁽¹⁸⁾ (1940) used a free vibration method developed by Fortsch (1938), for thin bars of rock clamped at one end

and heavily loaded to reduce the frequency to 20 - 30 cycles per sec. He used transverse and torsional vibrations, and also did static measurements. His dynamic results were smaller than the static ones, but he attributed this to a length effect (in the calculation of E, the power to which the length was raised was different for the static and dynamic calculations), and calculated the magnitude of the length correction by equating his corrected static and dynamic results. He compared his results with those of Ide⁽⁹⁾, and suggested that his own higher values were due to inadequate clamping.

A static-dynamic method at effectively very low frequencies (10^{-2} cycles per sec.) was used by Sokoloff and Skriabin⁽¹⁹⁾ (1937). They observed the time-deformation curves for centrally loaded cantilevers of over 200 specimens of rock. From their results, they formulated a memory function theory. They assumed the memory function χ (see before) to be given by

$$\chi = \beta e^{-\alpha t}$$

where β and α are constants depending on the rock type. The theory enabled them to deduce a relation between static and dynamic values for E, which is

$$E_S = \frac{E_D}{1+D}$$

where $D = \frac{\beta}{\alpha}$

This gave results for ($E_D - E_S$) of a similar order to those found by previous workers, but suggested that the dynamic value should be reached at very low frequencies (less than 1 cycle per sec.). They compared theoretical and experimental values for velocity as a function of frequency for sandstones and limestones, and found good agreement. However, they concluded that their theory was only an approximation to the actual behaviour of rocks.

Bruckshaw and Mahanta⁽²⁰⁾ (1954) used a forced vibration method in the 40 to 120 cycles per sec. frequency range using clamped rectangular specimens of dimensions 6 cm. x 2 cm. x 1 cm. One electromagnetic transducer was used to excite vibrations, and another to measure the forced amplitude as a function of frequency. A small ⁱⁿ decrease in E with frequency, of the order 2 per cent, was observed. This was considerably less than the variation (5 - 20 per cent) noted by Ide. However, in this type of experiment it is necessary to use relatively compact rocks (to avoid breakage), so a smaller decrease would be expected.

The results of the large number of dynamic measurements on rock specimens are summarized below:-

1. Dynamic values of E are greater than static values by up to 20 per cent., depending on the degree of compactness of the particular rock.

2. Young's modulus increases as the pressure applied increases, but for high pressures the behaviour of rocks approaches that of a perfectly elastic medium.
3. A small negative temperature effect occurs, which may be permanent for high temperatures.
4. The presence of moisture may produce a large decrease in the value of E , particularly in the case of sandstones.

3. Pulse methods

These methods provide the only means of measuring the bulk velocity directly in the laboratory. A large amount of work has been done recently in this manner, though mainly on materials other than rocks. Some of the experiments on rocks are mentioned below.

Hughes, Pondrum⁽²¹⁾ and Mins (1949) used an apparatus in which the pulse was propagated along the axis of a cylindrical specimen. It can be shown that reflections from the sides lead to a series of pulses reaching the other end of the rod, being propagated at the longitudinal velocity V_L and separated in time by an amount proportional to V_T .

Mxskimmin⁽²²⁾ (1950) used a similar apparatus, and Hughes and Jones⁽²³⁾ (1951) studied the effects of pressure and temperature on the velocity. They used frequencies of 2 - 6 megacycles per sec. and pressures up to 2000 kg/cm².

The velocity increased rapidly with pressure at first, and later more slowly. No evidence of dispersion was found up to 5 megacycles per sec. The specimens used were of sedimentary rocks - dolomite, sandstone, shale and limestone.

Hughes⁽²⁴⁾ (1956) studied four granites at pressures of 2000 - 5000 bars and 30 - 200°C. He obtained similar results and noted that the velocity in granite approached a fixed value as the pressure was increased. This was 6.25 km/sec. for Westerly granite.

Muller⁽²⁵⁾ (1954) used Schlieren photography for head waves passing into a specimen from a fluid medium. From the bending at the interface he was able to calculate the velocities involved. For igneous rocks he found V_L to be 5.1 to 8.0 km/sec., and for sedimentary rocks V_L was 2.4 - 6.0 km/sec.

Knopoff⁽²⁶⁾ (1954) used a model seismic method. This was essentially similar to that used in field work, time distance curves being obtained from first arrivals, and the velocities calculated from the slopes. He used a large block (37 cm. x 37 cm. x 27 cm.) of Westerly granite, with a crystal method for excitation and pick-up. Both longitudinal and Rayleigh waves were observed, but the transverse wave could not be identified. ρ and E were calculated from the ratio of velocities, and compared with the results of other workers (see table 1).

On account of the high frequencies (up to 15 megacycles per sec) used in pulse methods, deductions concerning seismic work (frequencies 0.5 - 300 cycles per sec.) are somewhat doubtful. However, a comparison is of some interest (see Table 1).

4. Seismic methods

An immense amount of work has been done in the last decade on the measurement of velocities by seismic means. It is not possible to give a comprehensive account of these results, but a few of the results which have more bearing on the present work will be mentioned.

The first important measurements were made by Leet and Ewing (1932)⁽⁶⁾. They made measurements on three types of granite in the U.S.A., using spreads of 50' - 4600' and using a three component seismograph. Their results indicated that they were dealing with a low-lying layer some 20 - 200 metres in thickness, as they obtained straight lines through the origin. An average value for V_L of 5.04 ± 0.03 km/sec. was obtained, and $\sigma = 0.33$ was assumed giving $E = 4.3 \times 10^{10}$ c.g.s.

Weatherby, Born and Harding⁽¹⁴⁾ (1934), following Gutenberg⁽²⁷⁾ (1932), studied Timoshingo granite using two spreads of 5 - 400' and 500 - 2500'. The short spread

gave $V_L = 14,900 - 17,150$ ft/sec., and the long one gave $V_L = 17,950$ ft/sec. They also used a laboratory electromagnetic method on specimens of the granite, and obtained somewhat higher values.

Oliphant⁽¹⁶⁾ (1950), used 3-component seismographs in investigations of Nevada limestones, following work by Birch and Obert, Windes and Duval^(see 16) (1946). Their results were similar to those of other workers. They noted a large spread in values, and also encountered some difficulty from layering.

Evison⁽²⁸⁾ (1951) used the falling weight method for chalk though he was more interested in internal friction, as were Collins and Lee⁽²⁹⁾ (1956). Brown and Robertshaw⁽³⁰⁾ (1953) also used the falling weight method.

Rothlisberger⁽³¹⁾ (1957) measured the velocities in sandstones, obtaining a value of $V_L = 2.2$ km/sec. However, he noticed that a spread of values of 7% occurred in a distance of only 10 metres, although he had chosen what he thought to be very homogeneous rocks.

The general conclusions from seismic measurements are in agreement with those obtained from other methods, the velocity increasing with depth etc. However, the results indicate that there may be a considerable variation in the values of velocity obtained in a particular area. This results

from local variations in the properties of the rocks, as well as from changes in velocity with depth. Layering produces considerable difficulties, and the often unknown values of the parameters such as pressure, temperature and moisture content further complicate the situation. The results obtained are thus average values with a relatively large variation, of a large volume of rock, and this must be borne in mind when a comparison with values obtained by other methods is attempted.

5. Earthquake methods

The work of Ewing and Press (1954) in which the earth waves of very long period from the Kamchatka earthquake were recorded, gave similar results to those obtained from normal seismic measurements. They showed that their results could be explained by assuming the existence of a layer of shear velocity 4.48 km/sec. and 516 km. thick over-lying a solid half-space of shear velocity 6.15 km/sec. with a density ratio of 1 : 1.1. They observed dispersion for periods less than 400 sec., their results being in accord with an increase of shear velocity with depth.

A comparison of these results with other methods is only possible in a qualitative manner.

Summary and Comparison of Results of Measurements of Elastic Constants.

The above discussion has shown that the elastic constants of rocks are dependent on the following parameters:-

1. Pressure (and Porosity)

The pressure effect is usually attributed to porosity. The presence of minute cracks and spaces in rocks will obviously affect the elastic constants. Under pressure, these cracks are to some extent closed up, making the rock more homogeneous. All the various methods above clearly demonstrate that the value of E increases with pressure, tending towards a constant value when the behaviour approaches that of a perfectly elastic medium.

2. Moisture content

Several workers have noted a decrease in E with moisture content. This appears to be greater in the case of sandstone and is probably absent in the case of igneous rocks.

3. Temperature

Small temperature effects have been observed; normally a negative temperature coefficient has been found. This may become permanent for relatively high temperatures but presumably a change in chemical composition then occurs.

4. Frequency

It has been pointed out that no dependence of E on frequency was observed ~~in~~ the range 140 - 60,000 cycles per sec., and also up to several megacycles. A small change of the order 2% was noted between 40 and 120 cycles per sec., but there are no measurements in the lower range, which is therefore of considerable interest. Measurements have suggested a difference between static and dynamic results of between 5% and 20%, so that the greater part of this difference may be expected to occur at less than 40 cycles per sec.

Some of the most important results of the measurements of elastic constants are given in ^{Table 1.} ~~Fig. 8.~~

In a comparison of the actual values obtained, it is necessary to specify the values of the above parameters. This is not possible in all cases, and quite impossible in most seismic measurements. In the laboratory measurements, it may be assumed (unless otherwise stated) that the temperature was room temperature and the moisture content that pertaining to room temperature. The pressure range has been stated when known; if not stated one may assume that no pressure was applied (i.e. atmospheric pressure). For the dynamic values, the frequency range is several kilocycles per sec.

The only rocks for which a detailed comparison is possible

are Quincy granite, Rock port granite, Westerly granite and Sudbury norite. Complete agreement cannot be expected, since only in the case of Zisman and Ide were the same specimen used. The values obtained for a few other rocks have been included in order that they may be compared later with values obtained by the present method.

It has been pointed out that the results of Zisman and Ide demonstrate that static values are definitely lower than dynamic ones. This is so for all the cases given in Table 1. In general the seismic results of Leet and Ewing are in reasonable agreement with the dynamic results, if some variation (as explained above) is allowed. In the final column of Table 1, average values for the various methods are given. The static value S is first shown (usually Zisman's), followed by the average dynamic value D^{*}, and finally the seismic value L of Leet and Ewing. The results of Birch and Bancroft (taking the pressure range into account) are of the same order as those of Ide, though a comparison is difficult on account of different specimens being used.

A rather large variation occurs in the values for Westerly granite, taken from a table by Knopoff. Knopoff's pulse values are consistently higher than those of other workers, though this is not thought to be related to the frequency difference involved. It has been suggested that this is due to his difficulty in identifying waves other than the longitudinal one

*Note. This is an average of many results.

TABLE 1

MEASUREMENTS OF ELASTIC CONSTANTS

Rock	Worker	Stress range (kg/cm ²)	Type of Measurement	E ($\times 10^{-11}$) (c.g.s.)	E(average) (c.g.s.)
Quincy granite	Zisman	3.1	Static	2.71)	3.5 S
	"	78.4	"	5.08)	
	Ide		Dynamic	4.16	4.3 L 4.3 D
	Leet & Ewing		Seismic	4.3	
	Birch & Bancroft	75' down	Dynamic	4.7	
	Obert	0	"	4.32)	
	"	500	"	6.41)	
	Adams & Coker	70-600	Static	4.64, 5.68	
Rockport granite	Zisman	11.2	Static	3.54)	3.9 S
	"	56	"	4.26)	
	Ide		Dynamic	4.34	4.7 D 5.0 L
	Leet & Ewing		Seismic	5.0	
	"		"	5.1	
	Birch & Bancroft	4000	Dynamic	8.36	
Sudbury norite	Zisman	36.5	Static	8.36	8.36 S
	Ide		Dynamic	9.06	
	Leet & Ewing		Seismic	8.82	9.1 L 8.8 D
	Muzzey		Dynamic	9.13	
	"		"	8.5	
	Birch & Bancroft	4000	"	9.67	
Westerly granite	Adams & Coker	70-600	Static	5.1 4.45	5.1 S
	Leet & Ewing		Seismic	5.28	
	Ide		Dynamic	3.99	5.3 L 3.8 D
			Dynamic	3.64	
	Birch & Bancroft	4000	Dynamic	9.67	
	Knopoff		Pulse	7.04	

TABLE 1 Contd.

Rock	Worker	Stress range (kg/cm ²)	Type of Measure- ment	E (x10 ⁻¹¹) (c.g.s.)	E(average) (c.g.s.)
Solenhofen limestone	Zisman	11.2 56	Static	5.77	5.7 S
	Ide		Dynamic	6.27	
Carboniferous limestone	Zisman		Static	6.06	7.0 D
	"		"	7.22	
Persian limestone	Ide(& Birch & Bancroft)	70-600	Dynamic	7.01	
	Richards		Static	5.7	
	"		Seismic	5.5	
Montreal limestone	Adams &		Static	6.35	
Quartzitic sandstone	Zisman	11.2 56	Static	6.36)	6.5 S
	"		"	6.65)	
Japanese sandstone	Ide		Dynamic	7.15	7.15D
	Kusakabe &		Static	.05-4	
Ohio sandstones	Nag aka	70-600	Static	1.58	
	Adams & Coker				
P.a. slate (Mass.)	Ide		Dynamic	11.25	
Everrett slate	Birch & Bancroft		Dynamic	4.87	
Japanese slates	Kusakabe & Nag aka		Static	1 - 11	

The primary purpose of the present work is to investigate the elastic behaviour of rocks over the frequency range 0 - 50 cycles per sec. It can be seen from the above summary that this range is of considerable interest, particularly as it includes the lower seismic frequencies. Unfortunately, the necessary experimental arrangement made the study of pressure and temperature effects almost impossible - however, these have already been fully investigated. It was possible to study the effect of moisture in some detail, though the frequency effect is thought to be of more practical importance.

Internal Friction

General measurements on solids

When setting up an equation of motion in which dissipation occurs, it is simplest to assume that the dissipative force depends on the velocity, as the equation is then easily solved. This corresponds to a viscous type of loss. For this reason it was at first common practice to assume that internal friction had a viscous origin. This being so, $\delta \propto f$ as in the case of the Voigt solid described above, or $\delta \propto \frac{1}{f}$ as in the case of the Maxwell solid; a combination of the two

would give a more complicated dependence of δ on f .

It was found that the properties of many solids, particularly organic polymers, may be explained by the above assumptions. The science of Rheology deals mainly with this type of material (generally non-metallic, and homogeneous). However, rocks are very different from this, so that even though the viscous mechanisms of rheology give an adequate account of the behaviour of many materials, such a simple account can hardly be expected in the case of rocks.

It was soon found that all solids did not obey the above viscous laws. Lord Kelvin⁽³³⁾ noted this. Later Kimball and Lovell⁽³⁴⁾ (1927) used the phase difference method, where the phase was exhibited as the deflection of an end-loaded revolving shaft. They used a frequency range of 10 - 30 cycles per sec. (in most cases) and a stress range of up to 6000 kg/cm². Most of their work was on metals, and it was found that the loss per cycle ΔW and the stress amplitude S were related by

$$\Delta W = c' S^2$$

($c' = \text{constant}$).

$\frac{\Delta W}{W}$ remained constant as the frequency was varied; the values of $\frac{\Delta W}{W}$ were in agreement with those of other workers.

Gemant and Jackson⁽³⁵⁾ (1937) used a free vibration method for torsional and lateral vibrations of reeds of various

dielectric materials (ebonite, glasses) and some metals. They used a stress range of up to 500 kg/cm² and observed no dependence of $\frac{\Delta W}{W}$ on f . In this stress range, $\frac{\Delta W}{W}$ was also independent of the stress. Their results agreed well with those of Kimball and Lovell in other respects. They suggested that "solid friction" would explain their results, though the actual mechanism was difficult to understand since their results were in the proportional stress-strain region and the slipping of grains doesnot occur till the yieldpoint is passed. They also pointed out that qualitative agreement could be obtained by considering a solido-fluid material, or a model in which several relaxation times were superimposed, though exact calculations are very difficult in both cases.

Wegel and Walther⁽¹¹⁾ (1935), using an electrodynamic method, studied several materials, including ferromagnetics and glasses. They found that $\frac{\Delta W}{W}$ was independent of frequency in the range $10^3 - 10^4$ cycles per sec. for most materials, though for ferromagnetics an increase or decrease occurred depending on the particular conditions of temperature etc.

Thus, for the more "rock-like" materials, it seems that a "solid-friction" mechanism applies. Owing to anelastic properties, a hysteresis loop is formed and this has a constant area, independent of frequency, so that the specific loss is

also independent of frequency. However, the actual mechanism is not known.

Zener⁽³⁶⁾ (1937) suggested that internal friction may have a thermal origin. Considering a vibrating reed of material, the compressed and stretched parts will be at different temperatures. At low temperatures the heat flow takes place isothermally, so that the process is reversible and no loss occurs. At high frequencies the process is adiabatic and again there is no loss. However, at intermediate frequencies mechanical energy is converted into heat, and for a reed of rectangular cross section,

$$\frac{\Delta W}{W} = \frac{2\pi (E_s - E_T) \cdot N \cdot N_0}{E_s (N^2 + N_0^2)}$$

where $N_0 = \frac{\pi K}{2c_p \rho d^2}$

where E_s is the adiabatic modulus,

E_T is the isothermal modulus,

N is the frequency,

ρ is the density

K is the thermal conductivity

c_p is the specific heat at constant pressure,

d is the thickness.

Bennewitz and Rotger⁽³⁷⁾ (1938) measured $\frac{\Delta W}{W}$ for German silver reeds in transverse vibration and obtained very good agreement with this theory. The maximum (given by N_0 above)

occurred as predicted at 10 cycles per sec.

The above formula applies both to homogeneous and heterogeneous solids, but additional mechanisms arise in the latter case. In polycrystalline materials (e.g. rocks), stress inhomogeneities arise from the different orientations of neighbouring grains, and from the presence of cavities etc. Again temperature fluctuations occur and there is a maximum dissipation at a low frequency given by

$$N_p = \frac{3\pi K}{f c_p a^2}$$

where a is the radius of grain.

This was checked by Randal, Rose and Zener⁽³⁸⁾ (1939) using a resonance method with brass specimens. At very high frequencies, the heat flow was confined to the grain boundaries and the loss was proportional to $\sqrt{f}$. At very low frequencies, the loss $\propto f$, and a maximum occurred as predicted.

Although the effect is apparent in thin reeds, this is not so with larger specimens. It has been suggested that this is because a large number of elementary processes occur, leading to a coalescing of the maxima and an apparent constancy of $\frac{\Delta v}{v}$.

Knopoff⁽³⁹⁾ (1956 and 1958) has worked out a general non-linear theory of internal friction, using thermodynamics, and has obtained an independence of $\frac{\Delta v}{v}$ and f over a large frequency range, though there is no direct confirmation

of his theory.

The importance of the grain boundaries was emphasised by Zener and also by Ke⁽⁴⁰⁾ (1947) who investigated the internal friction arising from viscous slip at the grain boundaries. He used a polycrystalline aluminium wire in torsional vibration using the free vibration method. The frequency was maintained at 0.8 cycles per sec. and the temperature varied. A maximum of $\frac{\Delta W}{W}$ occurred at 285°C. If internal friction is proportional to distance slipped multiplied by resistance to slip, $\frac{\Delta W}{W}$ should be small at low temperatures, when the distance slipped is small, and at high temperatures when the resistance to slip is small. Thus, a maximum should occur at some intermediate temperature. No maximum occurred with a single crystal. Ke's values were much greater than Zener's and he concluded that the phenomena could be adequately explained by viscous slip.

Taylor and Orowan⁽⁴¹⁾ (1941) supposed that the low $\frac{\Delta W}{W}$ of single crystals arose from the movements of dislocations in the crystal lattice, but no conclusive results were obtained. Snoek⁽⁴²⁾ (1941) considered the re-ordering of solute atoms which is a function of the stress applied. This effect should also be temperature sensitive.

It is difficult to see how the above thermal effects can apply to rocks, as the critical frequencies involved would be very small for the grain sizes and beam dimensions used. However, as suggested above, a superposition of effects may

occur.

Rocks are in general non-homogeneous polycrystalline substances, so it may be expected that several of the above processes may occur. More recent workers have tended to use much higher frequencies (pulse methods) so that their measurements are hardly relevant here since extra mechanisms (e.g. scattering) are involved. It can be seen that the position for solids in general is not very clear. Explanations exist for organic polymers (viscous), metals and dielectrics (solid friction) and thin metallic reeds (thermal effects), but clearly a more fundamental theory is desirable. The viscous and solid-friction explanations only remove the problem by one stage, without really explaining what is happening. In the case of rocks, it may be possible to separate the various mechanisms by a variation of the parameters involved.

Measurements on rocks

The most convenient parameter to choose in the case of rocks is the frequency f , since it has been shown that the various mechanisms generally lead to different relations between $\frac{\Delta\omega}{\omega}$ and f .

Most of the early workers on rocks used resonance methods, and determined their values for Q from the resonance curve. Ide⁽⁹⁾ (1937) used frequencies of about 3000 cycles per sec.

and obtained Q values between 100 and 200 . Birch and Bancroft⁽¹³⁾ (1938) used their large block of granite at frequencies between 120 cycles per sec. and 4500 cycles per sec; their values for Q ranged from 100 - 200, but with no frequency dependence. They noted a considerable pressure effect; $\frac{A\omega}{\omega}$ was reduced by factors up to one half for high pressures.

They pointed out that this constancy of $\frac{A\omega}{\omega}$ would cause the earth to act as a low pass filter. They correlated their work with that of Zener., pointing out the effect of a superposition of processes.

Sokoloff and Skriabin⁽¹⁹⁾ (1937) developed their theory (see above) and calculated the internal frictions of their specimens on this basis. However, their results did not agree with those of previous workers since ~~the attenuation coefficient~~ δ would approach zero for both low and high frequencies. However, a sum of relaxation times would overcome this difficulty.

Born⁽¹⁵⁾ (1941) used a resonance method for various sedimentary rocks. He found that Q was independent of f , but that the amount of moisture present was important. For Amherst sandstone the internal friction was the sum of two parts, one viscous ($\frac{A\omega}{\omega} \propto f$) and one solid-friction ($\frac{A\omega}{\omega}$ is independent of f). The viscous part depended on

the amount of moisture present. He covered the frequency range 2000 - 4000 cycles per sec. and his values of $\bar{\nu}$ for the various rocks ranged from 0.02 to 0.03. He compared these with field results, and obtained reasonable agreement though several assumptions were involved. Martin⁽⁴³⁾ (1948) did similar experiments on limestones, but did not observe any moisture effect, even on saturation.

Bruckshaw and Mahanta⁽²⁰⁾ (1954) investigated the 40 - 120 cycles per sec. frequency range. No correlation was possible, but a slight reduction of $\frac{\Delta\omega}{\omega}$ with decreasing frequency, particularly at the lower frequencies, was noted.

All these results were obtained in the laboratory. Seismic workers have sometimes obtained values of $\frac{\Delta\omega}{\omega}$, though these suffer from the same difficulties as do the seismic values of E (see above). A large spread of values occurs, and a knowledge of the various parameters involved is usually lacking. There are two general methods of checking the variation of internal friction with frequency by seismic means; the amplitude may be measured as a function of distance, or the frequency spectrum may be examined at various distances.

Gutenberg⁽²⁷⁾ (1932) examined the variation of the mean frequency at various distances from the shot point. Values of the distance D ranged from 10 to 10^5 metres and of the

period T from 0.1 to 0.01 secs. He obtained the relation

$$T^2 = T_0^2 + .0001 D.$$

This agrees very well with the solido-fluid or viscous mechanisms (as developed by Ricker) of the behaviour of rocks. For these mechanisms, $\frac{\Delta W}{W}$ increases with frequency up to a maximum. (ref. 52)

Kendal⁽⁴⁴⁾ (1941) did work on seismic reflection amplitudes, and found similar results. The distance-amplitude relation was in agreement with that predicted by a viscous theory. (ref 52)

Vestine and Forbush⁽⁴⁵⁾ (1953) observed the frequency distribution in the pulse produced by a shot fired 17 $\frac{1}{2}$ km. away and ^{it has been said that} their results were again in agreement with a viscous or solido-fluid theory. The centre frequency at 17 $\frac{1}{2}$ km. was about 7 cycles per sec., and they used an auto-correlation method of calculation. (ref. 52)

Ricker⁽⁴⁶⁾ did a large amount of theoretical work, mainly concerned with the width and shape of the seismic pulse at various distances from the shot. He assumed a Voigt type of material to occur. He did experiments on Pierre shale (1953) and obtained good agreement with the theory. However, a surprisingly large number of objections have been raised to his work, and it is by no means generally accepted.

Knopoff⁽³⁹⁾ (1956) checked Ricker's theory for a lossy material (wax) using a pulse method, and obtained good agreeene

with it. However, he suggested that for rocks a solid-friction theory should apply. He pointed out (1958) that a non-linear theory was required if $\frac{\Delta W}{W}$ was independent of f (as he said it was) and put forward the theory mentioned above.

Ewing and Press⁽³²⁾ (1954) studied the amplitudes in earthquakes, and obtained values of $1/Q = 670 \times 10^{-5}$ for mantle Rayleigh waves (periods 150 - 200 sec.) and $1/Q = 370 \times 10^{-5}$ for periods between 250 & 350 sec. They were unable to say whether this indicated a frequency dependence of $1/Q$ on f , or a greater penetration of the longer period waves. There are not very many measurements of internal friction using the normal seismic method. Born⁽¹⁵⁾ (1941) as mentioned above, obtained reasonable agreement between laboratory and seismic values, his seismic value being $\alpha = 0.036$ per wavelength. However, this is the average value in an unspecified frequency range.

Evison⁽²⁸⁾ (1951) obtained $\alpha = 0.020$ per wavelength for chalk at a frequency of 600 cycles per sec. Howell⁽⁴⁷⁾ (1954) used 10 - 3000 ft. spreads and obtained $\alpha = .0011 - .0090$ per ft., but with a large scatter. His results were rather inconclusive. He pointed out that plastic flow occurs near the shot point, but not at larger distances.

Menzel⁽⁴⁸⁾ (1954) attempted to check Sokoloff and

Skriabin's theory, by seismic means, but his results were disappointing, being perhaps more in accord with a solid friction theory.

A frequency analysis of the waveform at various distances was attempted by Collins and Lee⁽²⁹⁾ (1956) using the method of Fourier analysis. They attempted to correlate the absorption coefficients with frequency, but a large scatter occurred. They thought that the results favoured a linear relation between attenuation coefficient and frequency, rather than a square relation.

Macdonald⁽⁴⁹⁾ (1958) made seismic measurements on Pierre shale, and obtained a linear relation between α and f . He used frequencies between 100 and 500 cycles per sec., and found $\alpha = 0.12$ per wavelength

It seems that seismic results are somewhat inconclusive regarding the relation between internal friction and frequency, though there is perhaps more evidence of a viscous theory.

The pulse shape - distance results ^(of Richter) suggest a viscous mechanism, but the amplitude distance results are less conclusive and may suggest a solid-friction type of loss. However, the laboratory results are in better agreement and suggest that $\frac{\Delta W}{W}$ is constant over the frequency ranges used. It is very important to note that (for rocks) the frequency ranges used in the laboratory have generally been relatively high,

whereas the ranges used in seismic work are much lower. The importance of measurements in the lower seismic range is once more emphasized.

Table 2 shows values of internal friction obtained by various workers; the frequency range has been stated when known. A few seismic values have been included.

The following points emerge from the above discussion:

1. For various non-metallic polymers, the behaviour is adequately explained by viscous models.
2. For thin metal reeds, the values of internal friction agree with those obtained by a consideration of thermal effects.
3. For thicker metal specimens and some dielectric materials, $\frac{\Delta W}{W}$ appears to be constant down to low frequencies (0.1 cycles per sec.) and up to several megacycles per sec.
4. Laboratory measurements on rocks (at relatively high frequencies, greater than 10^2 cycles per sec.) also show this constancy of $\frac{\Delta W}{W}$.
5. Seismic results are often inconclusive, but pulse shape experiments suggest a viscous mechanism.

TABLE 2
MEASUREMENTS OF INTERNAL FRICTION OF ROCKS

Material	Worker	f range (c/s)	Internal Friction	$\frac{\Delta W}{W}$
Sylvan shale	Born	3000-12,000	$\bar{f} = 4.5 \times 10^{-2}$	.09
Hunton lime- stone	"	2000-10,000	$\bar{f} = 4.4-5.3$	.09
Amherst sand- stone	"	900-4000	$\bar{f} = 5.7-6.2$	.11
Cockfield- Vequa	"	3000-11,000	$\bar{f} = 4.5-4.8$	.09
Solenhofen limestone	Birch & Bancroft	About 10,000	$\frac{1}{f} = 670 \times 10^{-5}$	.042
Slate (Pa)	" "	" "	380	.024
Quincy granite	" "	" "	1000	.063
Rockport granite	" "	" "	770	.048
Diabase	" "	" "	170	.011
Quincy granite	" "	140-1600	500-1000	.03-.06
(very soft) Sandstone	Bruckshaw & Mahanta	40-120		.3
Oolite	" "	" "		.15
Shelly lime- stone	" "	" "		.10
Granite	" "	" "		.10
Dolerite	" "	" "		.06
Diorite	" "	" "		.045
Chalk	Evison(seismic)	600	$\alpha = .02 \text{ per } \lambda$	.08
Various sedi- mentary rocks	Born	Seismic	$.03 \text{ per } \lambda$	.08-.12
" " "	Howell	"	$\alpha = .0011-.0090 \text{ per ft}$	Not quoted
Pierre shale	Macdonald	-500	$0.12 \text{ per } \lambda$	0.48

SECTION 5

THE APPARATUS

A. The Method

As stated above, the purpose of this work was to develop a dynamic method of measuring Young's modulus and the internal friction of rock specimens, which would be suitable for use in the **low frequency** range of about 0 - 50 cycles per sec.

A resonant method is the most convenient, but for resonance to occur at low frequencies the specimen must be either very long and thin, or be very heavily loaded. The brittle nature of rocks almost rules out both these possibilities, if frequencies of the order of 1 cycle per sec. are required. The remaining dynamic method is the forced vibration method below resonance, as used by Kimball and Lovell (revolving rod method) and Bruckshaw and Mahanta (transverse bar method). Kimball and Lovell's method requires rather heavy loading, and is therefore unsuitable.

For a vibrating beam much below resonance, the equation involving the displacement y for a given force F is

$$y = SF \text{ or } y/F = (\text{const.}) \times E.$$

since S , a constant of the beam, depends on Young's modulus.

If the displacement y for a constant force is measured over a frequency range in which f is much less than $f(\text{resonant})$, the variation of Young's modulus will be given by the variation in the ratio y/F . The internal friction is exhibited as a phase difference ϕ between the vibration and the exciting force, and if f is small, is given by

$$\frac{\Delta W}{W} = 2\pi \phi$$

The variation of $\frac{\Delta W}{W}$ with frequency is thus easily obtained by measuring ϕ .

Since this is not a resonance method, the displacement y is not very large. To increase it sufficiently to make accurate measurement possible, either a larger force or a thinner (or longer) specimen must be used. A transverse method (as used by Bruckshaw and Mahanta) leads to a much larger amplitude for a given force than does a longitudinal or torsional method. A sufficiently large amplitude can be obtained by using thin beams in transverse vibration; the lower limit to specimen size is governed by the nature of the rocks to be investigated and since the resonant frequency is also lowered so that f is not very much less than $f(r)$, clearly an optimum beam size must be chosen. Also, a large exciting force might lead to fracture, and it is better to keep the strain small so as to approach more nearly seismic conditions.

Bruckshaw and Mahanta used a rectangular cross-section

beam of dimensions 6 cm. x 2 cm. x 1 cm., clamped securely at one end. An electrodynamic transducing system at the other end produced vibrations which were examined by a similar transducing system, the magnitude and phase of the pick-up signal being noted with respect to the input. E and $\frac{\Delta W}{W}$ were thus found. In such an electrodynamic method, the voltage output is directly proportional to frequency, and at 40 cycles per sec. the signal/noise ratio was too small for lower frequencies to be used.

It is clear that a frequency-independent method is required. An electrostatic method is unsuitable as condensers do not work efficiently at low frequencies. A F.M. method is possible, but introduces difficulties in both construction and calibration. An optical method suggests itself, and an adaptation of a method suggested by Bruce, Macinante and Kelly⁽⁵⁰⁾ has been used in this work. This involves the use of two optical flats with a small angle between them and very close together. Multiple interference Fizeau fringes are produced, and an amplitude of 3×10^{-7} cm. has been detected in this manner. This corresponds to 1/100 of the fringe separation; if only 1/10 of a fringe separation was detectable, an amplitude of 10^{-3} cm. would be sufficient to give an accuracy (in amplitude measurement) of 0.2% which is sufficient for the required purpose.

The method leads to the difficulty that fringes vibrating with a large amplitude cannot be seen; also the phase difference between the exciting force and the vibration cannot be measured. However, if, instead of one optical flat being rigidly fixed and the other one being attached to the specimen, the first one is attached to a similar specimen of a substance of well known properties (e.g. steel), the properties of the rock can be measured with respect to those of steel.

Two similar horizontal beams clamped at each end are used, one being of steel and the other of rock. Each beam has an optical flat attached to the centre, and the beams are parallel to one another, the flats being at a sufficient separation to produce interference fringes between them. Vibrators are attached to the centres of each beam, and these are excited in parallel. The resistance (governing amplitude) and inductance (governing phase) in each circuit are then adjusted until the two optical flats (and therefore the two beams) have identical amplitude and phase. Assuming that the values of E and $\frac{\Delta v}{v}$ are known for steel, a ratio of the various parameters involved then gives the required quantities for the rock.

Advantages of the Method

The main advantage of this method over most previous methods (excepting that of Bruckshaw and Mahanta) is that since resonance is not used, any frequency (sufficiently less than resonance to avoid very large corrections) can be used. Also, owing to the small size of the beams used and to the sensitivity of the optical system, the strains produced and the stress required are relatively small, being of a similar order of magnitude to those occurring in seismic work. The optical method of measurement has the additional advantage (as mentioned above) that it is not dependent on frequency, so there is no lower limit to the method.

One of the main difficulties in previous measurements, particularly regarding internal friction, has been that of allowing for losses to the air and to the clamps (or supports) involved. By using the above comparison method, these losses are almost entirely removed; the effect of stray vibrations (e.g. movements of building) is also much reduced. The time of operation of the method is small; a complete set of measurements can be made in 30 minutes and the results worked out in a further 15 minutes. Effects of temperature change etc. are thus reduced.

A few disadvantages will be mentioned. The beams are rather fragile on account of their size, and cutting them

is not very easy. Very soft rocks cannot be used, though quite soft samples of millstone grit have been measured. Also, very hard rocks are difficult to cut into uniform beams. The method is not suitable for pressure or temperature experiments, though moisture experiments can be done very easily.

B. The Apparatus.

It is convenient to describe the apparatus in three parts since it involves a mechanical system, an electrical system and an optical system. The requirements of each system are discussed separately at the beginning of each section.

(a) The mechanical system.

The main requirement is that the system should be very rigid, and reasonably massive to prevent stray vibrations from occurring. The vibrators and beams must be firmly held in fixed relative positions. It is important that the rock beam can be removed and replaced in exactly the same position, and with the same clamping pressure, and that it can be replaced by a similar steel beam. A device for producing the same clamping pressure, and for locating the clamps in position is therefore required. This refers both to the clamps at the ends of the beams, and to those

at the centre which attach the beams to the vibrators.

Allowance must be made in the various clamps to accomodate beams of different cross-section, and of different length. The optical flats must be firmly attached to the beams, and it must be possible to adjust them so that they are very close together and so that the required number of fringes appear in the field of view.

Photographs and sketches of the apparatus are shown in Figs. 2 to 8. The main material used in the construction is $\frac{1}{2}$ " dural sheet, chosen on account of its strength and ability to be milled easily to shape. A large horizontal base plate A measuring 24" x 8" x $\frac{1}{2}$ " has two similar pieces B of $\frac{1}{2}$ " dural attached perpendicularly to it as shown, by means of $\frac{1}{4}$ " angle steel lengths C. The two side plates B are parallel to one another, and are held in position by the angle steel C, two $\frac{3}{4}$ " diameter steel rods D which support the dural plate G (which holds the upper vibrator), a plate of $\frac{1}{2}$ " dural E which holds the lower vibrator, and the top F which is also parallel to the base plate. The side-plates B were milled to shape so that the top F was a good fit and accurately parallel to the base. Milling also ensured that the plates E and G holding the vibrators were in a parallel plane.

This represents the main part of the construction of the frame; it was found to be very rigid.

The top F is as shown in Fig. 2. It holds the four identical end-clamps, which hold the ends of the specimen in position. Two grooves were milled parallel to the long dimension of the top plate and enable the clamps to be adjusted for the required specimen length; each clamp is located in the groove by two pins (not shown), and can be rigidly fixed to F by four screws, passing through the slots shown and into threaded holes in F. The construction of an end-clamp can be seen from Fig. 3. It consists of a $\frac{1}{4}$ " steel plate I (with the locating pins attached), to which are attached four milled steel rods H. These are threaded at the upper end for 2BA nuts, and serve to locate a plate J parallel, and in a fixed relative position, to I. Semi-circular jaws to hold the specimen are attached to both I and J. Four springs between the nuts on the top of the rods and the plate J, enable the same clamping pressure to be applied in each case, so that a specimen can be removed and replaced so as to be in exactly the same condition.

The two vibrators are mounted on plates E and G. They are arranged so that their central shafts are directly below the grooves in F and are vertically below the centre of the specimens, when in position. The shafts of the vibrator are attached to the beam by means of 4 BA rods and the beam-clamps. These 4 BA rods are vertical (with respect to the

base) on account of the milling procedures mentioned above.

The beam clamps **I** are essentially similar to the end clamps, though flat clamps (rather than semi-circular ones) are used, and no provision for reproducing the clamping pressure is introduced. The lower part **L** was milled from a steel block, and four 2 BA rods locate a similar part **M** in position, so that the two clamping surfaces are also located. The threaded 4 BA rod in the lower part of **L** links up with the rod from the vibrator by means of a special cylinder (tapped on the inside) which was milled so it could be screwed flat against **L**. The nuts shown enable the various parts to be locked in position.

The optical flats are attached to the top of the beam clamps as shown. Each flat is held by a similar flat-clamp **N**. Four 10 BA screws are attached to a 1/16" steel plate **R**, and held at the top by a similar plate **P**. The optical flat is held between two thin dural plates **N**, and two similar steel plates **O** having holes through which the 10 BA screws pass. The flat can then slide up and down between the four screws; adjustment is aided by sponge rubber, and eight screws **Q** can fix the flat in position when adjustment is complete. A coarse adjustment to allow for beams of different dimensions is provided by the four screws **T** attached to another steel plate **S**; **S** is fastened as shown to the top

of the flat clamp.

It can be seen that the various requirements mentioned have been met. The method of operation is discussed in a later section, and several mechanical problems are also considered.

(b) The electrical system

The electrical system must supply sufficient power to drive the vibrators and produce the required amplitude of 10^{-3} cm. This meant in practice that a current of about 60 mA through a load of about 100Ω was required. Since power down to 1 cycle per sec. was required, a transformer could not be used, and a suitable commercial amplifier was not made. It was found to be important that the distortion in the amplifier be small; if the resonant frequency was a whole multiple of the operating frequency, harmonic distortion produced resonance of the beam, preventing measurement. A suitable L.F. oscillator and suitable variable resistors and inductances were required.

The electrical circuit arrangement is sketched in Fig. 4. The output from the amplifier is divided into two branches as shown. The upper branch is the "standard" branch and the vibrator V_1 is attached to the standard steel beam which

was never moved. The variable resistance R_1 and inductance L_1 extend the range of working. R_1 was variable between 100 Ω and 350 Ω in 50 Ω steps, and L_1 was normally 100 mH. The lower branch, which includes first a steel and then a rock beam, contains a variable resistance R_2 and inductance L_2 which were read in the course of an experiment. R_2 was variable from 0 to 1000 Ω in 0.1 Ω steps, and L_2 consisted of several air-core coils covering the range 0 - 200 mH. in 1 mH. steps. The coils were arranged to have a constant resistance so as not to affect the balance position.

A 200 - 500 V., 0 - 500 mA power-pack supplied the amplifier. The oscillator used was a Solartron L.F. model covering the frequency range 0.1 - 1110 cycles per sec., and providing an output of 10 volts; its stability was found to be very high. The amplifier circuit is shown in Fig. 4. A push-pull type reduces distortion. A double triode is used as a phase-splitter to supply each half of the push-pull arrangement. Each half consists of three 100 mA. power pentodes in parallel. Grid, anode and screen "stoppers" were used to remove oscillation troubles, but these have been omitted from the circuit shown.

This arrangement was arrived at partly by trial and error; an initial amplifier utilising five pentodes in parallel produced too much distortion. Since a transformer

could not be used, the load consisting of the two branches above was placed in the anode as shown. It was found that the distortion at 50 mA through a load of 100 Ω was not greater than 4% for both the second and third harmonics, and negligible for higher harmonics. This was found to be satisfactory (see later).

(c) The optical system.

The main requirement of this system was that the optical flats could be adjusted to produce a sufficient separation of the fringes seen in the field of view of the microscope. This adjustment mechanism has been described above.

The system is shown in Fig. 4. The light from a mercury lamp is reflected by a plane mirror, fixed between the two vibrators, so as to pass through the two optical flats. The microscope had an objective of about 1" and was focussed on a plane lying within the thin air gap between the two flats; a magnification of about x20 was used.

The brightness of the fringes was reduced by the relatively long path from the source to the flats - this could not easily be reduced. A reasonably powerful source was used and a filter was omitted. Only one bright line and one faint line were produced. The bright line was the main mercury green line ($\lambda = 5461 \text{ \AA}$); this was very sharp.

The percentage reflection of the optical flats was about 50 and produced a fringe separation of about 10 - 15 times the fringe line width.

At frequencies greater than 20 cycles per sec. a stroboscope was used to provide a frequency about 5 cycles per sec. different from the operating frequency. It was found that this procedure increased the accuracy at the higher frequencies used as the fringes then vibrated at about 5 cycles per sec. The strobe frequency could be varied continuously by means of a L.F. oscillator (Dawe .1 - 1000 cycles per sec. type), and the intensity and cut off time of the light was also variable.

A black screen on a dexion frame was placed over the apparatus to make observations easier; it was only necessary to use this on very bright days.

FIG. 2. THE APPARATUS

FIG. 3. CONSTRUCTION OF CLAMPS.

BEAM CLAMP (SIDE VIEW)

OPTICAL FLAT CLAMP (END VIEW)

END CLAMP (SIDE VIEW)

FIG. 4. ELECTRICAL AND OPTICAL SYSTEMS.

ELECTRICAL SYSTEM

OPTICAL SYSTEM

AMPLIFIER

FIG. 5. EXPERIMENTAL ARRANGEMENT.

FIG. 6. THE MECHANICAL SYSTEM .

FIG. 8. TYPICAL SPECIMENS .

SECTION 6

THEORY AND OPERATION OF THE METHOD

A. Theory

1. Bending of a cantilever

It can be shown that the displacement y of the centre of a cantilever of rectangular cross-section and length l , which is freely supported on knife edges at each end, and acted on at its centre by a static force F is given by

$$y = \frac{F}{48 EI} l^3 ,$$

I is the moment of inertia of the cross-section which is $\frac{wt^3}{12}$ for a rectangular beam of width w and thickness t . E is Young's modulus.

In the case of a beam firmly clamped at each end the corresponding formula is

$$y = \frac{F}{192 EI} l^3 .$$

The above formulae refer to the static case. It is generally assumed that the instantaneous deformation y_1 in the dynamic case if $f \ll f_R$ is given by

$$y_1 = y \sin (wt - \phi)$$

where y = static amplitude for force F ,

ϕ = phase lag of the vibration with respect to the
exciting force ,

w = the angular velocity (which is much less than the
resonant angular velocity).

In this particular experiment, the numerical values of the
above constants (e.g. $1/48$, $1/192$) are unimportant since a
comparison method is used.

2. The working formula

The experimental arrangement has been shown in Fig. 4.
In the following calculation, R_1 , L_1 etc. represent the
total resistance and inductance in the branches, obtained by
adding a constant amount to the actual readings.

For the deflection of the beam in branch 1, (i.e. "stan-
dard" branch)

$$y_1 = K_1 \cdot \frac{F_1}{E_1 I_1} \cdot l_1^3,$$

where K_1 is a constant, and for the deflection in branch 2
(i.e. steel or rock branch)

$$y_2 = K_2 \cdot \frac{F_2}{E_2 I_2} \cdot l_2^3$$

with a different constant K_2 .

At balance, $y_1 = y_2$.

Therefore

$$K_1 \cdot \frac{F_1}{E_1 I_1} \cdot l_1^3 = K_2 \cdot \frac{F_2}{E_2 I_2} \cdot l_2^3$$

Since the force F is proportional to the current i , and since $i_1 R_1 = i_2 R_2$ (neglecting reactance) and putting $I = \frac{wt^3}{12}$,

$$K_1 \frac{l_1^3}{E_1 I_1 R_1} = K_2 \frac{l_2^3}{E_2 I_2 R_2}$$

or

$$\frac{E_1}{E_2} = \frac{K_1 l_1^3 w_2 t_2^3 R_2}{K_2 l_2^3 w_1 t_1^3 R_1}$$

It can be shown that the reactive component is never more than 1/1000 of the resistive part.

If beam 2 (say steel) is now replaced by a similar beam 3 (say rock), and R_1 is kept constant and a new balance obtained,

$$\frac{E_1}{E_3} = \frac{K_1 l_1^3 w_3 t_3^3 R_3}{K_3 l_3^3 w_1 t_1^3 R_1}$$

Since the same clamping conditions apply to beams 2 and 3 $K_2 = K_3$.

Thus,

$$\frac{E_2}{E_3} = \frac{l_2^3 w_3 t_3^3 R_3}{l_3^3 w_2 t_2^3 R_2}$$

If $l_2 = l_3$ as in practice, replacing suffixes 2 by S (steel) and 3 by R (rock), and putting

$$F = \frac{w_S t_S^3}{w_R t_R^3} E_S ,$$

$$E_R = F \cdot \frac{R_S}{R_R} . \quad (1)$$

In the case of the phase, the specific loss $\frac{\Delta w}{w}$ in a specime is related to the phase lag ϕ by

$$\frac{\Delta w}{w} = 2 \pi \tan \phi \doteq 2 \pi \phi$$

In the case of the standard beam 1 and steel beam 2, if ϕ_2 is the phase difference between them, ϕ_2 is related to the necessary resistance R_2 and inductance L_2 to produce balance (in phase) by

$$\phi_2 = \omega \frac{L_2}{R_2}$$

For the rock beam 3 replacing the steel beam 2, if L_1 and R_1 are unchanged,

$$\phi_3 = \omega \frac{L_3}{R_3}$$

is the new phase difference.

The internal friction of the rock is given by $2 \pi (\phi_2 - \phi_3)$ if the loss in the steel beam is ignored. This is justified in a later section.

$$\text{Thus } \left(\frac{\Delta w}{w} \right) = 2 \pi (\phi_2 - \phi_3) = 4 \pi^2 \cdot f \left(\frac{L_2}{R_2} - \frac{L_3}{R_3} \right) .$$

Using the notation above, and writing the quantities

$\frac{fL}{R}$ as ϕ ,

$$\left(\frac{\Delta w}{w}\right) = 4\pi^2 (\phi_S - \phi_R) = 4\pi^2 \phi. \quad (2)$$

These expressions (1) and (2) are the simple working formulae.

3. Corrections

The above derivations assume that the formula $y = \frac{F}{S}$ is independent of frequency; this is only true if the operating frequency f is very much less than the resonant frequency f_R . In practice this could not be realised and values of $\frac{f}{f_R} < 1/3$ were used.

The general forced-vibration equation giving the amplitude of vibration y at an angular frequency w when w_0 is the resonant angular frequency is

$$y = \frac{A}{((w_0^2 - w^2)^2 + r^2 w^2)^{\frac{1}{2}}}$$

where r is a constant depending on the dissipation, and A a constant depending on the force applied.

It can be shown that $\frac{r^2 w^2}{w_0^4} \doteq (.005)^2$ so the term containing $r^2 w^2$ may be ignored.

$$\text{Then } y = \frac{A}{(w_0^2 - w^2)} = \frac{A}{w_0^2} \left(1 + \frac{w^2}{w_0^2} + \frac{w^4}{w_0^4} + \dots\right)$$

Since $\frac{w}{w_0} \leq 1/3$, terms greater than the third may be ignored.

The same argument applies to both amplitude and phase; the two working formulae derived above are thus true only if the corrected values of R_R , R_S , ϕ_R and ϕ_S are used.

The measured value of R_S is given by

$$R_S(\text{measured}) = R_S \left(1 + \frac{w^2}{w_0^2} + \frac{w^4}{w_0^4} \right)$$

and of ϕ_S by

$$\phi_S(\text{measured}) = \phi_S \left(1 + \frac{w^2}{w_0^2} + \frac{w^4}{w_0^4} \right)$$

A set of graphs were plotted so that percentage correction could be read off at once and the values of the corrections subtracted from the measured values of R or ϕ . Some typical results are included at the end of this section to illustrate the method of calculation.

B. Method of Operation

A suitable rock beam was first chosen; the method of cutting beams is described in a later section. The end clamps were then set at a suitable separation, to give an overhang of up to 1 cm. at each end, and such that they were equidistant from the connecting rod from the vibrator.

A suitable steel beam (i.e. one of similar dimensions) was then chosen and the experiment first performed with this

beam, and later with the rock beam in its place. The centre clamp, to which an optical flat was attached, was then placed in the centre of the steel beam, making sure that the beam was symmetrical with respect to the support, and the nuts were then tightened to a standard position. (Experiment showed that about $\frac{1}{2}$ turn more than fingertight was satisfactory see later). The thickness of the beam was noted and the nuts on the end clamps adjusted to give a standard value of spring pressure. This was obtained by unscrewing the nuts by two complete turns from the position of maximum tightness. The two end clamps were then simultaneously raised against the spring pressure by means of small levers, and the beam placed in position so that the stud from the centre clamp was vertically above the connecting rod from the vibrator. The connecting cylinder was then screwed into the stud and fixed in position by a nut.

Naturally, this same procedure was repeated in the case of the "standard" steel beam. This was not moved in the course of taking readings; it was tightly clamped in position to avoid accidental movement. It was a mild steel beam of dimensions approximately 1.3 x 0.3 x 20 cm. and had a resonant frequency of about 140 cycles per sec. It was chosen after considerations of resonant frequency, amplitude and the anticipated values of E for the various rocks used.

The next step, having the standard and steel beams in position, was to adjust the optical flats so that about seven or eight fringes occupied the field of view; this number was found to be the most convenient for measurements. The fringes were normally adjusted to lie parallel to the length of the beam, since this reduced the effect of stray vibration which could not be balanced out. Coarse adjustments were made by means of the four screws T, and the fine adjustments by the upper nuts Q in conjunction with the foam rubber. The flats were initially arranged parallel by eye, the microscope being used in the final stages. When in the correct adjustment, the lower nuts Q were tightened, clamping the flats in position.

After allowing a five-minute warming up period, the balance position was obtained by the following means. The amplifier was adjusted to give a current of about 10 mA. through each vibrator, and the resistance in the steel beam circuit was varied until the fringes ceased to vibrate to and fro; a frequency of 5 cycles per sec. was used at this stage. If a balance was not possible with the initial resistors, the resistance in the standard beam circuit. was changed by say 100 Ω . The current was then adjusted to about 80 mA. (40 mA per branch) and a more accurate balance obtained.* The inductance in the steel beam circuit was then

* Note. The strain was 10^{-5} to 10^{-6} .

changed to remove the remaining fringe vibration, by balancing the phases. In general a small phase difference appeared between the two steel beams; this could be removed by placing the required inductance in the steel beam circuit. Normally, however, an inductance of 100 mH. was placed in the standard circuit, and the necessary balancing inductance in the steel circuit.

After this preliminary experiment, a check was made to ensure that the apparatus was working properly. The steel beam was entirely removed, and then replaced; the fringes should reappear in approximately the same position and the balance position should be unchanged. Also, a balance position was obtained for different current and again the position should be unchanged. These conditions are discussed in a later section; if satisfactory, the experiment was then performed.

The oscillator frequency was set at 2, 5, 10, 15, 20, 25, 30, 35 and 40 cycles per sec. in this order, and the inductance value noted in each case; an approximate resistance value was also noted. The accuracy of these measurements is discussed later. The L values were then checked by repeating the frequencies in the reverse order. The resonant frequency was obtained by observing the fringes and changing the frequency near resonance; it could be obtained

to the nearest 1 cycle per sec. As explained above, care was taken to see that the second or third harmonics of the operating frequency were not very near to the resonant frequency; if so, the particular frequency was avoided. For the frequencies above 20 cycles per sec., it was more accurate to use the stroboscope, with a difference of frequency of about 5 cycles per sec. D.C. readings were made in some cases, but the accuracy was poor; this was **mainly** due to vibrations of the building.

The frequency range was then repeated in the same manner, the inductance being set to the previously measured value and the resistance value being noted to the nearest 0.1 ohm.

The above description refers to the calibration of a steel beam with which a rock beam was then compared. This calibration is unchanged as long as the distance between the end-clamps is the same. One calibration therefore suffices for the measurement of several rock beams.

The whole procedure was then repeated for the selected rock beam, and the results tabulated and worked out as shown below. The above procedure was that finally adopted; a summary of experiments to check the correct working of the apparatus is given in a later section.

C. Typical Calculation of Results

Typical results for a Diorite beam (L values) and an Old Red Sandstone beam (R values) are shown in the tables.

Referring to the R values first, column 2 gives the "measured R values", which are the average of the two readings on the resistance box (2-40 cycles per sec. 40-2 cycles per sec.). The "Total R" column is obtained by adding 68 Ω (the resistance of the vibrators and coils etc,) to the "measured R". From correction graphs previously prepared, the percentage correction was read off directly for the particular resonant frequency. In practice the correction was kept less than 10%. The "correction" is then subtracted from the "total R" to give the "corrected R". Finally E_R is obtained for each frequency by using the working formula derived above, using the "corrected R" values for the standard steel beam.

i.e. $E_R = F \times \frac{R_S}{R_R}$

The method of calculation of $\frac{\Delta \omega}{\omega}$ is similar. The phase ϕ is worked out for each frequency, and corrected in the same manner as for R. A value of ϕ is obtained by subtracting the rock phase from the steel phase, and $\frac{\Delta \omega}{\omega}$ finally obtained from the working formula, i.e.

$$\frac{\Delta w}{w} = 4 \pi^2 f \phi.$$

A graph of E versus f was made for each **rock beam**; for $\frac{\Delta w}{w}$ a plot was only made for the average values for each rock type, since individual errors were relatively large.

TABLE 3

Diorite (Beam Di.2). R. Values

f (c/s)	Measured R(Ω)	Total R(Ω)	(%) Correc- tions	(Ohms) Correc- tions	(Ohms) Correc- ted R_R	(Ohms) Correc- ted R_S	($\times 10^{-11}$)(cgs $E_R (=F \times R_S/R_R)$)
2	101.4	169.4	0	0	169.4	158.7	5.460
5	101.8	169.8	0.1	0.2	169.6	159.3	5.460
10	102.0	170.0	0.5	0.9	169.1	159.6	5.472
15	102.0	170.0	1.0	1.7	168.3	159.2	5.480
20	101.9	169.9	1.8	3.0	166.9	157.5	5.480
25	101.5	169.5	2.8	4.7	164.8	155.9	5.485
30	100.9	168.9	4.0	6.8	162.1	153.2	5.485
35	100.1	168.1	5.4	9.1	159.0	150.2	5.485
40	99.3	167.3	7.1	11.9	155.4	146.9	5.490

Resonant frequency $f_R = 155$ cycles per sec.

$$w_S = 0.50'' \quad t_S = 0.16''$$

$$w_R = 0.445'' \quad t_R = 0.223''$$

$$w_S t_S^3 = 204 \text{ units}$$

$$w_R t_R^3 = 493 \text{ units.}$$

$$F = 2.0 \times 10^{12} \times \frac{204}{493} = 830 \text{ units}$$

TABLE 4

OLD RED SANDSTONE (Beam ORS 3) L. VALUES

f (Hz)	Measured L (mH)	Total R (ohms)	($\phi = \frac{fL \times 10^3}{R}$)	% Corrn.	Corrn.	Corrd. ϕ_R	Steel Beam Corrd. ϕ_S	$\delta \phi$	$\frac{\Delta w}{w}$
2	72	140	1.03	0	0	1.03	2.25	1.22	.049
5	92	142	3.25	0	0	3.25	4.46	1.21	.048
10	90	141	6.39	0.5	<0.1	6.39	7.90	1.51	.060
20	88	141	12.5	2.0	0.3	12.2	13.7	1.50	.060
30	83.5	140	17.9	4.5	0.8	17.1	19.0	1.90	.076
40	82	140	23.5	8.0	2.2	21.3	23.6	2.10	.084

fR = 144 cycles per sec.

SECTION 7

MECHANICAL PROBLEMS AND ERRORS

A. Mechanical Sources of Error

There are a number of possible mechanical sources of error. These are enumerated below:

1. General vibration of apparatus.
2. Errors associated with vibrators.
3. Resonances of beams.
4. Inadequate optical flat clamps.
5. Inadequate centre clamps.
6. Inadequate end clamps.

1. Discussion

The vibration of the apparatus as a whole caused some initial difficulty, but this was greatly reduced by mounting the apparatus on sponge rubber. Measurements were then possible providing no very large movements in the laboratory occurred. As explained above, the apparatus was of a very rigid construction, and no resonance effects occurred in the rigid frame.

Since a comparison method was used, it is unlikely that any interaction effect between the vibrators will be important. It was noted that the vibrators could be removed and re-attached without appreciably affecting the balance position, provided the central shaft from the vibrator was vertical. As explained above, the shafts were arranged to be vertical in practice.

The effect of resonance has already been mentioned. The effect was eliminated by avoiding the use of operating frequencies of which the resonant frequency was the second or third harmonic. A frequency differing by about 2 cycles per sec. from the usual operating frequency was used in these cases.

No difficulties were noted with the optical-flat clamps or centre-clamps. It was possible to remove these clamps and replace them without disturbing the balance position. However, the adequacy of these clamps can be better seen from the "apparatus check" described later.

It was found that the effect of the end-clamps was of some importance, and a discussion of the various experiments performed in connection with this problem is given below. It was found that this effect could be almost eliminated, and this is demonstrated in the "apparatus check" section.

2. Effect of end-clamping

Initially it was hoped to rest the beams on knife edges at each end; however, the beams were very unstable and tended to rock from side to side. The formula for this case, already quoted is.

$$y = \frac{1}{48} \cdot \frac{Fl^3}{EI}$$

If to remove this effect, a stabilising force is applied (e.g. by an inverted knife edge), the resonant frequency and the formula for the displacement y are somewhat changed. The extreme case is that in which the beam is perfectly clamped at each end so that

$$y = \frac{1}{192} \cdot \frac{Fl^3}{EI}$$

In practice it is impossible to apply a perfect clamp.

At a position near the clamp there must be a definite strain, and this can hardly be supposed to become zero immediately at the clamp. An end-effect will occur, so that l in the above formula is slightly increased. The work of Regula⁽¹⁸⁾ and Fortsch underlined this. Regula attributed the difference between his static and dynamic results to this effect alone. Also Swift⁽⁵¹⁾, in experiments with steel beams of about 1" x 0.5" cross-section noted that errors were introduced when the clamping stress fell below a certain value depending on the actual cross-section.

For rocks, only relatively small clamping stresses are possible, on account of their brittle nature. The formula applying is then one involving a factor X where $48 < X < 192$ and $l = l_0 + \delta l$, so that

$$y = \frac{1}{X} \frac{Fl^3}{EI} .$$

It is thus important to investigate the effect of clamping the beams, and to see how the values of X or δl change with frequency and clamping. If δl changed with frequency, this would produce an apparent change in E . A similar result would occur if X changed with frequency, or if different effective clamping pressures were used for steel and rock beams.

Experiments were thus performed to check these effects, and these are briefly discussed below.

(a) R. values

Beams of steel, slate and old red sandstone were used, the slate being one of the hardest and the Old Red Sandstone one of the softest rocks investigated. The balance values were noted over the whole frequency range for various clamping pressures. These were obtained by unscrewing the 2 BA nuts by 1 turn ("1 up"), 2 turns ("2 up") etc. from the position of maximum tightness ("0 up"). The "1 up" position corresponds to a value of about 20 lb. clamping weight and the "5 up" position to a value of about 10 lb.

The values of the "corrected R" for the "2 up" positions were taken as standards, and the values at the other clamping pressures obtained with reference to this. The "2 up" columns thus contain the "corrected R" values and the other

columns give the ratios of R ("2 up") to R ("0 up", 4, 6 etc.)

TABLE 5

STEEL

f (c/s)	$\frac{R(2)}{R(0)}$ 0 up.	(Ohms) 2 up.	$\frac{R(2)}{R(4)}$ 4 up.	$\frac{R(2)}{R(6)}$ 6 up.
2	1.258	117.5	.927	.853
5	1.260	118.0	.925	.852
10	1.253	118.2	.924	.851
20	1.249	116.7	.923	.848
30	1.250	113.7	.927	.851
40	1.251	108.7	.930	.856
fR	171	153	147	144
Stan.	.41	--	.22	.28
Dev.				

SLATE

f (c/s)	0 up.	2 up. (ohms)	4 up.
2	1.430	134.3	.961
5	1.429	134.7	.959
10	1.429	134.9	.959
20	1.428	133.2	.962
30	1.415)?	128.9	.958
40	1.409)?	122.9	.957
fR	183	148	148
Stan.	?	--	.20
Dev.			

TABLE 5 Contd.

OLD RED SANDSTONE

f (c/s)	0 up.	1 up.	2 up. (Ohms)	4 up.
2	1.128	1.042	148.4	.985
5	1.124	1.042	148.9	.984
10	1.128	1.040	148.7	.986
20	1.123	1.041	146.6	.986
30	1.126	1.042	141.5	.986
40	1.121	1.038	134.5	.984
fR	151	144	140	139
Stan. Dev.	.28	.17	--	.10

The standard deviations from the mean value in each column are shown in the table. These are less than 0.30 in all cases except that of steel 0 and slate 0. Corresponding clamping positions for steel and the two rocks were then used to calculate the values of E; the percentage changes in E over the complete frequency range were then found. For slate the values were 0.8%, 1.1%, and 0.7% for the clampings 0, 2 and 4 respectively, and the corresponding values for Old Red Sandstone were 2.1%, 2.0%, and 2.1%. These results demonstrate that over the clamping range of say 1 - 4 the apparatus works in a satisfactory manner. The deviations involved are of the same order of magnitude as those

expected from theoretical consideration (see later). It appears that $\bar{f}_L$ does not vary with frequency, and that although the magnitude of the factor X varies with clamping pressure, no frequency-dependent effect is introduced.

However, the end-clamping effect has been further investigated for slate and steel beams. A steel beam was investigated over a wide range of clamping pressures. It was found that no effect occurred over the clamping range 1 - but that the R value was decreased slightly for very high pressures, at the higher frequencies (> 25 cycles per sec.). For very light clamping the beam was unstable and rocked on the knife-edges making measurement difficult. This high pressure effect could only just be produced with slate, since the beams tended to fracture. The effect appeared to be connected with the formation of a permanent groove in the specimen caused by the heavy clamping pressure. It is unimportant in the present context. Results for steel and slate beams at different clamping pressures were compared, and the results agreed with those above. However, it was noted that for a particular thin steel beam, under light clamping pressure, the value of R for higher frequencies was less than that expected. This caused an apparent decrease in the value calculated of E_{slate} at greater than 25 cycles per sec.

The effect was investigated by using steel and dural beams of different cross-section. For small cross-sections, the above decrease occurred for light clamping, but disappeared for heavier clamping. When a deliberately twisted beam was used, the effect was much more marked; however, a twisted beam of thick cross-section showed no effect. These results suggest that for light clamping and thin cross-sections, a change in $\bar{E}$ with frequency occurs, the effective length being smaller at the higher frequencies. This is what one would expect from purely mechanical considerations; the effect would produce an apparent increase in E with frequency. However, for the cross-sections and clamping ranges used in practice, the effect did not occur.

It was noted in the cross-section experiment, that when multiplying by the relevant factor ($w_1 t_1^3 / w_2 t_2^3$ etc.), the values obtained for all the beams of one material were not identical, though the ratios at the various frequencies were constant. This is due to the fact that the factor X changes for different cross-sections, even if the magnitude of the applied pressure is the same. However, this affected the frequency change in neither E nor $\frac{\Delta W}{W}$.

The above length effect was further studied by using beams of different lengths and varying the clamping pressures. The values were compared and little effect noticed except that

the actual magnitude ($\bar{f}_l$) of the length effect was less for heavy clamping than for light clamping. This was found by multiplying the corrected R values by the relevant ratios in length. Similar results were obtained for a slate beam, values of E_{slate} being found for each length; these were in good agreement. However, in practice the same lengths were always used for steel and rock beams.

The effect of overhang of the beams beyond the end-clamps was studied, in conjunction with clamping changes. No effect was observed for overhangs up to 5 cm. each side (clamping pressure 3) or 9 cm. each side (clamping pressure 0).

The following conclusions were reached, as a result of the above experiments, regarding the influence of the end-clamps on the R values:-

1. For clamping positions between 1 and 5, no frequency dependent effect on $\bar{f}_l$ or X occurs for either steel or rock, providing very thin beams are not used.
2. For light clamping, a reduction in R at the higher frequencies may occur. This is accentuated by twisting or by using small cross-sections. The result is that the effective length at frequencies greater than 25 cycles per sec. is less than that at lower frequencies.

Thus, it is seen that the various clamping difficulties are easily overcome.

(b) L values

A similar experiment to that for the R values was performed. Beams of steel, slate and old red sandstone were used over a similar range of clamping pressures and frequencies. It was noted that for steel, a very small change in L occurred between positions 1 and 4, and a larger change between 0 and 1, the change being greater at high frequencies. Slate behaved in a similar manner, but the change for old red sandstone was less, probably because the maximum pressure (position 0) was less in this case. Values of $\emptyset$ at various clamping pressures were obtained, and are shown below.

TABLE 6

STEEL

$(\emptyset = \frac{fL}{R} \times 10^3)$				
f (c/s)	0	2	4	6
2	2.6	2.5	2.3	1.7
5	5.1	4.7	4.6	4.2
10	8.5	8.0	8.0	7.3
20	14.1	13.5	13.3	13.3
30	19.6	19.0	18.6	18.4
40	24.6	23.7	23.2	22.7
fR	171	153	147	144

TABLE 6 Contd.

SLATE

f (c/s)	0	2	4
2	2.5	1.9	1.9
5	4.8	4.2	4.4
10	7.9	7.4	7.6
20	13.5	13.0	13.2
30	19.0	18.2	18.1
40	23.1	22.8	22.6
fR	183	150	148

OLD RED SANDSTONE

f (c/s)	0	1	2	4
2	1.2	1.1	1.1	1.05
5	3.4	3.3	3.3	3.1
10	6.4	6.4	6.4	6.2
20	12.1	12.0	11.9	11.9
30	17.7	16.9	17.0	16.9
40	21.9	21.2	21.2	21.0
fR	151	144	140	139

The values of $\frac{\Delta w}{w}$ were found from these results; the values obtained from corresponding pressures of clamping agreed with one another within the errors involved (about ± 0.1 in the above). These values are shown in the table below. The values over ranges 2 - 4 (slate) and 1 - 4

(Old Red Sandstone) were identical and are compared with the results found later.

TABLE 7

f (/s)	$\frac{\Delta w}{w}$	SLATE				$\frac{\Delta w}{w}$	OLD RED SANDSTONE	
	0	2-4	6	Later Value		0	1 - 4	Later Value
2	.004	.012	.012	.010		.056	.048	.049
5	.020	.016	.012	.016		.072	.052	.048
10	.016	.020	.016	.018		.076	.068	.060
20	.024	.020	.020	.020		.084	.060	.060
30	.024	.024	.016	.026		.084	.076	.076
40	.028	.032	.016	.028		.108	.096	.084

It can be seen that over a clamping range of about 1 - 6 up., no unexpected clamping effects occur. This range was always used in practice.

It is of considerable interest that these values of $\frac{\Delta w}{w}$ for slate and old red sandstone show a definite change with frequency. This effect was noticed at a very preliminary stage, and one of the purposes of the above clamping check was to see whether the effect could be changed by altering the clamping pressure. As can be seen, the results conclusively suggest that the effect is a real property of the rocks.

There are no other measurements of $\frac{\Delta w}{w}$ over this frequency

range for rocks. However, measurements on metals, glasses and some dielectric materials indicate that $\frac{\Delta W}{W}$ is independent of frequency over a wide range (down to 0.1 cycles per sec.). It is not possible to measure $\frac{\Delta W}{W}$ for metals by the present method; a material with an internal friction considerably larger than that for steel must be used. Beams of tufnol and wood were therefore measured, over the range 2 - 30 cycles per sec., and two sets of results were obtained in each case for the normal (2) and heavy (h) clamping positions. In all cases it was noted that $\frac{\Delta W}{W}$ was independent of frequency.* In the case of wood, the magnitude of $\frac{\Delta W}{W}$ under heavy clamping was about 25% less than that under normal clamping. This was probably due to the wood near the clamp being compressed; a marked impression was noted in the beam after clamping tightly.

Further experiments were performed on a beam of commercial asbestos. This was chosen so that extremely hard clamping pressures could be applied. The asbestos behaved like the slate and old red sandstone. Again about a 25% reduction occurred in $\frac{\Delta W}{W}$ on clamping heavily - an imprint was again produced in the beam. The experiment was repeated using even heavier clamping pressures, and very little further change occurred.

The graphs (Fig. 10) show the results for wood, tufnol,

* Note. Tufnol shows a small increase of $\frac{\Delta W}{W}$ with f for normal clamping.

asbestos, slate and old red sandstone under different clamping conditions. The conclusion is that the change in $\frac{\Delta W}{W}$ observed for rocks is a real effect, and not one due to clamping.

N.B.

The percentage changes in E were noted in all cases above. The clamping changes had no effect on this value. The values were 3.0% for wood, 2.5% for tufnol and 1.2% for asbestos.

3. Apparatus Check

Besides the check regarding clamping, a more general check was performed. Three steel beams and two dural beams, all of different cross-section, were measured in the usual manner. One of the steel beams was chosen as a standard, and E was found for each of the other beams with respect to this, and plotted against frequency. A straight horizontal line should be found if the experiment is working properly. The results are summarised in Table 8.

TABLE 8

STEEL BEAMS				DURAL BEAMS	
f (c/s)	(d) [R(b)/R(a)]	(b) [R value]	(e)	(a)	(b)
2	1.033	564	.931	.481	.417
10	1.031	572	.936	.479	.416
20	1.031	569	.933	.482	.417
30	1.027	551	.930	.482	.414
40	1.034	488	.931	.483	--
fR	204	133	122	142	125
Stan. Dev.	.24	Standard	.24	.14	.14

It can be seen that the standard deviation of any point from the mean is of the order 0.25% of the total value. In practice the resonant frequencies used were between 120 cycles per sec. and 250 cycles per sec., most values being about 180 cycles per sec. The above resonant frequencies are therefore typical. The accuracy of the corrections for resonance is evidently good, since the results for beams of widely differing resonant frequencies agree. In practice the correction was always less than 10%.

The cross-sections of the rock beams were usually somewhat larger than those used here, in order to obtain the required resonant frequencies. The above values of the factor proportional to E should be identical; the difference

FIG. 9. THEORETICAL AND PRACTICAL CLAMPS.

BEAM RESTING ON
KNIFE EDGES.

$$y = \frac{FL^3}{48EI}$$

PERFECT CLAMP.

$$y = \frac{FL^3}{192EI}$$

EXPERIMENTAL
ARRANGEMENT.

$$y = \frac{F(L_0 + 2dl)^3}{X \cdot E \cdot I}$$

end correction dl .

is due to the change in the factor X with thickness of specimen. The change is smaller for large thicknesses, and is not larger than $\pm 5\%$ in most practical cases. However, only the relative values of E are important in the present context.

(b) L. values

The phases of the five beams were also compared, as shown in Table 9. The average phases have been found and a standard deviation calculated as above.

TABLE 9

STEEL BEAMS				DURAL BEAMS			Stan. Dev.
f (c/s)	(d) ϕ	(e)	(b)	(a) ϕ	(b)	Average ϕ	
2	1.95	2.1	1.3*	1.9	1.9	1.95	$\pm .05$
5	3.8	3.75	3.1*	3.65	3.9	3.8	.08
10	5.55	5.6	5.1	5.2	5.5	5.45	.15
20	8.7	8.8	8.4	8.3	9.1	8.65	.2
30	11.9	11.9	11.5	11.5	11.8	11.7	.2
40	14.5	14.5	--	--	--	14.65	.15

The standard deviation can be seen to vary up to about .2. This corresponds to a percentage error of about $\pm 5\%$ at all frequencies.

The two steel (b) values marked * are of interest. The

low values are due to the beam being ineffectively clamped at the low frequencies due to twisting; this was verified by increasing the pressure slightly and consistent results were then obtained.

Again the experiments indicate decisively that the apparatus was working in a satisfactory manner, provided the clamping conditions mentioned above were used.

4. Method of Cutting Specimens

The apparatus was designed to use specimens of length between 15 and 22 cm., width up to 1.4 cm. and thickness up to 0.75 cm. These dimensions were decided upon after consideration of the resonant frequency, amplitude required and the strength of the specimens. The actual dimensions of a particular rock specimen were chosen in a similar manner.

A rock-cutting machine with a large horizontal wheel of 15" diameter was used, in order to obtain sufficiently long specimens. The original sample of rock was picked so as to measure about 1' x 6" x 3". This was first cut into slices measuring say 1' x 3" x 0.25"; in this size the specimens had sufficient strength to prevent easy fracture. The slice was then placed between two pieces of wood and held as loosely as possible in the jaws of the clamp on the cutting machine. Sponge rubber as well was used in some

difficult cases of very fragile rocks. The slice was then levelled and the required width chosen. Cutting was attempted only at a very slow rate. A small clamp was attached to the top of the beam to prevent it from flying off and fracturing when almost cut. By this method beams were obtained for most of the required rocks. The igneous specimens took considerable time on account of the slow cutting rate; the sedimentary ones were much easier, though more fragile.

The rock beams were not always perfectly smooth. Milling was attempted but this was not possible and always broke the beam. However, experiments on milled and unmilled steel and dural beams, and on even and uneven rock beams, indicated that no appreciable error was introduced ~~provided~~ the beam was not noticeably twisted when clamped in position.

Elastic properties of steel

The elastic moduli of various types of steel have been measured by several workers. A steel beam has sometimes been used as a check on the correct working of apparatus for the measurement of elastic moduli of rocks; this was done by Zisman⁽³⁾ and Birch and Bancroft⁽¹³⁾, and excellent agreement was obtained between static and dynamic measurements. Swift⁽⁵¹⁾ also made measurements statically and at about

3 cycles per sec. and obtained the same value for E. The fact that the value for E at several kilocycles per sec. and when measured statically is the same within about 0.2% shows conclusively that E for steel is independent of frequency. A change of 0.15% is expected theoretically from thermodynamic considerations (see Table 14). In the present calculations, E for steel has been taken as constant and of value 2×10^{12} dynes per cm^2 .

The same workers have also given values of $\frac{\Delta W}{W}$ for steel. The values were all low, being of the order 0.002 or less. The results again indicate that there is no variation with frequency. The value of $\frac{\Delta W}{W}$ for most rocks is about 20 times the above value for steel. It has therefore been assumed that $\frac{\Delta W}{W}$ for the steel beams used in the present work can be ignored at all frequencies.

B. Errors of the Experiment.

The various sources of error have been mentioned in this or previous chapters. Several types of mechanical error have been discussed and shown to be unimportant or avoidable, and the correct working of the apparatus has been demonstrated. In the absence of any such sources of error, there would still be an error in the balance

position, corresponding to the accuracy with which this can be obtained. The total error of the experiment is then the sum of these two types of error.

In the sections above, it was indicated that this total error could be deduced by averaging the values obtained for steel, dural and rock beams. In all cases a standard deviation of about $\pm 0.25\%$ for E was found. For the phase the error was about ± 0.15 in the value for ϕ . The results obtained for the various rock beams also suggest these values for the errors.

The value of E at any point on a graph of E versus f may thus be in error by $\pm 0.25\%$; these errors have not been indicated on the graphs. However, as explained above, the values of E found are relative and the absolute value of E may be different from the value quoted by $\pm 5\%$.

In the case of $\frac{\Delta W}{W}$, the total error is obtained by taking $\pm \sqrt{(5\%)^2 + (5\%)^2}$ since a comparison method is used. This suggests a value of $\pm 7\%$ in the final value for $\frac{\Delta W}{W}$.

It is interesting to compare the above values of error with those expected theoretically. The amplitude used is about 10^{-3} cm; this corresponds to

$$\frac{10^{-3}}{(2.1 \times 10^{-5})} = 40 \text{ fringe widths with } \lambda \doteq 5 \times 10^{-5} \text{ cm.}$$

if a balance can be obtained to 1/10 of a fringe width. An error of $\pm 0.25\%$ is thus expected. A balance is normally obtained to $\pm 0.2 \Omega$ in 100Ω , again about $\pm 0.2\%$.

In the case of $\frac{\Delta W}{W}$ the phase angle ϕ is of the order 1° so that the maximum amplitude displacement caused is

$$\left(\frac{1}{57}\right) \cdot 10^{-3} \text{ cm. } \doteq 1 \text{ fringe width.}$$

An error of about $\pm 10\%$ is thus to be expected.

Referring to Table 10 which gives the results obtained for E for all the rocks studied, the standard deviation of any value of $\sqrt{E}$ from the mean is about 0.25%. The following table, giving the results for $\frac{\Delta W}{W}$ suggests an average error of very nearly 10%.

The errors in the experiment are thus:-

E:	Error in absolute magnitude	$\pm 5\%$
	Error in relative variation	$\pm 0.2\%$
$\frac{\Delta W}{W}$:	Error in absolute magnitude	$\pm 10\%$

SECTION 8

RESULTS

Measurements have been made on the following types of rock:

1. Slate: Green. Origin unknown. Very homogeneous, little sign of cleavage and bedding.
2. Diorite: Cockermouth, Cumberland. Typical coarse-grained specimen.
3. ~~Diorite~~: ~~Analcime~~, Clee Hills, Salop. Typical fine-grained specimen.
4. Limestone: Shelly. Carboniferous, Derbyshire. Small shells.
5. Limestone: Upper oolite. Portland. Typical specimen.
6. Chalk: Middle. Flamborough head pebble. Very hard specimen.
7. Sandstone: Old Red, Perthshire. Typical, some clay pellets.
8. Sandstone: Micaceous, Weald. Noticeable bedding.
9. Millstone grit: Carboniferous, Derbyshire. Very soft and weathered specimen.

No attempt was made to cut specimens in different perpendicular directions or with respect to the bedding. Those in which bedding was apparent were cut parallel to it.

The following experiments were performed:-

- A: Variation of E with frequency: 3 beams of each rock .
- B: Variation of $\frac{\Delta W}{W}$ with frequency: 2 beams of each rock .
- D: Effect of moisture on $\frac{\Delta W}{W}$ and E: 1 beam each of dolerite, oolitic limestone, millstone grit, old red sandstone and micaceous sandstone.
- C: Other materials: Variation of E and $\frac{\Delta W}{W}$ with frequency for asbestos, wood and tufnol.
- E: Effect of oil on $\frac{\Delta W}{W}$ and E: 1 beam of old red sandstone.

A. Variation of Young's Modulus with Frequency

The method of operation of the apparatus and the method of calculation of the results have been described in detail above. Several beams were cut of each rock type, and three beams of each then measured. The dimensions of the beams were similar, and were chosen for each rock type so as to permit a reasonable amplitude of vibration to be obtained. Different lengths were sometimes used, and this entailed the calibration of further steel beams. It was found that

the actual calibration values altered slightly (by about 1%) in the course of several days, and in practice calibrations were made about every two days.

Table 10 summarises the results obtained. The reference number and dimensions of each specimen are given in column: 2.

Column 3 contains the resonant frequencies - the choice of specimen size was usually such that the resonant frequency was between 100 and 250 cycles per sec. The actual magnitude of Young's modulus E at 2 cycles per sec. is given in column 4 (the error is $\pm 5\%$ as explained above). Column 5 contains the percentage change in E over the 2 - 40 cycles per sec. range, based on the 2 cycles per sec. value and columns 6 and 7 give average values of the three results for the three beams in columns 4 and 5.

The graphs, Fig. 11 and Fig. 12, are plots of E versus f for each rock type. Each graph is an average of the results of measurements on the three beams of each type. In these graphs, the actual magnitudes of E are in arbitrary units and have been adjusted to be roughly equal to make comparison easier.

The temperature and pressure may be assumed to be normal room values. The specimens were not dried before measurement; as explained later, only the old red sandstone, millstone grit and micaceous sandstone results would be affected.

B. Variation of Internal Friction with Frequency.

The same rock types were used as in A, in the manner described previously; the same beams were used but only two of each type in this case. Frequent calibrations of the steel beam were made, one calibration serving for both E and $\frac{\Delta W}{W}$ measurements.

Some difficulty was experienced in making accurate measurements at frequencies below 5 cycles per sec. Sometimes a measurement was not possible because of the very large value of inductance required. As a result, there is some uncertainty in the values of $\frac{\Delta W}{W}$ at low frequencies.

Table 11 summarises the results. The first three columns are similar to the corresponding ones in Table 10. Column 4 gives the measured value of $\frac{\Delta W}{W}$ at 2 cycles per sec and column 5 the value at 40 cycles per sec. Column 6 gives average values of the two beams for $\frac{\Delta W}{W}$ at 2 cycles per sec; owing to the above uncertainty, these values have been obtained by drawing the most reasonable line through the points, making allowances for the errors involved. This is discussed later. Column 7 contains average values of $\frac{\Delta W}{W}$ at 40 cycles per sec. The graphs, Figs. 13, 14 and 15 are plots of $\frac{\Delta W}{W}$ versus f for all the rock types. Each graph is an average of the two beams of each type. The same scale has been used in each case. Fig. 16 is a plot of the

percentage variation in E (from 2 - 40 cycles per sec.) from Table 10, versus the change in $\frac{\Delta W}{W}$ (from 2 - 40 cycles per sec.) from Table 11.

C. Measurements on Other Materials

It has been mentioned above that a check was made on the working of the apparatus by using beams of wood, tufnol and asbestos. Measurements were made in the usual manner on both the variation of E and $\frac{\Delta W}{W}$ with frequency. Different clamping conditions were used in the measurements, but results for the normal condition (i.e. as used for rocks) are given below in Table 12.

Columns 4 and 5 contain $\frac{\Delta W}{W}$ at 2 cycles per sec. and 40 cycles per sec. - these are the same for wood and tufnol within the errors of the experiment. Column 6 gives the actual magnitude of E at 2 cycles per sec. and column 7 gives δE , the percentage variation in E from 2 - 40 cycles per sec. Figs. 17 and 18 show plots of $\frac{\Delta W}{W}$ versus f, and E versus f for these three substances. The values of E have been adjusted to be approximately the same to aid comparison.

D. Effect of Moisture on Elastic Behaviour

Measurements on specimens containing a known percentage (by weight) of water were made on one beam each of dolerite,

oolitic limestone, old red sandstone, micaceous sandstone and millstone grit. A preliminary experiment indicated that the specimens could be effectively dried by leaving overnight in an oven at about 100°C ; also, soaking overnight in a bath of water was found to be sufficient to produce almost complete saturation.

A saturated beam was measured in the usual manner, except that frequencies of 2, 5, 10, 20, 30 and 40 cycles per sec. only were used to reduce the time of the experiment (because of evaporation difficulties). The weight of the beam was noted immediately before and after the experiment. The beam was then allowed to dry a little in air and the experiment repeated. This process was continued until the weight of the beam remained constant - the percentage of water was then equal to that of a normal undried specimen.

A final experiment was performed after leaving overnight in an oven. Starting with a saturated specimen, measurements could be made at about hourly intervals, about 4 hours being required to attain the laboratory weight. Extra points were then obtained in the graphs by repeating if required.

Dolerite showed no effect and oolitic limestone showed only a small effect so it was decided to concentrate on sandstones. The results are summarised in Table 13. Column 3 gives the total water content (by weight) at saturation. The percentage

variations in E (from 2 - 40 cycles per sec.) for dry and saturated beams are given in columns 4 and 5 and the values of $\frac{\Delta W}{W}$ (at 40 cycles per sec.) for dry and saturated beams in columns 6 and 7. Finally column 8 gives the percentage decrease in E (at 2 cycles per sec.) on saturation, based on the oven-dry value. The variations in E were not measured for dolerite, oolitic limestone and millstone grit.

Figs. 19 and 20 show plots of $\frac{\Delta W}{W}$ versus f for millstone grit, old red sandstone and micaceous sandstone for different moisture contents. Figs. 23 and 24 show plots of percentage E change (from 2 - 40 cycles per sec.) versus change in $\frac{\Delta W}{W}$ (2 - 40 cycles per sec.) for micaceous sandstone and old red sandstone for different moisture contents.

E. Effect of Oil on Elastic Behaviour

On account of the large changes in both E and $\frac{\Delta W}{W}$ produced by the addition of water to sandstones, an experiment was performed using a thick motor oil instead of water, for the same beam of old red sandstone as was used in the water experiment.

The experiment was performed in a similar manner to the water experiment, but in the reverse direction. An oven dry specimen was measured first and progressively saturated with oil. Preliminary experiments indicated that it was possible

to saturate the beams by simply soaking in oil. Oil contents up to 1% were possible in two or three hours, but several days soaking was required for the final 3% saturation. It was possible to remove the oil by soaking in paraffin for a few days and then heating in an oven.

The results are summarised in the last line of Table 13 below the results for water. Fig. 22 shows a plot of E versus f for various oil contents. Fig. 20 contains a plot of $\frac{\Delta W}{W}$ versus f for various oil contents, and Fig. 24 shows a plot of the variation in E (2 - 40 cycles per sec.) versus the variation in $\frac{\Delta W}{W}$ (2 - 40 cycles per sec.) as a function of oil content.

TABLE 10

Rock Type	Dimensions	Res. f (c/s)	E_{-11} ($\times 10^6$ cgs) (2c/s)	δE (%) (2-40 c/s)	Avg. E ($\times 10^6$ cgs)	A. . (%) δE
Dolerite	7.5"x.31"x.15" 7.5"x.37"x.16" 7" x.44"x.27"	82 92 206	8.8 9.0 8.3	0.8 1.0 0.7	8.7	0.8
Old Red Sandstone	7" x.59"x.21" 7" x.51"x.27" 7" x.47"x.28"	106 119 139	1.8 2.1 1.6	2.5 2.2 3.1	1.8	2.6
Mica Sandstone	6"x.53"x.29" 6"x.52"x.27" 6"x.50"x.25"	277 242 218	3.2 3.3 3.6	1.7 1.9 2.0	3.4	1.9
Oolitic Limestone	7"x.40"x.26" 7"x.47"x.28" 7"x.57"x.28"	133 169 146	3.5 3.0 3.0	1.3 1.0 1.1	3.2	1.2
Slate	7.5"x.36"x.14" 7.5"x.46"x.20" 7" x.46"x.18"	96 172 125	8.5 7.6 7.9	1.0 1.05 1.1	8.0	1.0
Chalk	7"x.38"x.30" 7.5"x.40"x.30" 6"x.49"x.30"	183 180 256	3.8 3.2 3.6	1.1 1.3 1.1	3.5	1.2
Shelly Limestone	6"x.49"x.27" 7"x.46"x.25" 7"x.47"x.25"	237 164 161	4.1 4.2 4.8	1.1 0.9 0.9	4.4	1.0

TABLE 10 Contd.

Rock Type	Dimensions	Res. f (c/s)	E ($\times 10^6$ cgs) (2c/s)	δE (%) (2-40 c/s)	Av. E ($\times 10^6$ cgs)	Av. (%) δE
Dio-rite	6"x.41"x.23"	307	6.7	1.0	6.5	0.9
	7"x.45"x.22"	155	6.8	0.8		
	7"x.47"x.23"	154	5.9	0.9		
Mill-stone Grit	6"x.54"x.30"	115	0.70	5.8	0.83	6.6
	6"x.57"x.30"	137	0.92	7.0		
	6"x.47"x.30"	122	0.87	7.1		

TABLE 11

Rock Type	Dimensions ins.	fR (c/s)	$\frac{\Delta W}{W}$ (2c/s)	$\frac{\Delta W}{W}$ (40c/s)	Av. $\frac{\Delta W}{W}$ (2c/s)	Av. $\frac{\Delta W}{W}$ (40c/s)
Dolerite	7x.37x.16 7x.44x.27	93 211	.012 .004	.026 .038	.014	.034
Old Red Sandstone	7x.51x.27 7x.47x.28	131 144	.044 .049	.085 .084	.050	.085
Mica Sandstone	6x.50x.25 6x.52x.27	220 243	.028 .022	.074 .074	.035	.074
Oolitic Limestone	7x.40x.26 7x.57x.28	132 153	.018 .012	.048 .035	.020	.042
Slate	7x.46x.20 7x.36x.14	162 95	.010 .005	.028 .022	.012	.026
Chalk	7x.38x.30 7x.40x.30	200 175	.015 .040	.056 .058	.035	.057
Shelly Limestone	7x.46x.25 7x.47x.25	163 163	.005 .006	.020 .028	.010	.024
Diorite	7x.47x.23 7x.45x.22	156 144	.011 .016	.032 .032	.015	.032
Millstone Grit	6x.54x.30 6x.57x.30	125 142	.12 .08	.16 .14	.10	.15

TABLE 12

Material	Dimensions ins.	fR (c/s)	$\frac{\Delta W}{W}$ (2c/s)	$\frac{\Delta W}{W}$ (40c/s)	E (2c/s) ($\times 10^{-11}$ cgs)	δE (40c/s) (%)
Tuf- nol	7x.48x.24	109	.048	.056	2.6	2.4
Wood	7x.52x.23	102	.040	.044	2.20	2.3
Asbe- stos	7x.56x.25	136	.024	.056	5.00	1.2

TABLE 13

Rock	Beam & fR	Saturation Moisture (%)	%E varn. (dry)	%E varn. (satd.)	$\frac{\Delta W}{W}$ (40c/s) (dry)	$\frac{\Delta W}{W}$ (40 c/s) (satd)	Change in E (26/s) (%)
Doler- ite	D4. fR = 108	0.25	--- No measurable change ---				
Ooli- tic Lime- stone	04 fR = 120	5	Not measured		.032	.06	-15
Mill- stone Grit	M5 fR = 140	9	Not measured		.14	.30	-50
Mica- ceous sand- stone	MS2 fR = 250	1.0	1.6	5.5	.08	.32	-30
Old Red Sand- stone	ORS3 fR = 130	3.5 4.0	1.2	4.5	.07	.24	-40
Old Red Sand- stone (OIL)	ORS3 fR = 130	3.0	1.2	11.5	.07	.41	0

FIG.10.

CLAMPING CHECK. GRAPHS OF $\frac{\Delta W}{W}$ VERSUS f
FOR OLD RED SANDSTONE, SLATE, ASBESTOS, WOOD
AND TUFNOL.

FIG. II.

GRAPHS OF E VERSUS f

[Dolerite, Diorite, Shelly Limestone, Chalk, Slate]

FIG. 12.

GRAPHS OF E VERSUS f .

[Millstone Grit, Old Red Sandstone, Micaceous Sandstone and Oolitic Limestone.]

FIG. 13.

GRAPHS OF $\frac{\Delta v}{v}$ VERSUS f .

[MILLSTONE GRIT, OLD RED SANDSTONE, MICACEOUS SANDSTONE.]

FIG. 14.
 GRAPHS OF $\frac{\Delta \lambda}{\lambda}$ VERSUS f .
 [DOLERITE, SLATE, DIORITE.]

FIG. 15.

GRAPHS OF $\frac{A}{V}$ VERSUS f .

[SHELLY LIMESTONE, DOLITIC LIMESTONE, CHALK]

FIG. 16.

GRAPH OF δE VERSUS $\delta \left(\frac{\Delta W}{W} \right)$
FOR ALL ROCKS STUDIED.

FIG. 17.

GRAPHS OF E VERSUS f.

[TUFNOL, WOOD, ASBESTOS]

FIG. 18. GRAPHS OF $\Delta w/w$ VERSUS f .
[TUFNOL, WOOD, ASBESTOS]

FIG. 19. GRAPHS OF $\frac{\Delta W}{W}$ VERSUS f AS
FUNCTION OF WATER CONTENT.
 [MILLSTONE GRIT, MICACEOUS SANDSTONE]

FIG. 20.

GRAPHS OF $\frac{\Delta W}{W}$ VERSUS f AS FUNCTION
OF OIL AND WATER CONTENT IN SAME
SPECIMEN OF OLD RED SANDSTONE.

FIG. 21

GRAPHS OF E VERSUS f AS FUNCTION OF
WATER CONTENT IN OLD RED SANDSTONE.

FIG. 22.

GRAPHS OF E VERSUS f AS FUNCTION OF
OIL CONTENT IN OLD RED SANDSTONE.

FIG. 23.

GRAPHS OF δE VERSUS $\delta(\frac{\Delta W}{W})$ AS FUNCTION
OF WATER CONTENT IN MICACEOUS SANDSTONE.

FIG. 24.

GRAPHS OF δE VERSUS $\delta(\frac{\Delta W}{W})$ AS FUNCTION OF
OIL AND WATER CONTENT IN SAME SPECIMEN
OF OLD RED SANDSTONE.

F. Discussion of Results

Several important features can be seen in the above results; these are discussed below, and a comparison with the results of previous workers follows.

1. Variation of E with frequency

A variation definitely larger than that which could be accounted for by the errors of the experiment was noted in all cases. This ranged from about 1% for the igneous and harder sedimentary rocks, to 2.6% in the case of old red sandstone. A very high value of about 7% was found in the case of millstone grit. This was a particularly soft and very weathered specimen, and is certainly not typical of rocks in general. The plots of E versus f suggest that in most cases E is almost constant above frequencies of about 10 cycles per sec; a relatively rapid change in E occurs between 2 cycles per sec. and 10 cycles per sec. It was not possible to obtain a D.C. point, but the results obtained in attempted D.C. experiments suggested that the actual D.C. value of E was that to which the curves could be extrapolated.

For Dolerite, Diorite, Shelly limestone, Chalk and slate, E appeared to be constant above 10 cycles per sec. However, for the softer igneous rocks, E continued to increase slowly with frequency, though the most rapid change was

below 10 cycles per sec. E for Oolitic limestone and micaceous sandstone increased very slowly, whereas the increase for old red sandstone and millstone was much more pronounced.

The graphs of E versus f for the other materials (tufnol wood and asbestos) were similar to those for rocks. Variations of about 2.5% occurred for wood and tufnol, the variation again being greater at lower frequencies. The asbestos had a variation of about 1% and E was constant above 10 cycles per sec.

Generally then, most rocks showed a variation of 1% - 3% over the 2 - 40 cycles per sec. range, the change being more rapid at the lower frequencies.

2. Variation of internal friction with frequency

The errors in $\frac{\Delta W}{W}$ are of the order $\pm 10\%$; they have not been indicated on the graphs. Despite this relatively large error, the results show a very definite increase of $\frac{\Delta W}{W}$ with frequency, particularly at the lower frequencies. As explained above, very careful experiments were performed to check the validity of this effect. The increase occurred for all the rocks investigated, and the variation appears to follow that in E since in several cases $\frac{\Delta W}{W}$ is approximately constant above about 20 cycles per sec. The errors at the

lower frequencies are somewhat larger than at the higher ones, and in plotting the graphs the curve has been put in by eye in what appears to be the most reasonable position. On account of the errors involved and the scatter of points, it is not possible to be more precise than this.

The results for shelly limestone, slate, chalk and oolitic limestone suggest that the constant value of $\frac{\Delta W}{W}$ has been reached ^{before} ~~at~~ 40 cycles per sec. There is a considerable scatter for dolerite and diorite, probably due to poor specimens since cutting was difficult, and it seems reasonable to suppose that the constant value is reached quickly in these cases too. Micaceous sandstone still shows a pronounced rate of increase at 40 cycles per sec., and millstone grit and old red sandstone show smaller rates of increase. For all rocks measured $\frac{\Delta W}{W}$ almost doubles between 2 cycles per sec. and 40 cycles per sec., but, as stated above most of this change is at the lower frequencies. The magnitude of $\frac{\Delta W}{W}$ at 40 cycles per sec. range from about 0.03 for the igneous and harder sedimentary rocks, to 0.085 for old red sandstone and 0.15 for millstone grit.

Asbestos apparently behaves as most rocks, whereas Tufnol and Wood have internal frictions ~~definitely~~ independent of frequency over the range considered. These results have been discussed above in the "apparatus check" section,

and are of considerable importance since they are in agreement with those of previous workers.

The plot of percentage change in E (from 2 - 40 cycles per sec.) versus the change in $\frac{\Delta W}{W}$ (2 - 40 cycles per sec.) for all the rocks measured is interesting in that it suggests a relation between the mechanism responsible for change in internal friction and that responsible for the variation of E with f. A straight line can be drawn through the points for the softer sedimentary rocks (oolitic limestone, chalk, micaceous sandstone, old red sandstone and millstone grit) but the other rocks do not lie very well on the line.

Thus, a definite increase of $\frac{\Delta W}{W}$ occurs with frequency for all the rocks measured, the rate of increase being greater at the lower frequencies.

3. Effect of moisture on elastic behaviour

A considerable effect occurred when moisture was introduced into specimens of old red sandstone, micaceous sandstone, and millstone grit; Oolitic limestone showed a smaller effect and dolerite showed no effect at all. Experiments on the rocks not investigated showed that the absorptions were very small (less than 0.2%), so that no effect could be expected. Oven dried specimens of old red sandstone, micaceous sandstone and millstone grit showed a small variation of $\frac{\Delta W}{W}$ with frequency, but for small amounts of moisture (0.3%)

a considerable increase in $\frac{\Delta W}{W}$ occurred at all frequencies. The increase, however, was much larger at higher frequencies, suggesting a viscous type of mechanism. $\frac{\Delta W}{W}$ continued to increase up to saturation of the rock; the curves for higher moisture contents suggested that a maximum value of $\frac{\Delta W}{W}$ had almost been reached at 40 cycles per sec.

Young's modulus was also considerably affected in the rocks showing this effect. E was only measured for old red sandstone and micaceous sandstone, but in both cases the value of E for the oven-dry specimen was halved on saturation. Graphs of E versus f for various moisture contents in old red sandstone suggest that for oven dry specimens the maximum value of E is reached quite quickly (10 cycles per sec.) whereas for saturated specimens the value of E continues to increase with frequency at 40 cycles per sec. The percentage change in E from 2 - 40 cycles per sec. for old red sandstone increased from 1.2% (oven dry) to 4.5% (saturated).

Graphs of δE versus $\delta \frac{\Delta W}{W}$ were plotted for old red sandstone and micaceous sandstone for the various moisture contents. A good straight line was obtained for old red sandstone, again suggesting a linking of the loss process and the mechanism producing the variation in E. The line for micaceous sandstone was not very good, though this may have been due to inaccuracies at 2 cycles per sec. and

5 cycles per sec. where the large value of $\frac{\Delta W}{W}$ made measurement difficult.

Thus, the effect of moisture is to produce a large increase in $\frac{\Delta W}{W}$, the values at the higher frequencies being most changed; a corresponding decrease in E occurs and the percentage change in E over the frequency range also increases.

4. Effect of oil on elastic behaviour

On account of the high viscosity and high surface tension of oil (relative to water) considerable time was taken for the oil to be absorbed into the specimens. One beam of old red sandstone was used, the same beam as in the moisture experiments. The effect on $\frac{\Delta W}{W}$ was essentially the same as that for water, i.e. a large increase in $\frac{\Delta W}{W}$ occurred and the increase was more pronounced at the higher frequencies. The maximum value of $\frac{\Delta W}{W}$ was reached more quickly than in the moisture case, and again the frequency at which this occurred was smaller for higher oil concentration. Measurements could not be made at 2 cycles per sec. for high oil concentrations, since a sufficiently large inductance was not available. The results suggested a relatively rapid change in the 2 - 10 cycles per sec. region, but this was not so pronounced

as for the moisture case. The actual increase in $\frac{\Delta W}{W}$ was about 30% greater for oil than at a corresponding concentration for water.

The effect on the value of E was investigated, but no definite effect was observed. As explained, there is an uncertainty of about 5% in the actual value of E, but the values for E of oven dry and oil saturated specimens were in agreement to this accuracy. This is a very important difference from the effect of water. Graphs of E versus f for various oil contents suggest a similarity to the water effect, but the percentage change in E on saturation was 11.5% compared with 4.5% for water. A graph of δE versus $\delta\left(\frac{\Delta W}{W}\right)$ was again a good straight line.

Generally then, the effect of oil is similar to that for water, except that there is no change in the magnitude of E at low frequencies.

G. Comparison of Results with those of Previous Workers

An account of previous work of a similar nature has been given in the review section. However, the points particularly relevant to the present work are stated again below.

1. Variation of E with frequency

There are few measurements available here. Birch

and Bancroft could observe no variation over the range from 140 - 4,500 cycles per sec. for their large block of granite. Bruckshaw and Mahanta covered the range 40 - 120 cycles per sec. and observed a variation of about 2% for igneous rocks and about 2.5% for sedimentary rocks; the rate of change of E was slightly greater at the lower frequencies. The present results are only in qualitative agreement, as they suggest that for most rocks E is approximately constant above about 10 cycles per sec., and that the variation may be only 1% for igneous rocks.

A comparison of actual magnitudes of E is of little value, but it can be seen from Tables 1 and 10 that the present values are in reasonable agreement with those of previous workers.

2. Variation of $\frac{\Delta W}{W}$ with frequency

Birch and Bancroft⁽¹³⁾ observed no variation of their values of $\frac{\Delta W}{W}$ with frequency; however, in most measurements of this type the errors are relatively large, being of the order of $\pm 20\%$. Bruckshaw and Mahanta⁽²⁰⁾ found a small decrease in $\frac{\Delta W}{W}$ with decreasing frequency - the rate of decrease seemed to be larger at the lower frequencies, which again is in qualitative agreement with the present results.

As explained above, the seismic results are of two types. Firstly the experiments on pulse-shape and distance (using frequencies below 50 cycles per sec. in most cases) suggest a linear relation between $\frac{\Delta W}{W}$ and frequency (viscous mechanism) and secondly the normal amplitude-distance experiments suggest, though they are very inconclusive that $\frac{\Delta W}{W}$ may be independent of frequency (solid friction). The frequency values in the latter case are normally between 100 cycles per sec. and 500 cycles per sec. Experiments by Gutenberg⁽²⁷⁾, Kendal⁽⁴⁴⁾ and Ricker⁽⁴⁶⁾ were of the first type; experiments by Evison⁽²⁸⁾, Collins and Lee⁽²⁹⁾ and Macdonald⁽⁴⁹⁾ were of the second type.

Ewing and Press⁽³²⁾, in studies on earthquakes, obtained values of $1/Q$ of 370×10^{-5} (periods 250 - 350 sec.) and $1/Q = 630 \times 10^{-5}$ (150 - 200 sec.). They were unable to state whether this was due to an increase in $1/Q$ with frequency, or to a greater penetration of the waves of longer period.

As pointed out previously, measurements on metals and some dielectrics have suggested that $\frac{\Delta W}{W}$ is independent of frequency over a wide range. However, these materials are very different from rocks, being generally very homogeneous. Kimball and Lovell⁽³⁴⁾ examined several metals, but also maple wood, celluloid, glass and rubber over the range

1 cycle per sec. to 30 cycles per sec. Their graphs of celluloid and glass show a tendency towards a decrease in $\frac{\Delta W}{W}$ below 5 cycles per sec., of the order of 10%; however, this was not regarded as significant. Gemant and Jackson⁽³⁵⁾ studied the range 0.1 - 10 cycles per sec. for several dielectrics, metals and glasses, but observed no frequency dependence of $\frac{\Delta W}{W}$. The present results for wood and tufnol are in excellent agreement with this constancy of $\frac{\Delta W}{W}$.

A comparison of the present results with those of previous workers is difficult. However, since the present results suggest that $\frac{\Delta W}{W}$ increases with frequency up to say 40 cycles per sec. and then remains constant, it may be possible that this would account for the results of the pulse-shape experiments (using low frequencies) and for the amplitude-distance experiments (using higher frequencies).

The actual magnitudes of $\frac{\Delta W}{W}$ have been given in the review section. They can be compared with those given above; complete agreement is not, of course, expected owing to differences in samples etc. It can be seen that the present results are of the same order of magnitude as those of previous workers, though generally smaller than those of Bruckshaw and Mahanta.

3. Effect of moisture on elastic behaviour

The experiments of Weatherby, Born and Harding⁽¹⁴⁾ 1935 on Amherst sandstone showed that a decrease in E of up to 50 per cent could be produced by saturation. Born⁽¹⁵⁾ (1941) noted a similar effect (also with Amherst sandstone) and also studied $\frac{\Delta W}{W}$. He found that $\frac{\Delta W}{W}$ was constant for oven dry specimens (600 - 4000 cycles per sec.) but that a marked viscous effect occurred when moisture was present; he suggested that both solid friction and viscous mechanisms were present, and that the viscous mechanism was increased in the presence of moisture.

However, some results by Martin⁽⁴³⁾ on limestones showed no effect either on E or on $\frac{\Delta W}{W}$. The present results for oolitic limestone showed only a small effect on saturation, so that it appears that sandstones are more readily affected than other rocks.

The present results can be seen to be in reasonable agreement with those of earlier workers; the main difference from the work of Born is that his plot of $\frac{\Delta W}{W}$ versus f was a straight line whereas a maximum is reached in the present case. Born used frequencies between 1000 cycles per sec. and 4000 cycles per sec. and water contents up to 0.4%. It has been pointed out above that the maximum value of $\frac{\Delta W}{W}$ is reached at a lower frequency for higher water content; for

low moisture contents a straight line was obtained. It thus appears possible to reconcile the present results with those of Born.

4. Effect of oil on elastic behaviour

As pointed out above, there are few measurements on the effect of water on elastic constants; no previous results on the effect of oil on elastic behaviour have been found in the literature.

SECTION 9

CONCLUSIONS

The following important features arise from the above discussion.

1. An increase in E of between 1% and 3% occurs for most rocks in the frequency range 2 - 40 cycles per sec. The rate of increase is more pronounced at the lower frequencies and a constant value is normally reached by 10 cycles per sec.
2. An increase in $\frac{\Delta W}{W}$ occurs at low frequencies for all the rocks investigated; a constant value is normally reached below 40 cycles per sec.
3. The E variation appears to be related to the $\frac{\Delta W}{W}$ variation.
4. The elastic properties of sandstones are considerably affected by the presence of water. $\frac{\Delta W}{W}$ is considerably increased in a frequency-dependent manner and E is considerably decreased. There is a close correlation between the change in E (ΔE) and that in $\frac{\Delta W}{W}$ ($\Delta(\frac{\Delta W}{W})$).

5. The effect of oil on sandstones is similar to that of water, but the effects are more pronounced. The magnitude of E at the low frequencies is little changed.

An explanation is required for the above results. Zener⁽³⁶⁾ worked out the relation between the adiabatic and isothermal values of Young's modulus. He obtained

$$\frac{E_S - E_T}{E_T} = \frac{E_S T \alpha^2}{C_p \rho}$$

where E_S is the adiabatic modulus,

E_T is the isothermal modulus,

T is the absolute temperature,

α is the linear thermal expansion coefficient,

C_p is the specific heat at constant pressure in ergs per gm. and

ρ is the density in gm/cc.

In the Table 14 below the measured values of the percentage change in E (over the 2 - 40 cycles per sec. range) are compared with the theoretical values obtained from this formula.

It is clear that for rocks the theoretical changes in E (obtained from purely thermodynamical considerations) are much less than the measured changes. However, the two values

for wood and tufnol are in reasonable agreement (the error in the measured values in these cases is 0.5%). Thus, it appears that for rocks another mechanism is in operation and that this is associated with a change in $\frac{\Delta W}{W}$ since $\frac{\Delta W}{W}$ was constant for wood and tufnol, but increased with increasing frequency for rocks.

TABLE 14

Material	E_s ($\times 10^{-11}$ cgs)	α ($\times 10^{+6}$ cgs)	C ($\times 10^{-7}$ cgs)	ρ gm/cc	%E change (theo- retical)	%E change (obser- ved)
Mild Steel	20	11	0.42	7.9	.15	not measured
Sandstone	2	10	0.80	2.4	.05	3.0
Granite	5	8	0.72	2.7	.05	not measured
Wood	2	50	1.7	0.6	1.5	2.2
Tufnol	1.5	85	1.3	1.4	1.9	2.4
Asbestos	5	15	0.84	2.4	0.25	1.1

The various models which have been proposed to explain the behaviour of solids have been discussed in a previous section. The Voigt model (firmo-viscous material) and solido-fluid model both predict dispersion only at high frequencies. The Sokoloff-Skriabin model is the only one which actually predicts a change in E at low frequencies and their results show a qualitative agreement with the present values. Their theory predicts that the maximum value of E should be reached at very low frequencies (less than 1 cycle per sec.). They worked at very low frequencies and since they could only measure long relaxation times, the extrapolation of their theory to higher frequencies is of doubtful validity.

Zener's theory of thermodynamic absorption (which explains very well the behaviour of thin metal reeds) certainly does not appear to be the main effect for rocks, as it predicts a very small $\frac{\Delta W}{W}$ with a maximum at less than 1 cycle per sec. It is difficult to see how a summation of such effects could account for the above results. The solid-friction model accounts for the results for tufnol and wood, which have constant $\frac{\Delta W}{W}$, but fails for the results for rocks. The Voigt solid shows a linear increase in $\frac{\Delta W}{W}$ up to a maximum which occurs at very high frequencies (unless the damping is very large) and is of a very large value. The solido-fluid material is similar at low frequencies but a maximum is reached

beyond which a decrease occurs. The Sokoloff-Scriabin theory predicts an increase in $\frac{\Delta W}{W}$ up to a maximum at very low frequencies (less than 1 cycle per sec.) followed by a decrease.

Knopoff^(26 & 39) noticed the constancy of $\frac{\Delta W}{W}$ measured for rocks and other materials (mostly at high frequencies) and developed a thermodynamic theory using tensors to account for this. The theory was for homogeneous materials. However, rocks are by no means homogeneous and it seems that the present results may be due to inhomogeneities.

It is clear from the above summary that none of the well known models can completely explain the observed results for E or $\frac{\Delta W}{W}$. However, an "explanation" could perhaps be obtained by using a summation of expressions of the Sokoloff-Scriabin or similar type. This procedure has often been used in this type of case. However, it is extremely doubtful whether such an approach is of any value. There is little point in fitting the theory to the results if the physical processes occurring are not thereby clarified. This will not be attempted here. The same objection does in fact apply to the whole series of "models" which have been discussed above; if a solid acts as if it were a certain arrangement of springs and dashpots this again fails to

help in an understanding of the actual physical nature of the solid concerned. What is evidently required is a theory of Knopoff's type, but which takes into account the effects of inhomogeneities. This cannot be attempted here, but the results will be further discussed to emphasise the effects of inhomogeneities and to attempt to form a qualitative idea of the mechanisms occurring.

The experiments on sandstones using oil and water are particularly interesting. E was unaffected by oil but considerably reduced by water. The value did not change during the actual measurement so it is unlikely that the cement actually dissolved. A slide was cut of old red sandstone and showed a number of relatively large pores and numerous smaller pores and cracks in the vicinity of the individual grains. Since the surface tension of oil is much greater than that of water, the oil would penetrate the pores less easily. The oil may thus be unable to come into close contact with the cement, though the water is able to do so and weakens the cement. The results did show a slight decrease in E (by 10%) for the saturation value of oil (3% by weight) after several days soaking. The change in $\sqrt{E}$ occurring for both water and oil presumably depends on the presence of fluid in the larger

pores, and is therefore similar in both cases.

Both the oil and water experiments gave similar results for the effect on $\frac{\Delta W}{W}$, the oil effect being about 30% greater. The mechanism of loss appears to be that the fluid moves through the pores from the compressed to the dilated regions, dissipating energy by viscous action. The larger pores, i.e. regions penetrated by both oil and water, appear to produce the main effect. It was at first thought that an explanation might be that small movements at the grain boundaries might be enhanced by the action of oil or water, producing increased dissipation. However, the constancy of E for oil experiments rules this out since E should decrease as for water. The effect thus seems to be of a viscous nature and thus increases with frequency. It can be seen qualitatively that an optimum frequency (corresponding to maximum $\frac{\Delta W}{W}$) would occur when the fluid flowed by a maximum amount. This would probably be lower for the larger fluid content because of greater inertia, as the results indicate. The constancy at higher frequencies can be attributed to the greater viscous forces off-setting the smaller amount of flow. The same explanation accounts for the increase in E ; there is a greater resistance to bending at higher frequencies. It is a property of

viscous processes that a change in E with frequency is related to the corresponding change in $\frac{\Delta W}{W}$; this is strikingly shown by the graphs of $\sqrt{E}$ versus $\sqrt{\left(\frac{\Delta W}{W}\right)}$.

The actual dependence of the results on viscosity is not easy to determine. The oil values for $\frac{\Delta W}{W}$ were about 30% greater than for water. This is much less than the difference in viscosities, but clearly surface tension plays a large part in the effect.

A qualitative explanation of the observed results for sandstones containing fluid is thus possible. As stated above, the results are in general agreement with those of Born, except that a maximum $\frac{\Delta W}{W}$ was reached here. This is probably due to the fact that his frequency range (in octaves) was much less than in the present case and that greater water concentrations were used here.

The general trend of the results for the igneous and harder sedimentary oven dried specimens is similar to that for sandstones containing fluid; E and $\frac{\Delta W}{W}$ increase with frequency up to a constant value and the two changes appear to be related. The small values of both $\sqrt{E}$ and $\sqrt{\left(\frac{\Delta W}{W}\right)}$ in these measurements make it difficult to say whether the plotted points lie significantly on a straight line. However, since E and $\frac{\Delta W}{W}$ do increase with frequency, it is

reasonable to assume that a relation between them does exist, and to look for a viscous-type mechanism as the explanation. It should be pointed out that although these results closely follow those for sandstones containing fluid, the effects were definitely not due to the presence of small quantities of moisture - the experiments on dolerite proved this conclusively. Since no fluid is present, and movements of air could produce no appreciable dissipation, the mechanism must be associated with the various inhomogeneities present. The term "inhomogeneities" can be taken to include cracks, pores and effects in the neighbourhood of grain boundaries. The importance of pores etc. was clearly demonstrated by the results of the fluid experiments. The observed results may well be due to very small "viscous" movements at centres of inhomogeneity. The viscous element leads to a stress-strain diagram of the Sokoloff-Skriabin type with a time increase of strain for constant stress.

The mechanism of dissipation thus appears to be as follows. At very low frequencies (say less than 1 cycle per sec.) a thermodynamic relaxation of the type suggested by Knopoff probably occurs, leading to a small constant $\frac{\Delta W}{W}$. At higher frequencies the above viscous element becomes important, and is superimposed on the "static" $\frac{\Delta W}{W}$.

Above about 40 cycles per sec., the "viscous" $\frac{\Delta W}{W}$ becomes constant so that the measured internal friction is independent of frequency. It is not possible to attempt a mathematical treatment here; further experimental work would probably be required first. However, the necessary type of approach is clear; a general theory of Knopoff's type must be adapted to take into account the effect of inhomogeneities, the importance of which cannot be over-emphasized.

It is unfortunate that the experiments could not be performed under pressure. As pointed out above, rocks under pressure behave like ideal elastic bodies, and since the above results are largely due to effects of inhomogeneity, it is probable that these effects would be reduced by pressure. For this reason, field results should show less variations than measured here, especially for deep sediments. This may account for the results of Macdonald⁽⁴⁹⁾ but it is more likely that he did not reach sufficiently low frequencies.

The purpose of the present work was to study the elastic behaviour of rocks in the frequency range 2 - 40 cycles per sec. The main points arising have been discussed and possible qualitative explanations suggested. Certain factors deserve

further study.

1. The dependence of the fluid effect for sandstones on viscosity, surface tension and other factors.
2. The exact nature of the viscous-like mechanism associated with inhomogeneities.

An investigation of the pressure effect is also important; this cannot be done with the present apparatus.

Very careful experimentation is required for the solution of these problems; it is difficult to increase further the sensitivity of the present apparatus, but this method appears to be the most convenient for this frequency range. The range below 10 cycles per sec. is clearly of vital importance in the behaviour of rocks. After an investigation of the above problems, a mathematical approach should be possible. As stated above, a theory of the type developed by Knopoff but taking account of inhomogeneities is what is required.

Finally the following features are stated as being applicable to seismic field work.

1. At low frequencies the values of $\frac{\Delta w}{w}$ for most rocks increase with frequency up to a maximum normally reached ~~at~~ ^{before} 40 cycles per sec. This may account for the discrepancy mentioned above (p. 59) between the results obtained from pulse-width measurements (low frequency)

and those from direct attenuation methods (higher frequency). Because of the frequencies used in seismic work generally, the effect will probably be small in most cases.

2. Most rocks show a slight increase in E with frequency up to a maximum normally reached by 10 cycles per sec. This will, of course, produce a slight dispersion, but it will be very small in most cases.
 3. For porous rocks, particularly sandstones, $\frac{\Delta W}{W}$ may be considerably increased and E considerably decreased if water is present. Considerable dispersion and frequency selection may occur if low frequencies are used. Oil has a similar effect on $\frac{\Delta W}{W}$ but no effect on E ; this is unlikely to produce an observable difference in the properties of oil-containing and water-containing rocks because of the pressures occurring at great depths. However, the effects should be easily observable in shallow rocks.
 4. The elastic behaviour of rocks is considerably influenced by their inhomogeneity. Dispersion and frequency selection are almost entirely due to this.
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ACKNOWLEDGEMENTS

The author wishes to express his thanks to Professor J.McG. Bruckshaw for his advice and encouragement throughout the work, to various members of the staff and of the workshops of Imperial College, and to the British Petroleum Company who supplied the grant for this research.

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