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INVESTIGATION OF GAS DISCHARGES

WITH A

ROTATIONAL COMPONENT

by

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Abstract

This thesis embodies the experimental results which prove that circulating currents have been obtained in a plasma, that is currents which close back on themselves entirely within the plasma, and the theory evolved to explain this. The apparatus also behaves as a plasma gun.

The apparatus produces a gas discharge in 4-19 μ Hg of hydrogen in an axial magnetic field of 480 gauss and a radial electric field between 6 anodes and one central cathode. A bank of twelve 1 μ f condensers is used, charged to 4.4 kV and giving 12 kiloamp altogether.

The oscillograms of the voltage induced in a small search coil inside the discharge tube show that the magnetic field on the axis is reduced by 2100 gauss (about 4.4 times the applied field). This can only be due to a circulating current of about 4900 amperes which always opposes the applied magnetic field. This was only observed between the pressures of 4 μ Hg and 14 μ Hg. Later an electromagnetic valve was used to admit hydrogen into the evacuated tube. There are also currents in the plasma at least 8 cm above the electrodes and the plasma is expelled at a velocity of $1.1 \cdot 10^7$ cm/sec. It fills a tube 60 cm high.

It was not always found possible to explain the sign of the results in terms of the usual concepts of plasma physics. It was, therefore, assumed that the discharge occurs in well defined channels, which permit azimuthal electric fields in the discharge. An expression was found for the curvature of the channel, and the equation of its path. The axial magnetic field and the circulating current were calculated. In one case the correct magnitude is obtained for both. With the electric field reversed both are too low by a factor of 2-3. The sign is correct in all cases.

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CHAPTER 1

INTRODUCTION

1.1 Purpose and Method of the Research.

The aim of this research was to form a plasma and in that plasma to create currents which would close back on themselves. This should be done in such a way that there is no metallic material inside the plasma once the circulating currents have been set up in order that nothing remains inside the plasma to cool it. The principle being that once one has set up a circulating current inside a plasma - that is a current which closes back on itself within the plasma - one can exert forces on the plasma by the action of suitable magnetic fields. These fields can expel the plasma from the region where it has been created, can accelerate it, bring it to a stop or alternatively contain and compress it.

The research can therefore be divided into three parts. The creation of a circulating current in a plasma; its expulsion away from the region where it was formed; and finally, its acceleration, or alternatively its trapping and confinement.

The reason for wanting a circulating current have already been explained. There are two reasons for wanting to expel the plasma from the region where it was formed. The first is to get it away from the electrodes which were used in making it and which are liable to cool it and the second is that this gives it a velocity. This brings us to the third point, the reasons for accelerating it. One reason is to give it momentum - for use say in propulsion or for further experiments with fast plasmoids - the other is to give it kinetic energy prior to firing it into some form of thermonuclear furnace or firing it at another plasmoid.

In this respect the research was inspired by the experiments of W.H. Bostick¹⁵ in the United States, who has found that he can produce plasmoids with circulating currents and accelerate them and that, firing them at each other, these plasmoids behave very curiously forming spirals and other such geometries. Though in the present research the method of forming the circulating current is quite different.

Only the first two parts of this outline program were carried out: the setting up of circulating currents and the expulsion of the plasmoid.

The method proposed to create circulating currents was to use a radial electric field between one central cathode and six peripheral anodes on a circle together with a magnetic field perpendicular to the plane of the electric field - an axial magnetic field. In this way the magnetic field is everywhere perpendicular to the electric field and any current perpendicular to both fields (i.e. parallel to $\underline{E} \times \underline{B}$) will be in a tangential direction and therefore circulating. The discharge is struck between cathode and anode to create a plasma. There is thus a radial component to the current, parallel to $\underline{E}$, but if there is also a cross-component parallel to $\underline{E} \times \underline{B}$, this will be circulating. A circulating current once set up can run independently of the electrodes and the radial component of the current. It may well last very much longer than the radial component.

It is well known that a charged particle in crossed electric and magnetic fields will drift parallel to $\underline{E} \times \underline{B}$ and produce a current in that direction. In a plasma with equal numbers of positive and negative charges, both components (ions and electrons) will drift at the same velocity and the net current will be zero, unless some other effect occurs as well. One such effect is different collision frequencies for the two components, another is the effect of the centrifugal force. These and several others are examined in chapter 10. On the strength of this the research was started in the hope that one or other factor would come into play. As it was, the circulating currents were shown to have been set up, but in some cases with a sign contrary to that expected. This could only be explained by starting with a totally different model, and assuming that the discharge was occurring in well defined channels. This is examined in chapter 11.

The circulating current could, of course, have been created by electromagnetic induction, that is by having a time varying magnetic field. Great care was taken, however, in the final results to eliminate this source of current, as it has been used in many other projects and we were wanting to show that a different effect was the cause of the circulating current observed.

To expel the plasma initially it is sufficient to make the magnetic field stronger below the discharge than above. The radial component of the field together with the circulating current (if the right way) will give an axial force.

This will be helped by the magnetic field of the electrodes acting on the radial component of the current.

Subsequent acceleration of the plasmoid would have been obtained by having a magnetic wave front travelling up the tube, by using a long magnetic coil and loading it with condensers thereby making it into a delay line. The speed of the wave front can then be varied by suitably changing the loading and the number of turns of wire per centimeter of the coil. Containment or compression would have been obtained by mirror fields or cusped fields.

1.2 Summary of Results, Discussion and Conclusion.

Circulating currents were certainly observed by the magnetic field which they produce at the axis of the discharge. In fact, changes of magnetic field of three or four times the applied field were observed. The circulating current set up in the plasma always opposes the applied magnetic field. That is the plasma is "diamagnetic", and the direction of the circulating current changes with the direction of the magnetic field, but not when the electric field alone is changed.

We quote an example from section 7.5.4. This is the maximum circulating current observed and the other quantities involved are:

Applied voltage	= 4400 volts
Applied magnetic field (centre)	= 480 gauss
Change of magnetic field at the plane of the electrodes	= - 2100 gauss
Circulating current	= 4900 amps
Radial current (per branch)	= 2000 amps
" " for the discharge	= 12000 amps
Ratio of circulating current to radial current	= 0.41

In the expulsion experiments the velocity of expulsion was measured with external search coils.

It is $1.1 \cdot 10^7$ cm/sec.

Currents were measured 8 cm above the plane of the electrodes, but the exact shape of the trace varies so much from discharge to discharge and so much like the way it does without a magnetic field that the plasmoid must have lost most of its organised structure by the time it reached this level.

The whole of chapters 10 and 11 are devoted to deriving a theoretical explanation of the circulating currents. Several causes could produce them. The property which was most difficult to explain was the fact that the circulating current always opposed the applied magnetic field, whereas the simplest explanation, that the circulating current was due to collision forces, and other explanations predicted a current depending on the direction of the electric field - opposing the magnetic field when the cathode is at the centre, strengthening the magnetic field when the anode is at the centre.

These unsuccessful theories all assume that the plasma is continuous and that there are no variations of the properties with rotation around the cathode at a given radius. That is they assume cylindrical symmetry and allow only changes with the radius. They are unsuccessful in predicting the right sign in both cases.

The next step, which proved successful, was to assume that the arcs take place along well defined channels, with relatively little conduction anywhere else - thus automatically dropping the restriction of zero azimuthal electric field. As a first result this theory produces the correct direction for the change in the magnetic field in both cases, always diamagnetic.

It predicts a cut-off value. This was not shown to exist experimentally, mainly through lack of sufficient practical results. However this might shed some light on the otherwise incomprehensible phenomenon of ignition delay (Section 5.2).

The channel theory predicts the right order of magnitude for the change of magnetic flux. The values of chapter 9 suggest very good agreement. They were arrived at independently without foreknowledge that they would be so close, but in fact many of the quantities involved are not known to a high degree of accuracy. However, this certainly shows that this theory does not predict results which are wildly out.

It would be very rash to conclude without further data that channels have actually formed, but the theory as it stands now certainly seems to infer this.

If the discharge has formed channels, which is not as originally intended, this does not prevent it from setting up a closed loop of current. Figure 11.10 in chapter 11 shows that the channels run at almost constant radius for

Button Source

Mechanism of Projection

Fig 1.1 Bostick's Button Source

Fig 1.2 Ixion

Fig 1.3 Alfvén's Plasma Rings

an appreciable length. They come very close to each other and this suggests that they might run into each other and form a complete circulating current, unless each channel is very narrow indeed. If a complete ring has formed we have achieved what we have set out to do, even if the plasma looks like a wheel with rim and spokes rather than like a disc.

Traces were obtained from outside the tube, 8 cm above the plane of the electrodes. They are not very explicit but they do show that there are currents at this level. It is just conceivable, but unlikely, that these arcs are still connected to the electrodes and that they do not form a closed loop, but six independent segments. In this case each arc of 2 cm in length has been stretched to go up 8 cm and down again, a total of 16 cm. So in all probability, the channels have joined into a complete ring - or several successive rings to account for the several peaks of the trace 8 cm above the electrodes.

Whether there is a continuous plasma or whether channels have actually been formed, the experimental evidence proves that there is a complete loop of circulating current.

1.3 Comparison with other work.

The whole subject of plasma physics is expanding because of the interest in the thermonuclear reaction.¹⁻⁵ These, by necessity, can only take place in plasmas contained by magnetic fields to keep them away from the cold walls. There is, however, no one route which is guaranteed to succeed and so the various experiments performed all over the world are all more or less individuals even if they can be loosely classified by some of their characteristics. This makes it very difficult to place the present experiment relative to other work, especially as it could be compared by some of its characteristics to some widely different experiments. The present research will be briefly compared to three such experiments which show some similarity to it.

It has been said above in this introduction that to a certain extent this research was inspired by the work of W.H. Bostick.¹⁵ Bostick creates a plasmoid by what he calls a "button" source of plasma and expels it into a magnetic field (Fig. 1.1). There he either accelerates it or he observes its behaviour when meeting one or more similar plasmoids. This work does not go as far as Bostick's, as it only creates a circulating current and

expels it. However the method of creating a circulating current is much more involved than Bostick's. Bostick takes an arc, bends it into a circle using a magnetic field, and joins the two ends to form a loop - his electrodes are very close to each other. The present experiment obtains circulating currents by forming a current perpendicular to the electric field and uses a radial electric field, not a dipole field.

The emphasis in the present experiment is more on the method by which the circulating current is formed than on its subsequent use.

By its geometry this apparatus resembles homopolar machines¹¹, such as Ixion¹ in the United States, (Fig. 1.2). Ixion acts as a condenser, because of its rotating plasma, but this apparatus has no tendency to behave as a condenser, even though in both apparatuses circulating currents are noticed. This is because in the present apparatus the magnetic field is not strong enough to prevent the ions reaching the cathode at an early stage in their cycloidal motion. The plasma thus cannot rotate as a whole. This aspect is considered in more detail in section 10.4.

There exists in Sweden an apparatus described by Alfvén^{12,1} (Fig. 1.3) which shows very similar results, both of circulating currents with fields several times the applied magnetic field and of the velocity of expulsion. Alfvén's apparatus has a radial magnetic field. It forms the plasma outside the magnetic field and then shoots it through the field, whereas the present apparatus forms the plasma in an axial magnetic field.

It is interesting that two apparatuses which are so dissimilar should give results which have the appearance of being the same.

1.4 New Contributions.

In conclusion it can be seen that, to the author's knowledge, no other apparatus is really similar to the present one. Apart from the value of the investigation as a whole, there are four points which stand out as being particularly new contributions to the field of plasma physics.

1. A plasma was set up in a radial electric field and an axial magnetic field and was proved to be "diamagnetic" whatever the sign of either field. This is not in itself new, but in such diamagnetic plasmas set up to date, the plasma as a whole rotates and the diamagnetism is caused by the MV^2/R term. This has been shown not to be the case here.
2. This plasma has been shot out at least 8 cm, at a velocity of $1.1 \cdot 10^7$ cm/sec. This again is not new but had not yet been done with this geometry of field.
3. It is strongly suspected that channels were formed inside the discharge and that this is the cause of the diamagnetism. (New contribution)
4. The theory of the dynamics of channels set out in chapter 11 is, to the author's knowledge, not to be found anywhere else. (New contribution)

1.5 Possible further uses

The results obtained in this research could be put to further uses. First of all, the apparatus has been shown to be successful in one of its purposes. It has been shown that the plasma was expelled from the region of the electrodes. It thus behaves as a plasma gun and could be used as such for further research, for propulsion, or for fuelling a large thermonuclear reactor.

The magnetic ignition of the discharge, discussed in detail in chapter 5, could well prove to be a novel way of switching large currents with a predetermined delay. This can be made into a multiswitch with several currents switched by the same tube, or several such switches could be fired simultaneously by having the magnetic fields in series.

Circulating currents of course are the starting point of many lines of further research. The fact that they can be obtained by using a radial electric field and an axial magnetic field can be incorporated in the design of further apparatus.

Probably this present research has more bearing on the theory of plasmas in crossed fields than on that of plasmoids.

1.6 Bibliography.

It is as difficult, through lack of space, to give a balanced bibliography in a subject like this one, which is in full expansion and therefore includes many apparently different lines of progress, as it is difficult to define to which domain this particular research belongs. The problems will be tackled under three headings.

(1) General sources of references: The most general sources of references are the proceedings of recent conferences on thermonuclear work and plasma physics, both as regards experiments taking place in other places as for theoretical treatments. Amongst these are the 2nd Geneva conference¹ and the Uppsala conference³ and nearer home those of the Institution of Electrical Engineers² and of the Physical Society⁴. Good lists of references are also included in Bishop⁵, who reviews work in the United States, Landshoff⁶, work on Magnetohydrodynamics and Linhart¹⁴ work in Plasma Physics.

(2) The theoreticians: Those whose work were consulted in developing a theoretical approach to this apparatus are listed in section 10.9. To these, one must add Denisse and Delcroix's most recent work¹³. A new line of approach to plasmas is through irreversible thermodynamics. This was not used in this case as the collision frequencies were considered to be too low. The latest paper on the subject is by Kihara and Midzuno⁷ who quotes Maecker and Peters⁸ as being the first to apply irreversible thermodynamics to plasmas. They, in turn follow the approach of De Groot⁹ and Prigogine¹⁰.

(3) Work related to this research: This work was initially inspired by Bostick's work on shooting plasmoids¹⁵, but by its configuration it resembles Ixion and other rotating plasma experiments. These are reviewed by Wilcox¹¹. Curiously enough, a totally different approach leads to results apparently similar to our case. This is the plasma gun described by Alfvén¹² et alii.

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CHAPTER 2

THE APPARATUS: INTRODUCTION

2.1 Initial Apparatus.

There were two aims to the research: the first to set up a circulating current in the plasma of the discharge, the second to expel this plasma at high speed as an independent plasma or plasmoid. As outlined in the introduction, to produce circulating currents one requires a radial electric field and an axial magnetic field in a direction perpendicular to the electric field. The radial electric field can be considered as that between two concentric cylinders, but it was feared that in this geometry one arc might strike between the two electrodes and merely rotate at high speed in the gap. To force the discharge to occur all around the central electrode, six discrete electrodes were used, each connected to its own circuit, in place of the one outer electrode. This does not give a rigorously cylindrical field, but with six anodes the field should approach circular symmetry sufficiently and the glass-ware still remain feasible in practice. At the same time the currents in the six circuits must start together and remain the same if symmetry of the electric field is to be maintained. This was achieved by using six identical circuits and six equalising transformers - one to one transformers. Each current goes through two transformers and so is kept equal to two other currents (Fig. 2.1). The axial magnetic field was obtained by coils around the outside of the discharge tube.

Both the magnetic field and the electric field were obtained from banks of condensers. The discharge of the field condenser bank was started by the spontaneous breakdown of the air across a variable spark gap in air, set to spark at the required voltage. The main condenser banks were discharged in six identical parallel circuits through the six transformers and the six anodes and back through the central cathode and common earth lead. The discharge medium is hydrogen (or sometimes neon) at a pressure of about 15 microns of mercury.

To trigger the main discharge, it was contemplated to use six triggatrons, but these proved expensive. A preliminary investigation showed that the discharge could be triggered by a very sharp pointed electrode inside the main cathode. This process of itself triggers the six circuits approximately together. The further synchronization of the six circuits was left to the equalizing transformers.

Fig 2.1 Initial Circuit

Fig 2.2 Final Circuit Cathode Central

Fig 2.3 Final Circuit Anode Central

FIG 2.4 THE COMPLETE APPARATUS

To synchronize the main discharge with the magnetic field, the voltage for the striking electrode was taken from the terminals of the field coil and taken through a delay line to the striking electrode to give the magnetic field a chance to rise to its peak value. As high a voltage as possible was needed for the striking electrode, so the full field voltage was used.

The process responsible for starting the discharge is discussed in detail in section 5.1, and the process of equalization in section 5.3.

2.2 Changes to the circuit.

In the same two sections, 5.1 and 5.3, it is also shown why the trigger electrode was discovered to be unnecessary and hence why it was disconnected and the delay line removed from the circuit. It was also discovered that the transformers were only needed in about 2% of the cases. As they accounted for 70% of the inductance of the circuit they were removed as well, this permitting larger currents for the same condenser capacity.

On the other hand the oscillatory nature of the discharge proved very disturbing so six resistances were incorporated in the circuit - one per branch to damp the current and avoid reverse currents. After these three changes, the circuit was as in Fig. 2.2 when the central electrode is the cathode and Fig. 2.3 when the discharge is reversed and the central electrode is the anode, with six cathodes.

2.3 Injection of hydrogen into a vacuum.

Although the initial orientating measurements were done in a steady atmosphere of hydrogen - with any expelled plasmoid slowed down by the gas ahead of it, it was obvious that to get a high velocity of expulsion of the plasmoid - and may be to accelerate it later - the plasmoid had to be expelled into a vacuum. This meant that somehow a suitable quantity of hydrogen had to be let into the tube locally to the electrodes at the required moment. Two methods were tried. The first one, using titanium hydride, is described in section 4.1. It failed because it acted as a disturbing electrode. The second method, this time successful, was the use of an electro-magnetically operated valve allowing hydrogen to enter the evacuated tube just below the electrode area (section 4.2).

At the time the valve was put in, there were difficulties in obtaining circulating currents. It was thought that using a d.c. magnetic field might improve the chances of circulating currents occurring. It would certainly make their diagnosis easier (section 7.1). It was also thought that if the discharge could be made to take place at a higher pressure, then this too might improve the chances of circulating currents. Letting the hydrogen into a vacuum had the extra advantage of making both these things possible.

With a steady pressure of hydrogen in the region of 15μ of Hg, the pressure is so near the pressure of spontaneous ignition (19μ of Hg and above) that a steady (d.c.) magnetic field makes the gas permanently conductive in an electric field (by forcing the electrons to spiral and increasing their chances of collision). The only way then to build up the voltage across the electrode gap is to have a good vacuum and to inject the hydrogen only at the moment of the discharge. Similarly a pressure of hydrogen greater than 19μ of Hg. can only be used if the hydrogen is injected at the moment of the discharge. As it was, the valve gave trouble initially and in the mean time circulating currents were observed in static hydrogen at pressures lower than had been used previously, but not before the period of the magnetic field had been radically increased from 42μ sec to 8.3 msec.

It can be said that without this lengthening of the period of the magnetic field, the circulating currents would not have been noticed. It is however not known whether a steadier field (longer period) was actually necessary for the circulating current to exist or whether it merely made the detection of small changes of field easier against a reduced rate of change of applied magnetic field.

The use of a valve was amply justified, especially as it was successful. It made the use of a d.c. field possible, and even if this was found not to be essential for circulating currents, it was essential to devise an apparatus which was not limited to a.c. fields. Admittedly this meant using a smaller magnetic field, as to save time, the same coil (no 12) was used as for a.c. and the 200v d.c. mains used. Had a high pulsed potential been available, a much larger magnetic field could have been obtained as coil no 12 was insulated for 4 kV. working.

It is in the expulsion experiment that the valve really showed its value, especially once a large glass tube had been installed instead of the upper part of the discharge tube, to help lessen the pressure build-up ahead of the

plasmoid. The presence of a good vacuum ahead of the plasmoid meant it could proceed without having to sweep up unionised gas ahead of it. The proof is that without the vacuum (i.e. with about 10μ of Hg of hydrogen), the discharge does travel upwards but only as a short flame of a few inches whereas with the vacuum, and with the directed velocity due to the pressure of the hydrogen behind the valve, the luminosity of the discharge fills the large tube (23 inches high) (Figs 8.2 and 8.3). This aspect is taken up in more detail in chapter 8. The effect of the vacuum would have been even more important had experiments to accelerate the travelling plasmoid been attempted.

CHAPTER 3

THE INITIAL APPARATUS

3.1 The Discharge Tube.

The central part of the apparatus is the discharge tube Fig. 3.1 which can be evacuated and filled with hydrogen, neon or argon to a suitable pressure. Eight electrodes are sealed into the lower part of the tube, Figs 3.2, 3.3 and 3.5. Six anodes are used on a circle 2 cm radius around a central cathode to produce a radial electric field. To ensure that all six anodes carry the same current, their six circuits are identical. The central electrode is the common return of all six circuits. An eighth electrode passes through the centre of the tubular cathode. It is used to strike the discharge. The main cathode ends in a hollow cone, the top of which is cut so as to provide a hole of one millimeter in diameter, and the point of the striking electrode is adjusted to be level with the edge of the cone. A spark is then struck between the point and the cone by applying several kilovolts on the striking electrode. The glass tube itself is made in two parts jointed by greased flanges. The lower part holds the electrodes, the upper part is just a plain tube 50 cm long, to which later on a cone and socket side arm was added to lodge first the titanium hydride injector, and later the leads for the search coil.

The six anodes and the striking electrode are tungsten rods of one millimeter in diameter. They can be unscrewed and replaced. A short piece of NiloK is sealed through the glass and has a screw thread on its end. A glass tube slides over the electrodes, to prevent discharges occurring except at the top of the electrodes. The striking electrode terminates in a very sharp point sharpened electrolytically. The cathode is hollow to accommodate the striking electrode inside itself. Consequently there must be two glass-to-metal seals on the cathode. Molybdenum was originally used for it, but because of its high heat conductivity it was found impossible to make a second seal on it without cracking the first, so nickel was used instead with the cone, removable, also of nickel. The bulk of the glass is pyrex with graded seals to each electrode. Two glass tubes emerge from the base, one, with a cone, connects the discharge tube to the rest of the vacuum system, the other is a blank which was later used for the valve inlet.

FIG 3.1 DISCHARGE TUBE

FIG 3.2 DISCHARGE TUBE-DETAIL

Fig 3.4 Transformer

Fig 3.3 Isometric View of the Electrodes

FIG 3.6 GLASSWORK - FRONT

FIG 3.7 GLASSWORK - BACK

The complexity of the tube made it difficult to anneal and it cracked several times in the process. The tube was entirely remade after a breakage, but on exactly the same plan. Apart from these, it gave no troubles. With all electrodes carefully cleaned in HF/HNO₃ acid it was easy to outgas. There was no difficulty in obtaining discharges with the various gases, either by triggering or by spontaneous breakdown, as the dimensions had been previously tested with a small pilot tube. The pilot tube also had a striking electrode but only one anode.

3.2 The Glass work.

The rest of the glass work, Figs 3.6 and 3.7, fulfils two purposes, the first to evacuate the discharge tube and the other to obtain a suitable pressure of gas. On the one side we have a backing pump and its motor, a tap, a mercury diffusion pump, a liquid air trap and a further tap before the discharge tube. On the other side we have three gas bottles, each fitted with two taps so that small volumes of gas can be measured, leading to a mains and a Penning pressure gauge. The three gas bottles contain Neon, Argon, and Hydrogen respectively. All these are connected between the last tap in the pumping line and the discharge tube. The tubing which was later added for the electromagnetic valve goes from the space between the two taps on the hydrogen bottle directly to the second inlet at the base of the discharge tube.

3.3 The Power Pack.

The power pack was already available. It had been designed by a third year student, to produce 125 watts at voltages up to 10 kv. It gave good service at 5 kv once most components had been replaced by ones of larger rating, though at 10 kv it was liable to give trouble. It has been described as "using a class C oscillator as power source and a double tuned, over coupled, air cored, radio frequency transformer as the voltage step up device" (laboratory report by D.S. Froome 1958) and can be seen at the top of the apparatus (Fig. 2.4).

Briefly it consists of a 500 v d.c. power pack, a class C oscillator with six STC 58/254 M valves in parallel, an air cored radio frequency transformer for voltage step up and a rectifying, voltage doubling, circuit, with two R.10 high voltage (10 kv) rectifying valves. The voltage is changed by changing the capacity of the primary (or secondary) circuit.

FIG 3.8 CONDENSERS

FIG 3.9 TRANSFORMERS

3.4 The Condensers.

The condensers used were $1\mu\text{f}$, 7 kv working voltage, condensers specially ordered from Wego Condensers for use on discharge work (Fig. 3.8). The same condensers were used for the discharge circuit and for the field circuit. Their inductance had been kept especially low so that rapid discharges could be obtained. In fact their inductance was so low that it was never thought necessary to measure it, although it must have been only a fraction of a microhenry as the total circuit inductance (without transformers) was only $2.1\mu\text{h}$. Meant for research work they were relatively cheap and uncased. When new, their leakage resistance was high (about $3000\text{ M}\Omega$).

They were supplied from the power pack through one main switch and a chain of resistors. The resistors were used (a) to limit the charging current drawn from the power pack to 1 mA, for each condenser and (b) to make the six circuits independent when discharging. The main isolating switch handle is fully insulated so that the switch can be operated manually under all conditions.

3.5 The Transformers.

With six separate anode circuits to give a circular symmetric field, the problem is how to get all six circuits striking at the same time and to keep the six currents equal afterwards. Admittedly, the problem is simplified by assuming that the striking electrode has an equal effect on each anode and that, the circuits being all as exactly alike as possible, the currents will be equal too. But some form of guarantee was thought necessary, (a) to force the six currents to start together, (b) to keep them equal thereafter - bearing in mind that some discharges have characteristics which look as negative resistances to currents of certain frequencies which thus start to grow if the total effective resistance of the circuit to them is negative.

With this in mind, it was decided to use six one-to-one transformers of high mutual inductance and very low leakage flux (Figs 3.4 and 3.9). This is done by having bifilar windings. When these are used with transient currents, they have the property of preventing changes in the difference between two currents. As all currents are initially zero, and hence equal, the transformer will reduce the rate at which the currents become unequal. As each current goes through two transformers in series, each current is kept

equal to two other currents. (Incidentally, these are not its neighbours in space to prevent the possible coupling of mechanical and electrical instabilities.) By keeping the leakage flux very low, one ensures that the inductance of the circuit stays small.

Ultimately it was discovered that the discharge was well-behaved and that the transformers were needed in only 2% of the cases. Consequently they were removed to reduce the total inductance of the circuit. This did not unduly affect the balance of the six currents (see section 5.3). The transformers act in both the cases of inequality mentioned above, for, in both cases, a voltage is induced which, by Lenz's law, either tends to reduce the growth in current difference, or if some branches of the discharge have not struck, increases the voltage across the gap until an arc does strike.

The time constant or e-folding time, for the growth of a current inequality is of the order of M/R , where M is the mutual inductance of the transformer and R the total resistance of that circuit. For the transformer to be effective, this time must be greater than the duration of the discharge. To obtain discharges of a few microseconds with condensers of one microfarad, the circuit resistance must be less than a few ohms, so $M = 0.1$ mh would be sufficient and $M = 1$ mh should be ample. At the same time the leakage inductance, should not be more than about $1 \mu\text{h}$.

If Δi is the difference between two currents and i the value of one of them, then $\frac{\Delta i}{i} \approx \frac{L}{M}$, where L is the inductance of the circuit. So it is advantageous that M should be as large as possible.

If a transformer has an air gap of m cm then

$$M = \frac{0.4 \pi N^2 A \mu}{\ell + \mu m} \cdot 10^{-8} \text{ henrys}$$

and the difference between the currents at saturation of the core is

$$\Delta I_{\text{sat}} = \frac{B_{\text{max}} (\ell + \mu m)}{0.4 \pi \mu N}$$

so if the mean mutual inductance for the maximum total effect is M_m

$$M_m \Delta I_{\text{sat}} = NAB_{\text{max}} \cdot 10^{-8}$$

So the maximum mutual inductance can only be obtained at the expense of limiting the saturation current difference.

Yet $M \Delta I / \Delta t \approx V$, the available voltage

so the time at which saturation occurs is given approximately by

$$\Delta t_{\text{sat}} \approx \frac{M \Delta I_{\text{sat}}}{V} \approx \frac{NAB_{\text{sat}}}{V} \cdot 10^{-8}, \text{ (2-3 } \mu \text{ sec in our case)}$$

After this the transformer ceases to be effective independently of the size of the gap. So it is best to make the mutual inductance as large as possible and therefore to have no gap.

The actual design of the coil is limited by some rather stringent factors. To get a rapid discharge the leakage inductance of the transformer must be kept less than 1 or 2 μ h and its resistance less than $\frac{1}{2} \Omega$. To reduce the leakage flux, use was made of bifilar windings, but as every turn has to be insulated against the full voltage, coils of one layer were used to make repairs easier. At the same time, the skin depth in copper at 1 megacycle per second is so small that flat wire of rectangular cross-section must be used. To make the best use of the space available the conductors were wound on edge. To obtain a mutual induction of at least 1 mh in these conditions, one must use as long a coil as possible as well as a large core cross-section. Because of the small skin depth in steel at 1 Mc/s Ferroxcube can be shown to be far better than laminated steel, even though its saturation flux is lower. The largest blocks commercially available were used, each core being built up of four 4 in by 1 in by 1 in blocks.

The two windings were insulated from each other by putting Systoflex tubing between them to keep them apart and impregnating both coils together in paraffin wax. Occasional breakdown of the insulation was repaired with paraffin wax.

Each transformer was tested by using it to obtain two distinct arcs from only one condenser, under conditions where only one arc would otherwise have been struck. This proves that they are at least sufficient to force all arcs to strike.

Specifications

Core of Ferroxcube grade B2

$B_{\max} = 3200 \text{ lines / cm}^2$

Resistivity = 10^5 ohms - cm

Initial $\mu = 200$

Maximum $\mu = 800$

Nominal $\mu = 600$

4 pieces of ferroxcube 4" x 1" x 1"

Window 4" x 2"

Cross-section of ferroxcube 1 sq in = 6.5 cm^2

Length of path = 43.2 cm

No of turn = 60

With no air gap $L = 4.0 \text{ mh}$

saturation current = $I_{\text{sat}} = 3.0 \text{ amp.}$

$LI_{\text{sat}} = 12.5 \text{ mh amp.}$

With bifilar windings there is little lost flux, so $M \approx L = 4.0 \text{ mh}$

and leakage inductance = $1.4 \mu\text{h}$

Insulating cylinder Tufnol $3\frac{3}{4}"$ long, $1\frac{1}{2}"$ I.D., $1\frac{3}{4}"$ O.D.

Copper wire of rectangular cross-section, wound on edge

$0.036" \times 0.128"$

Resistance = 0.34 ohms

Results

Interwinding resistance = 80-100 Megohms

Interwinding capacitance = 3000-5000 pf

Mutual inductance = 1.4 mh

Leakage inductance = $1.4 \mu\text{h.}$

3.6 The Delay line.

The discharge must strike at or near the maximum of the magnetic field. In order to do that, a delay line was designed to provide a voltage on the striking electrode after a suitable delay. As the voltage needed on the striking electrode is several kilovolts, it was decided to use the 5 kv step voltage which occurs across the field coil when the spark gap strikes, and to take it directly through the delay line to the striking electrode. This meant insulating the delay line to full working voltage (10 kv, as that voltage was contemplated for the field coils). The delay line used was a simple cascade of low-pass L-C units (Fig. 2.1). This also acts as a voltage doubler at the striking electrode, by reflection at the end. The delay line is pictured in Fig. 3.10. It was, however, demonstrated later that the striking of the main discharge was due to the direct action of the magnetic field on the discharge and that the striking electrode and the delay line were redundant; both were therefore disconnected (see section 5.1).

The periods of the magnetic fields considered initially were between 30 and 90 μ s. For the discharge to strike at the peak of the field, the delay introduced by the delay line must be a quarter period, i.e. from 8 to 22 μ sec or more if longer field periods are used. A magnetic coil as well as having some resistance, has some self capacitance and must be represented as an inductance, L, with a resistance, r, in series with it and a capacitance, c, in parallel, across both L and r. When used in conjunction with a condenser, C, in a delay unit, the value of c must be very much smaller than C or the pulse will not be delayed, but will pass straight through. A preliminary calculation was carried out which suggested that the coil would have to be specially wound in single layers separated from each other by an insulator to keep the self capacitance low and that it would be difficult to obtain a sufficiently high value for the inductance. A minimum delay time of 10 μ sec was chosen to be obtained in 5 units of 2 μ sec. As the largest condensers available commercially for 10 kv working voltages were of 0.001 μ f and since smaller condensers could easily be obtained, it was decided to wind the coils for as high an inductance as possible consistent with keeping their self-capacitance low but at least with 8 mh to keep a safety factor of 2. This was pessimistic. It was possible to wind coils of 49 mh giving a maximum of 7 μ s delay per unit. 8 units were built giving a total of 56 μ sec which was considered satisfactory.

FIG 3.10 DELAY LINE

In designing the coil, its self-capacitance had to be kept low and also its resistance to prevent excessive attenuation. A rough optimization was done. Assuming constant skin-depth and for a given inductance, to keep the resistance down the internal diameter must be small and the outer diameter large, to leave as much insulation between layers as possible. This gives a thin flat coil. To stay within reasonable dimensions and get the required inductance a trial design was calculated with the coil 3 cm thick 5 cm inside diameter and 13 cm outside diameter.

To keep the resistance below that giving 25% voltage drop for the delay line 4 conductors were wound in parallel. This design was then adapted to fit the space available. Use was made of a long tube of $2\frac{1}{4}$ inch outside diameter Tufnol, which was available, to support and insulate the coils.

On completion of the delay line, it was tested together with the striking electrode and the magnetic field, producing a spark between the striking electrode and the cathode at the appropriate pressure.

Delay line

Centre insulating tube 2". I.D. $2\frac{1}{4}$ ". O.D. Tufnol

Spacers : wood $\frac{7}{8}$ " thick

Insulating cheeks $\frac{1}{16}$ " thick, $5\frac{1}{2}$ " O.D.

Winding space $2\frac{1}{4}$ ". I.D. $5\frac{1}{2}$ ". O.D. 1" wide

No of coils 8

Overall length $16\frac{5}{8}$ ".

Condenser stand. Paxolin $\frac{3}{16}$ " thick

Condensers 0.001 μ f.

Coils (mean values, each)

Inside radius = 2.86 cm

Winding space = 2.54 cm wide

3.79 cm deep

4 wires side by side of gauge 30 copper wire

No of turns per layer: 17

No of layers: 45

Total number of turns: 771

Insulation between layers 0.020" of 1" wide polythene tape

Total depth of winding 3.96 cm

Results

Inductance = 49 mh each
 Self capacitance = 34 pf "
 Resistance = 14.5 Ω "
 Total delay time of line (8 elements) = 65 μ sec
 Attenuation 15% in 65 μ sec
 Mutual inductance between successive coils 15 mh.

3.7 The Magnetic Coils

In this section we shall discuss all the field coils used, although some coils, 12 and 13 especially, were not used until much later. The first point which one must keep in mind is that the magnetic field was generally obtained by discharging a bank of condensers (2 or 4 μ f) and that therefore the peak field obtained does not depend on the actual number of turns but only on the geometric shape and size of the coil. The number of turns only influences the period of the discharge. Two ways of looking at the problem will make this fact clear.

The energy of a magnetic field in air is proportional to the volume integral of B^2 , irrespective of how it is created. As with charged condensers a fixed quantity of energy is available, the volume integral of B^2 is fixed. So with a fixed size and shape of coil B^2 will be fixed independently of whether it is caused by many turns and a low current or vice-versa. Alternatively

$$H \propto i n, i = V_0 \sqrt{C/L}, L \propto n^2, \text{ therefore}$$

H is a function neither of n nor of i . In fact B depends mainly on the volume of field created.

The first field (O1) to be investigated was one where the field is slightly less at the centre than further up or down to give a so-called "mirror" configuration with two coils a slight distance apart. With a tube of outside diameter 5.2 cm two equal coils were designed, length a cm, and distance $2b$ cm between them. The problem is to get the highest central flux for a given condenser energy. Under these conditions, with n turns per coil.

$$H = 0.4\pi \frac{in}{a} \left[\frac{a+b}{\sqrt{(a^2+b^2)+r^2}} - \frac{b}{\sqrt{b^2+r^2}} \right]; i = \frac{V \cdot \sqrt{c}}{\sqrt{L}} \text{ amps}$$

and L is given from tables¹ as

$$L = 0.4 \cdot 10^{-8} \pi r^2 (n/a)^2 \cdot 2 \cdot ((a+b)k_{123} + b k_2 - (a+2b)k_{12} + a k_1)$$

henrys

where the k's are shape factors, so

$$H = \frac{0.4 \pi V_0 \sqrt{c}}{\sqrt{0.4} \pi r \sqrt{2}} \cdot 10^4 K$$

$$\text{where } K = \frac{\left[\frac{a+b}{\sqrt{(a^2+b^2)+r^2}} - \frac{b}{\sqrt{b^2+r^2}} \right]}{\sqrt{a(k_{123} + k_1 - k_{12}) + b(k_{123} + k_2 - 2k_{12})}}$$

and it is obvious that K must be optimized. So K was worked out numerically for a few cases and plotted.

In conclusion K is maximum for $a = 1$ cm to 1.5 cm, and K is larger the smaller b is. Obviously to get less flux at the centre than further up or down b must not be zero. We start diminishing H seriously if b is made greater than 0.5 cm. From this, we also conclude at once that with $b = 0$ and just one continuous coil, length 2a, the optimum length of coil is about 2-3 cm. This value was used for all coils apart from the above pair.

Coil 01

This is the "mirror" double coil just mentioned and is made of two coils 1.5 cm long, 1 cm apart. The same wire was used as for the transformer, to keep the resistance down, rectangular wire gauge 10 by 20, wound on edge. It is double cotton insulated and was impregnated in paraffin. This gave a total inductance of 48 μ h and a peak central field of approximately 5.5 kilo gauss for 2 μ f condenser charged at 5 kv. This coil was in fact seldom used.

Coil 03

This was extensively used. It was wound exactly as 01, but being designed to be used alone and not in a pair, it was 2 cm long. Its measured inductance is 14.5 μ h but with

its leads, the total inductance of its circuit is 21.5 μ h. With 2 μ f condensers charged at 4 kv, the peak current is 1.22 kiloamps. Its central field is given by $H = 3.12 i$ (gauss, amps) so the peak field is 3.8 kilogauss. The resistance is so low that the damping is negligible.

Coil 11

This was a trial coil wound very roughly with about a hundred and twenty turns to obtain design data for gauge 30 wire and a greater number of turns as a preliminary to designing a coil of about 1 henry, in order to get fields of 9 msec period. It was only used in a very few experiments in particular those of section 5.2

Measured inductance = 1.28 mh length 1.5 cm.
With condensers 0.94 μ f each

No of condensers	1	2	3
Period (μ sec)	218	308	378
Current (amps)	109	153	189
Central field (kilogauss)	2.7	3.7	4.6

Coils 12 and 13

These two are identical electrically, No 13 was wound on a much thinner internal cylinder to pass over the cone of the large discharge tube, when assembling the tube. They were used for all successful experiments on circulating current and were designed to give much longer periods to the field.

Nominal design values

Internal insulation Tufnol cylinder 2" I.D. 2 $\frac{1}{4}$ " O.D.

Winding space 1" by 1"

No of turns 3500 turns of gauge 30 wire

Insulated for 7 kv

Estimated inductance about 1 henry.

Measured values

Between cheeks 2.5 cm
 Depth of winding 2.2 cm
 Int. diameter of insulation (No. 13) 5.6 cm
 Int. diameter of winding = 2.3" = 5.84 cm
 No of turns = 3500
 Resistance = 189 ohm
 Inductance = 0.87 henry
 Period with 4 μ f condenser = 11.7 ms

The central field was calculated from the measured dimensions and $H = 0.2 \pi i \rho \ell [\sinh^{-1}(2r_2/\ell) - \sinh^{-1}(2r_1/\ell)]$

where ρ is the number of turns per cm^2

ℓ = the length of the coil

r_1, r_2 = the inner and outer winding diameters
 respectively

so at the centre

$$H = 529 i \quad (\text{gauss, amp.})$$

With 1 μ f condensers charged to 4 kv the central field is

4.5 kilogauss with 4 condensers

3.2 " " 2 "

and 560 gauss when used across the D.C. 200 volts mains.
 With discharges in static hydrogen the discharge occurs after 200 μ sec

so with 4 condensers the field is 480 gauss

whereas with the hydrogen injected by the valve
 the field is d.c. and is 560 gauss

Ref. 1. Campbell and Childs - Inductance.

3.8 The Pressure gauge.

A normal Penning gauge and its associated power pack were used to measure the pressure of the gas in the discharge tube. The gauge was calibrated against a standard McLeod gauge, using hydrogen.

3.9 The Resistance.

Initially the discharge was oscillatory. This complicated the measurements of the axial magnetic field, so a resistance was introduced in each circuit to limit the discharge to the first half cycle. The resistance of the circuit can be less than that for critical damping because the resistance of the arc increases as the current passes through zero, and the current in the second half cycle becomes negligible. In this way a larger peak current is possible than with a critically damped circuit. The resistance was adjusted until the current, measured with an oscilloscope, was negligible after the first half period.

With several kiloamps going through the resistance a fine wire could not be used and special precautions had to be taken at the terminal of the resistance wire, to prevent sparking, as this fuses the connection after one or two discharges.

The length of the resistance was reduced by loose winding in order to keep the area of the circuit down and hence its total inductance as well as for convenience, though care was taken to make the self-inductance as small as possible. With nichrome the resistivity is high enough for the skin effect to be negligible even at 1 Mc/s.

Resistance

Material : Nichrome wire 0.027" diameter

D.C. Resistance : 0.85 ohm

Inductance : 0.28 microhenrys

CHAPTER 4

DEVELOPMENT OF THE APPARATUS

4.1 The Titanium Hydride Injector.

Despite its final lack of success, because it shorted the field coil to the discharge, the titanium hydride injector was successful in its first purpose which was to release hydrogen rapidly into a vacuum, doing so in 10-40 μ sec from the beginning of the magnetic field. In this respect it proved more successful than the valve as it is very much more rapid than the valve. Yet, looking back, it was possibly the most troublesome piece of equipment used.

In principle the injector depends on an electric spark from a sharp point causing the temperature of some titanium hydride to rise sufficiently for the hydride to decompose and release hydrogen. The problem is thus divided into two parts (i) obtaining a spark and (ii) forming the titanium hydride in the first place, by heating titanium foil in hydrogen. A rough calculation showed that the titanium should absorb enough hydrogen to give a good discharge. In fact it worked the first time, but it took a great deal of trial and error before it worked reliably.

The injector was connected across the magnetic field coil (Fig. 4.2) so as to be acted on by a voltage immediately the field starts to rise. To limit the amount of hydrogen released a blocking condenser of 1000 pf was needed in series with the injector. The spark between the fine point and the titanium foil cylinder burnt the point away, so a replaceable point of copper wire (gauge 44) was used instead. This had to be placed carefully within 0.5 mm of the titanium foil or the spark would not strike properly.

The hydride was formed with the titanium foil already mounted inside a small removable tube of molybdenum as titanium is very brittle when it contains hydrogen. The molybdenum tube and the titanium foil were heated together in a stream of hydrogen by a high frequency induction coil for 30 mins at full power, cooled slowly for 4 mins and left another 5 mins before switching off the hydrogen. Like this up to 30 discharges could be obtained from one hydrogenating treatment.

Two cones and sockets are used (Fig. 4.1), so that the point can be turned relative to the titanium foil and both

Scale: Half Full Size
Dimensions in millimeters

Fig 4.1 Titanium Hydride
Firing Electrodes

Fig 4.3 Valve

Fig 4.2 Titanium Hydride
Injector: Circuit

relative to the discharge, so that use can be made of the whole surface of the foil. In practice all the discharges could be obtained from one or two points, but use was made of this ability to turn and of a slight excentricity of the point to adjust the gap between the point and the cylinder without opening the discharge tube to air. The molybdenum tube is 2.5 cm long, 0.7 cm outside diameter and 0.5 to 0.7 mm thick. The titanium foil is mounted as an annulus inside the molybdenum tube and is 0.8 cm deep and 0.1 mm thick. The temperature of absorption of hydrogen is approximately 300°C and that of release 700°C.

The delay between the start of the magnetic field and the start of the discharge was found to be 10-40 μ sec. This is the time it takes the injector to spark and release the hydrogen and for the hydrogen to reach the electrode area and to ionize, and has nothing to do with the ignition delay discussed in section (5.2). In fact the injector works just as well in the absence of a magnetic field so long as the circuit of Fig. 4.2 is not changed.

Unfortunately both this delay and the quantity of hydrogen released are very variable. In fact the device in its present state is very tedious to use. It needs constant readjustment of the spark gap and renewal of the point. The hydrogenating process has to be repeated every 30 or so discharges and this necessitates opening the discharge tube to the atmosphere. Very often attempts do not produce the release of hydrogen. However the major weakness of the titanium hydride injector is that it shorts the field to earth.

4.1.1 The effect of the plasma on the injector.

By its very nature the injector is inside the vacuum tube and unfortunately the plasma reaches the injector's electrodes and shorts them to the main cathode of the discharge. In doing so it provides a direct path to earth for the field current. This not only causes the field current to vanish rapidly but radically upsets the discharge.

This was discovered when investigating the cause of an unusually rapid damping of the field current and was suggested by the observation of arcing spots on the injector in places where there ought not to have been any. Of course it had to be proved that this was a short between the injector and the main cathode and not a genuine transfer of energy from the magnetic field to the plasma by means of circulating currents.

The effect noted was a more rapid damping and a change of the frequency of the field suggesting a change in the resistance and the inductance of the field circuit. That this was not due to the apparent circuit (Fig. 4.2) was shown by replacing the injector by a short circuit in Fig. 4.2. The blocking condenser prevents serious damping. Neither is the hydrogen the cause, as this effect was not observed with static (i.e. non-injected) hydrogen.

To prove the explanation given, a way had to be found to cause the damping to disappear. The impedance of the shorting path had to be increased without changing the resistance of AD (Fig. 4.2) in order not to interfere with the injection process. An inductance of $34 \mu\text{h}$ was inserted in series between A and D. This changed both the damping and the frequency of the field as expected, showing that the injector was definitely involved. A direct transfer of energy from the magnetic coil to the plasma was ruled out by observing precisely the same rapid damping with the magnetic field coils removed well away from the discharge. The effect could therefore not be due to the direct effect of the field on the plasma. Finally the current from the cathode was shown to contain a component at the frequency and damping of the field long after the discharge current should normally have ceased, showing quite conclusively that the field current had shorted to the cathode. This made the injector useless without further development and it was preferred to develop the electromagnetic valve instead.

4.2 The valve.

To inject hydrogen rapidly at the time of the discharge an electromagnetic valve was placed in a tube joining the hydrogen bottle to the discharge tube. The connecting tube starts between the two taps above the hydrogen bottle, seen clearly in Fig. 3.6, and goes via a tap, a set of bellows, the valve, a cone and socket joint to the second glass inlet which has been especially put there when making the discharge tube and left sealed. The third tap was used either to meter a quantity of hydrogen gas or as a throttle.

The valve itself consists of a glass stopper with a mild steel inset and is placed inside the connecting tube (Fig. 4.3). Normally the valve which terminates in a ground glass cone rests on a conical ground glass seat and is held closed by a spring. The cone was later greased to improve the seal. A magnetic coil around the tube lifts the mild steel inset. Initially a gap was left between the mild steel and the glass in order that the mild steel should lift the glass by impulse.

The glass broke after a while, and was repaired without leaving any play between the steel and the glass.

The inset is also tapered, with an enlarged section at the top to increase the lifting force of the coil. It is located by three bosses in the tube. The spring fits into a recess in the stopper and is held by three bosses in the glass tube. A cone and socket joint was used between the valve and the discharge tube, so that the discharge tube could be removed.

4.2.1 The Coils.

Two coils were designed to operate the valve one to operate at 200 volts (coil no 22) and the other at a much lower voltage but built for higher currents (no 21), should the need for it arise. The coils were mounted so that either could be used at will. In practice only coil 22 was found convenient for lifting the valve, but coil 21 was sometimes used from a 6 v. battery for holding the valve down. It was not strong enough at that voltage and was replaced by a copy of coil 22, coil no 23, used with the 200 volts mains.

A short calculation showed that there was no advantage in using a mild steel core for the field coils, so an air coil was designed to fit the expected characteristics of the valve. (The actual values are quoted in brackets.)

Period of main magnetic field = 11.7 msec

Pull of coil = 5 x weight of inset, so the time to
open 0.1 mm = 2.3 ms

Estimated mass of steel inset = 2.5 gms (7.2 gms but the
area of the head is larger too)

so magnetic field required = 1560 gauss

and ampere turns = 7700

with an inner diameter of $1\frac{1}{2}$ cm an outer diameter of
 $5\frac{1}{2}$ cm and a length of $2\frac{1}{2}$ cm

winding space = 5 cm^2 ; copper space $\approx 2 \text{ cm}^2$

Coil 22 and 23:-

with gauge 30 copper wire available,

voltage needed = 190 volts (200 volts)

and number of turns = 2560 turns (2600 turns)

hence resistance ≈ 63 ohms, (68 ohms)

current = 3 amps.

inductance $\approx$ (flux $\times 10^{-8}$ /current) = 260 mh (160 mh)

flux rise time = 4.2 ms.

The coil was used (a) in series with the main field coil with a variable shunt across the valve coil so as to obtain 200-400 volts across the coil. This shunt was usually about 40 ohms and the valve opening time of 9-17 msec. With a 220 ohms short the rise time is only 6.6 msec. (b) with the main magnetic field constant and connected across the 200 volts d.c. mains, and with the valve coil using the -200 volts d.c. mains.

Coil 21

with copper wire 1 mm²

Number of turns = 200

Voltage needed = 14.7 volts

resistance = 0.4 ohms (0.55 ohms)

current = 38.6 amp.

4.2.2 Performance.

The valve did not work satisfactorily the first time but after some trial and error, it was made to operate in a way which gave complete satisfaction.

Initially the valve was synchronized with the magnetic field simply by putting its lifting coil (22) in series with the magnetic field coil. To adjust the rise time of the valve so as to get the discharge at a suitable value of the magnetic field, a shunt was put across the valve's coil, usually of about 40 ohms resistance.

However the rise time of the valve was not constant. It depended on such things as the exact position of the valve with respect to the magnetic field, stiction and the pressure of hydrogen behind it. It was not difficult to synchronise the valve with the magnetic field by putting both coils in series, as the rise time of the valve was short enough, compared to the life of the field, yet in practice the discharge occurred erratically in the first four half cycles of the magnetic field, and there was no way of predicting what the value of the magnetic field would be nor its sign. The only way out of this dilemma was to use a d.c. field switched on well before the discharge

and switched off afterwards but this meant losing the benefit of the use of condensers for the field and dropping from a peak field of $4\frac{1}{2}$ kilogauss to a d.c. field of 560 gauss.

Another source of trouble was that the valve leaked. A coil (no 21) below it was insufficient to hold it down (despite the spring inside the valve). The leakage was not finally stopped until the valve was repaired for a crack, a new coil put in (no 23) to hold it down, and the cone lightly greased. The coil was not needed to hold the valve permanently but only to bed it into the grease and make it gas tight and to hold the valve whilst loading the gas behind it. The volume above the discharge was also increased to make the leak relatively less important.

By this time I had learned that the valve would not take the full pressure of the hydrogen in the bottle. The standard procedure was to measure a small quantity of gas between taps 1 and 2, to switch the discharge d.c. magnetic field on, to release the gas into the space behind the valve (with the holding coil (23) on) thereby increasing the volume by 16 to 1, switching off the holding coil and lifting the valve (coil 22) as soon afterwards as possible. This meant having the discharge condensers charged in advance and the measuring equipment ready. This was used as standard procedure and gave good results. With the valve well seated and the magnetic field on, the discharge can now fill the large glass container.

4.3 The large discharge tube.

Near the end of the research the upper part of the discharge tube was replaced by a much longer tube 16 cm in diameter, to reduce the build up of pressure ahead of the plasmoid. The tube above the flange was changed but nothing below.

The new part was actually made in two parts connected by a 55 mm diameter cone and socket, so that the magnetic field coils could be slipped on. The lower part, at the level of the electrodes, is still 50 mm in diameter as before and has a side arm for the leads of the search coil. The upper part was made from a Dewar flask blank and is a large tube 16 cm in diameter ending in a hemispherical dome. It is 35 cm long, and is jointed to the neck of the 55 mm socket. The whole is protected by wire meshing, which does not go the whole way round the tube so as not to allow induction currents. The new tube can be seen photographed by the light of its own discharge in figs 8.2 and 8.3.

CHAPTER 5

IGNITION PHENOMENA AND EQUALIZATION

5.0 Summary.

From 4μ of Hg. to 19μ of Hg. pressure, the initiation of the discharge is a function of the field coil, yet the magnetic field does not have to reach any critical value before ignition occurs. The problem of ignition has not been fully investigated.

The problem of the synchronization and equalization of the six currents, however, has been neatly tied up. Except for rare occasions, it can be shown that the currents are equal to within 10%. This is satisfactory.

5.1 Ignition.

With the hydrogen pressure below that at which a spontaneous discharge will occur in the applied electric field, a way had to be found to initiate the discharge with the electric field already present, and in such a way as to synchronize the discharge and the magnetic field. Originally it was decided to initiate the discharge by producing a spark between a very sharp electrode, inside the cathode, and the cathode itself; the cathode having a 1 mm diameter aperture at its apex. To do this the 4 kv voltage from condensers which is suddenly applied to the field coil was also applied to a delay line whose output was connected to the trigger electrode, (Fig. 2.1). In this way the discharge could be started at a suitable time of the sinusoidal magnetic field.

Although a spark was produced on the trigger electrode, it was discovered that the magnetic field was alone responsible for initiating the discharge in the range of pressure of 4μ of Hg to 19μ of Hg. On the one hand the discharge would not start without a magnetic field and on the other, the time at which the discharge started was not affected by the trigger. Above 19μ of Hg the discharge occurs spontaneously and below about 4μ of Hg the available fields were insufficient to cause a breakdown. So the delay line and trigger were abandoned and the discharge initiated by means of the magnetic field.

A more detailed observation later showed that the function of the magnetic coil is two-fold. Not only does it produce a magnetic field, it also provides a constant potential surface at the potential of the centre electrode. This prevents early spontaneous discharge. Without this coil, or with it not connected to the circuit or earth, the discharge will ignite spontaneously at pressures well below 19μ of Hg.

The action of the field coil in initiating the discharge is also two-fold. From a pressure of 19μ of Hg down to $14\frac{1}{2}\mu$ of Hg, the discharge is initiated by electric induction, a rapid change in the potential of the coil being sufficient. Below this pressure, $14\frac{1}{2}\mu$ of Hg, down to 4μ of Hg the magnetic field is actually needed. This can be demonstrated by putting the 0.88 henry field coil on the discharge tube and in parallel the 17μ h coil somewhere else. The conductors of the 0.88 henry coil are thus at the electric potential they would normally have when acting as a field coil, but the inductance ratio ensures that the current in the 0.88 henry coil is negligible and hence that the magnetic field is negligible too. This arrangement is sufficient to initiate the discharge from 19μ of Hg down to $14\frac{1}{2}\mu$ of Hg, but not below that. It is interesting to note that the threshold pressure below which circulating currents are noticed is also about 14μ of Hg. We have thus four ranges to the discharge in hydrogen.

- (1) above 19μ of Hg spontaneous discharge,
- (2) between 14μ of Hg and 19μ of Hg initiated discharge without circulating current,
- (3) between 4μ of Hg and 14μ of Hg initiated discharge with circulating current,
- (4) below 4μ of Hg no discharge possible with the fields available.

5.2 Ignition Delay.

The actual time which elapses between the start of the magnetic field and the start of the discharge when it is initiated by the magnetic coil will be referred to as ignition delay. It was investigated (a) to get some idea of the initiating mechanism, (b) to find out the value of the magnetic field at the time of the discharge.

The ignition delay is a function of the inductance of the field coil:

with a 17 microhenrys coil	ignition delay	=	10-20 μ sec
88 microhenrys	" " "	=	20-40 μ sec
1.28 millihenrys	" " "	=	15-55 μ sec
0.88 henrys	" " "	=	110-300 μ sec.

However the ignition delay does not depend on the actual period of the field circuit, for using the same coil with 1, 2, or 3 similar condensers and hence with different field periods does not give a systematic difference in the ignition delay. That the ignition delay is a function of the coil is in itself a further proof that the field coil initiates the discharge.

As the magnetic field is sinusoidal in time (being obtained by the discharge of condensers) the longer the ignition delay, the greater the value of the magnetic field when the discharge strikes, for a given circuit, depending on the rise time of the field. Although it has been conclusively shown that the discharge between the pressures of 4μ of Hg and $14\frac{1}{2}\mu$ of Hg will not start without a magnetic field, it does not seem that the magnetic field needs to reach a critical value before the discharge strikes. Not only is the value of the magnetic field spread widely about a mean for a given coil but this mean varies from 2000 gauss for the 1.28 mh coil to 480 gauss for the 0.88 h coil. (Peak magnetic field available: $4\frac{1}{2}$ kilogauss). So one has the queer result that the discharge needs a magnetic field to strike, but that the value of this field is immaterial and yet in the few cases when the discharge does not strike in the first quarter cycle of the magnetic field, the discharge waits before striking until the magnetic field has dropped again to approximately the same value.

The position as regards the electric field is not clear either. In one case at least, the 1.28 mh coil, it has been shown that a minimum voltage of 4 kv is needed (except above 15μ of Hg where less is needed).

As unfortunately neither topic had the slightest chance of demonstrating the presence of circulating currents, further investigations on the mechanism of ignition were not carried out.

My impression, often, was that the ignition delay was pressure dependent, that the tendency was for it to be longer the lower the pressure, and very small at pressures approaching 19μ of Hg, but when all the results for a particular coil are plotted against pressure, the points

are so widely distributed that this conclusion cannot be substantiated.

In conclusion it must be admitted that very little was found out about the mechanism of initiating the discharge except that the coil was necessary. This lack of information on the ignition delay made it impossible to vary it at will. As the magnetic field at the moment of discharge depends on the ignition delay, it also made it impossible to vary the magnetic field easily. If V is the available voltage, L the inductance of the coil, k is a constant of the coil and t_1 the ignition delay, the field H is given by

$$H = ki \approx k\left(\frac{di}{dt}\right)_0 t_1 = k\left(\frac{V}{L}\right) t_1 \quad (\text{approximately})$$

which is independent of the peak field available. With a fixed maximum voltage V , this limits the field to well below the peak.

We pointed out that the threshold value of the pressure for circulating currents was the same as the limit to which electric induction alone is sufficient to initiate a discharge. Far from suggesting that the circulating currents are the means whereby the arc strikes below $14\frac{1}{2} \mu$ of Hg by providing a longer path, it may well be that the ignition delay above that pressure is so short that the magnetic field has not enough time to build up to a sufficient value and in this prevents the circulating current from arising. There would thus be a threshold magnetic field, but no threshold for the pressure. Unfortunately no reliable measurements of the ignition delay were taken above 15μ of Hg.

5.3 Equalization.

As the object of the research was the setting up of a plasmoid, it was necessary to keep the electric field symmetrical and to ensure that the discharge occurred in the six branches simultaneously. The six branches were made as strictly as possible identical and each branch was connected to two of its neighbours through an equalizing transformer which would ensure that the currents stayed equal for as long as possible. Investigation showed that the six currents were well synchronized. The flux in the transformer was measured and only in 2% of the cases was it not negligible so that the transformers were removed for they represented considerable extra inductance. This did not cause appreciable loss of equalization.

Bearing in mind what has just been said (section 5.1) about the initiating of the discharge by the magnetic coil, it is not surprising that this effect should be precisely the same on six identical circuits and that the six currents should be simultaneous.

The main evidence for this assertion is that the trace of the total current is always very smooth. If the six currents were not simultaneous there would be either six separate traces, or a ragged front edge due to six starts, or six peaks. "The current wave form shows in a decisive manner that the currents occur together. (There is only one total current trace and there are no violent secondary waves superposed on the mean current.)" - section 6.4. The total separation is thought to be about 1 μ sec or about 10% of the discharge time. The removal of the transformers did not roughen the traces.

A check was made on this method. A few tests were carried out on a discharge known not to be equalized, at well over a millimeter of mercury of pressure. The current trace showed up to six distinct starts.

With the transformers in circuit the discharge current suggests a circuit inductance of 10 μ h. If the arcs were not balanced but firing independently there would be an unbalanced inductance of 2 mh due to the transformer. The discharge would then have a period ten times longer. So the six arcs must be at least partially balanced. Without transformer the current is consistent with the remaining inductance. Were the currents not simultaneous, the current trace would last much longer.

When a damping resistance is included in the circuit, the smoothest current is obtained if a resistance is used in each branch rather than if one resistance is used in the common return.

Finally the currents were measured in two branches simultaneously and displayed on a two beam Cossor oscilloscope. The separation between the traces, except for rare exceptions, was within 25% of the discharge time (damped discharge) and usually about 10% of the discharge time. Although not exhaustive these measurements were entirely satisfactory.

CHAPTER 6

MEASUREMENTS OF THE CURRENT

6.1 Introduction.

Measuring the current was the first aspect of the discharge performance to be considered and it remained of prime importance right through. The problem was to measure the discharge current by inserting a small resistance either in the lead to the cathode, the common electrode, or to each anode individually. The discharge time was originally estimated at 1 μ second. With condenser voltage of 5 kilovolts, measurements of 5 kilo-amperes per branch had to be contemplated or of 30 kilo-amps in the return lead.

The first results obtained showed that the current was of an oscillatory nature, there being 3 or 4 full periods. Varying the conditions of the discharge, such as varying the pressure of the gas, the magnetic field, or the method of injection of the gas, will change the damping coefficient (hence the resistance of the arc), but not the period of the oscillation. In other words changing conditions do not affect either the capacitance or the inductance of the circuit significantly. Any changes in arc capacitance or inductance are insignificant relative to the external parameters. Furthermore the changes in the resistance could not be correlated with the absence or presence of a magnetic field.

As the primary aim of the research was the detection of circulating currents, the fact that the discharge current is a function only of the external circuit, makes its measurement useless for the purpose of diagnosis. The current was only measured to pilot changes in the circuit and to determine the value of the radial component of the current.

As was said in chapter 2.2, it was found necessary to damp out all except the first half-period so that the electric force should not take negative values. At first a resistance was introduced into the circuit which would give critical damping of the current. However the current was then too small; but, if the current is sufficiently reduced at the end of the first half-period, the arc will not re-form in the reverse direction, or only with a much increased arc resistance, so that the current in the second half-period can be neglected. Thus an external resistance of lower value can be used.

Relying on the changes in the arc resistance to extinguish the discharge at the end of half a cycle, however, has the disadvantage that the changes of resistance are important during a half-cycle. The time graph of the current will approach the form $e^{-\alpha t} \sin \beta t$ but only approximately. Nevertheless, in order to get at least an estimate of the resistances and circuit components, the waveform will be compared with $e^{-\alpha t} \sin \beta t$.

6.2 Theory of $e^{-\alpha t} \sin \beta t$.

Having not found a sufficiently explicit reference, I shall quote here the most important properties, so as to be able to refer to them later. Although the circuit is composed of six equal branches in parallel, we can lump mutual and self parameters into a simple damped oscillatory circuit for each branch. At any rate this will be a good model for the total cathode current.

The circuit is governed by equation:-

$$Ri + L \cdot di/dt + (1/c) \int i \, dt = 0 \quad (1)$$

with the boundary conditions $i_0 = 0$, $i_\infty = 0$,

$$(di/dt)_0 = V/L \quad (2)$$

In determining the form of the solution to equation (1) only the ratios of R,L,C are important, so we can define:

$$\tau = \sqrt{LC}, \quad \omega_0 = 1/\tau, \quad Q = \omega_0 L/R \text{ and } Z = \sqrt{L/C} \quad (3)$$

and for critical damping we have

$$R_c = 2\sqrt{L/C} = 2Z \text{ and } Q = 1/2 \quad (4)$$

So for convenience we measure R in terms of R_c such that

$$\frac{R}{R_c} = \frac{R}{2\sqrt{L/C}} = u \quad (5)$$

$$\text{so that} \quad Q = 1/2u \quad (6)$$

with this notation the solution to equation (1) is

$$i(t) = \frac{V}{\omega_0 L} \cdot \frac{1}{\sqrt{1-u^2}} \cdot e^{-ut/\tau} \cdot \sin(\sqrt{1-u^2} \cdot t/\tau) \quad (7)$$

which gives us limit solutions, for an undamped circuit,
 $u = 0$

$$i = (V/\omega_0 L) \sin(t/\tau) \quad (8)$$

and for a critically damped current $u = 1$

$$i = (V/\omega_0 L) \cdot (t/\tau) \cdot e^{-t/\tau} \quad (9)$$

The maximum current occurs at time t_m where

$$t_m = \frac{\cos^{-1} u}{\sqrt{1-u^2}} \cdot \tau = \frac{\sin^{-1} \sqrt{1-u^2}}{\sqrt{1-u^2}} \cdot \tau \quad (10)$$

and is given by

$$i_m = \frac{V}{\omega_0 L} e^{-ut_m/\tau} \quad (11)$$

If we define $\sin \phi = \sqrt{1-u^2}$, $\cos \phi = u$ (12)
 and differentiate equation (7) we get

$$\frac{di}{dt} = \frac{V}{L} \cdot \frac{1}{\sqrt{1-u^2}} \cdot e^{-ut/\tau} \cdot \sin(\phi - \sqrt{1-u^2} \cdot t/\tau) \quad (13)$$

If we put $t'/\tau = t/\tau - \phi/\sqrt{1-u^2}$, (14)

the first zero of di/dt occurs for

$$i = i_{\max}, t' = 0, t = t_m \quad (15)$$

and equation (13) becomes

$$\begin{aligned} \frac{di}{dt} &= -\frac{V}{L} \cdot \frac{1}{\sqrt{1-u^2}} e^{-u\phi/\sqrt{1-u^2}} \cdot e^{-ut'/\tau} \cdot \sin(\sqrt{1-u^2} \cdot t'/\tau) \\ &= -\omega_0 e^{-ut_m/\tau} \cdot i(t') \end{aligned} \quad (16)$$

where $i(t')$ is given by equation (7) for $i(t)$ with t replaced by t' .

We shall now consider the voltage S developed across an impedance $r + p\ell$. We shall put $y = r/\omega_0 \ell$ and define:

$$\begin{aligned}\sin\theta &= (\sin\phi)/\sqrt{y^2 - 2y\cos\phi + 1} \\ \text{and } \cos\theta &= -(y - \cos\phi)/\sqrt{y^2 - 2y\cos\phi + 1}\end{aligned}\quad (17)$$

$$\begin{aligned}S &= \ell \, di/dt + r i \\ &= \frac{V}{L} \frac{1}{\sqrt{1-u^2}} \cdot e^{-ut/\tau} \left\{ \ell \sin(\phi - \sqrt{1-u^2}t/\tau) + \frac{r}{\omega_0} \sin(\sqrt{1-u^2}t/\tau) \right\} \\ &= \frac{V\ell}{L} \frac{1}{\sqrt{1-u^2}} e^{-ut/\tau} \cdot \sqrt{y^2 - 2y\cos\phi + 1} \cdot \sin(\theta - \sqrt{1-u^2}t/\tau)\end{aligned}\quad (18)$$

If we put $t''/\tau = t/\tau - \theta/\sqrt{1-u^2}$ (19)
equation (18) becomes

$$S = -\omega_0 \ell \sqrt{y^2 - 2y\cos\phi + 1} \cdot e^{-u\theta/\sqrt{1-u^2}} \cdot i(t'') \quad (20)$$

Thus one family of curves of $i = i(t)$, graph 6.1, with u as a parameter, will suffice to describe the properties of equations (7), (16), (20). This is evident as both i and di/dt satisfy the same differential equation (1), and so must a linear combination of them.

The time from the 1st zero of S to the minimum of S is thus also given by t_m in equation (10).

$$t_m/\tau = \phi/\sqrt{1-u^2} = \phi/\sin\phi \quad (21)$$

and the period T (from one zero to a corresponding zero) is given by

$$T/\tau = 2\pi/\sqrt{1-u^2} = 2\pi/\sin\phi \quad (22)$$

Thus
$$\frac{t_m}{T} = \frac{\phi}{2\pi} \quad (23)$$

so that u and τ can both be found by measurements of t_m and T in an oscilloscope trace of S without knowing either r or ℓ , and hence the peak current can be calculated from equation (11). L can be found from equation (3) as C is known accurately.

Fig 6.1 Current v. Time in a damped oscillatory circuit (equation 6.7)

When the circuit is critically damped $u = 1$ and we have equation (9)

$$i = (V/\omega_0 L) \cdot (t/\tau) \cdot e^{-t/\tau} \quad (9)$$

so that
$$\frac{di}{dt} = \frac{V}{L} \left(1 - \frac{t}{\tau}\right) e^{-t/\tau} \quad (24)$$

and
$$S = \frac{V}{L} \left(1 - \frac{t}{\tau} + y \frac{t}{\tau}\right) e^{-t/\tau} \quad (25)$$

this is zero for $t/\tau = 1 + y/(1-y)$ (26)

and is minimum for $t/\tau = 2 + y/(1-y)$ (27)

so the time from the 1st zero to the minimum of S is t_m where

$$t_m = \tau \quad (28)$$

6.3 Difficulties.

Because the current to be measured is ten thousand amperes it is essential that the measuring resistance be of the order of the milliohm so as not to give unwieldy signals. Furthermore the resistance must be kept of that order because the total resistance round the circuit is 400 milliohms and the measuring resistance should be small compared to that. By trial and error, a resistance of $1/4$ milliohm was chosen.

The resistance is of copper three by one millimeter rectangular cross-section, 3 cm long, and is folded back on itself to reduce its inductance. This wire was used as it was available, having been especially chosen for the transformers for its low resistance. At high frequency, a rectangular section is more economic, and larger dimensions would be wasteful because the skin depth is 0.45 mm for specific times of 10 μ s and 0.15 mm at 1 μ s.

The disadvantage of using such a low value for the resistance is that its inductance is relatively high. With a time scale of 1 μ s, 10^{-9} henrys are equivalent to 1 milliohm. To decrease the L/R ratio would necessitate a decrease of the cross-section and a corresponding decrease of length. As the resistance is only 3 cm long, it was not deemed wise to shorten it even further because of end effects.

The choice of resistance was very unsatisfactory, but at the time it was the best that could be thought of, for

such a low value of the resistance. A very serious lack of time prevented improving the resistance. Considering the magnitude of the current conducted, even for a very short time, the resistance must be robust, especially at solder points. Flimsy construction using several fine wires in parallel had been tried, but did not survive more than a few discharges.

Grover¹, p. 35, gives for a single conductor:

$$L = 2 \ell \left[\ln(2\ell/(B+C)) + 1/4 \right] \cdot 10^{-9} \text{h}$$
 at high frequency. This gives $18 \cdot 10^{-9}$ henrys. Folding the wire back on itself reduced this to $2.8 \cdot 10^{-9}$ henrys inclusive of the mutual inductance of the vector distance of the ends of the resistance with the rest of the circuit. This is small, and is only that of a straight conductor of the length of the vector distance. The total inductance cannot be further reduced as a substantial part of the remaining magnetic field is very close to the surface of the conductor and cannot be removed.

The resistance was calibrated under conditions as near as possible those of operation using the sinusoidal discharge of two field condensers in parallel through two known inductances also in parallel. Oscilloscope traces were photographed. Table I gives the results obtained. The effect of the inductance is several times that of the resistance. To illustrate this, the curve in Fig. 6.2 has been calculated from equation (18). It gives the theoretical value of the voltage signal δ across the measurement resistance as a function of time. The measured values of r and ℓ have been used and u and τ are the mean values obtained with 12 condensers (table VI). Fig. 6.3 shows an actual oscilloscope trace under these conditions.

This calibration must however be treated with suspicion. Firstly with such a large cross-section the skin effects are not negligible. They affect both the resistance and the inductance. Secondly, and more important, there is the presence of induced voltages. As far as possible, one end of the resistance is kept at earth potential and the voltage signal is taken through a coaxial cable to the oscilloscope. The lead to the earth of the resistance is the outside conductor and is connected to the oscilloscope earth. However with measured currents of up to 13 kiloamps at 100 kc/s, it is very improbable that the oscilloscope earth follows exactly that of the resistance despite the fact that the latter is earthed to the frame of the apparatus through a thick lead. Voltages can thus be

Fig 6.2 The theoretical voltage S , developed across the measurement resistance

Fig 6.3 Typical current oscillogram with 12 condensers

induced in the loop formed by the "earth" leads, and this will vary from experiment to experiment. Unfortunately no oscilloscope differential amplifier was available for the given voltage and frequency.

Experimentally the amplitude of the signal was very erratic. Nevertheless if the amplitude seemed unreliable, the period was astonishingly constant even when conditions were varied. As it is possible to obtain the frequency and the damping factor of the current from time measurements alone without considering the amplitude (section 6.2), it was decided to neglect the amplitude. With this method $T/2$ and t_m only are measured, Fig. 6.3, and the value of neither the resistance nor the inductance of the measurement resistance need be known.

As has just been said a coaxial cable is used for the signal. It was terminated by a 75 ohm resistance at the oscilloscope. This is negligible with respect to $1/4$ ohm and damps the 50 Mc/s auto-oscillations otherwise set up in the leads.

With the high voltages involved there is no question of putting the resistance at a potential other than earth. When the current to individual anodes is measured, the resistance is placed on the earth side of the condenser in its individual return to the common earth bus-bar. With the reversed d.c. electric field, no measurements have been taken as the whole of the discharge and leads are at a high voltage. No point of the circuit is always at earth, because the power pack will not give a negative voltage without internal changes.

Looking back on all the difficulties encountered it would probably have been better to measure di/dt with a small toroidal coil encircling the current-bearing conductor; and completely insulated from it.

If a critically damped current is measured with a resistance with a large self-inductance the resulting signal will have one zero. (See section 6.2 equations (26) and (27).) Because a homopolar generator behaves as a condenser it was imperative to check that this zero was due to the inductance of the test resistance and not a genuine current zero. In the case of an oscillatory discharge there was no problem as removing the magnetic field left the period unchanged, thus there could be no measurable condenser effect in the discharge. When the current was damped the magnetic field was again removed without change in the shape or the size of the trace. There is thus no

measurable capacitive effect in the arc due specially to the magnetic field. To ensure similar conditions of voltage and pressure, the discharge was ignited in this case either by electric induction, with both ends of the field coil connected together to one end of a dummy field coil placed well away from the discharge, or with the hydrogen injected by the valve.

Table I

Period = 27.1 μ sec (measured on trace)
 Capacitance, (total) = 2 μ f. (Standard measurement)
 Hence total inductance = 9.6(2) μ h
 Coil inductance (total) = 6.8(7) μ h
 Hence lead inductance = 2.7(5) μ h
 Condenser charging voltage 4 kv (electrostatic voltmeter)
 Hence current = $V \sqrt{\frac{C}{L}} = 1.79$ kA.
 Phase at zero time = 69° electrical (measured on trace)
 1st peak = 1.23 volts (" " ")
 Damping factor negligible
 Therefore resistance $r = 0.25$ milliohm
 and inductance $L = 2.8 \cdot 10^{-9}$ henrys.

6.4 Results.

In the first series of experiments the equalizing transformers were still being used. Despite the fact that bifilar windings were used, there remained a total of 3 μ h of leakage inductance per branch. The current trace was extremely simple, corresponding to a lightly damped oscillation with a total resistance of 10% to 18% of the critical damping resistance (Table II). As can be seen the inductance of the rest of the circuit is higher than that of the transformers.

It was found that for pressures in the range 4-30 μ Hg of Neon the transformers are not needed for the currents to be approximately equal. As there was difficulty with the insulation between the windings, this range was chosen for experimentation and the transformers dispensed with. This, together with the more compact circuit which this permits, reduces the total circuit inductance to 2.3 μ h and resistance to about 0.4 ohm (Tables III and IV).

That the current is oscillatory is surprising. One would have thought that the resistance of the arc would have been too high for this to be possible, especially as the circuit was designed to have as low an inductance as possible. Furthermore this necessitates a current reversal and it was certainly not obvious that the plasma would keep its conductivity long enough to cover the zero of current.

In the above investigation the conditions were varied as much as possible. Neither changing the pressure nor the period of the magnetic field nor its absence had any effect on the period and hence on the peak current. The damping factor on the other hand is variable but it has not been found possible to correlate these changes with changes in the conditions. The ignition phenomenon, itself badly understood, has probably a great deal of influence on the resistance. The nature of the gas does not seem to affect the period either.

When a resistance is introduced in each branch to prevent the electric field in the discharge taking negative values the observed current wave-form no longer exactly corresponds to a damped oscillation. The observed value at $t = 0$ is less than theory. The time observed to the first zero is too long compared with τ . This is ascribed to two causes. First of all with more resistance in the circuit and the same stored energy, the total power dissipated is less and only a part of it is dissipated in the arc itself. In this less powerful condition the resistance of the discharge changes with time in a more noticeable fashion, or at least that is what a study of the current wave-form leads one to conclude. It is in fact this effect which allows one to use an external resistance of lesser value than would make the resistance critical at the beginning, the current being critically damped, or even killed, by a rapid increase of resistance only in the later part of the discharge. Whilst we are on this subject it is of interest to note that the actual value of this external resistance used was determined by trial and error from the current oscilloscopic trace till the reverse current was judged negligible.

Secondly the only mutual element of the six branches is the discharge itself. The current wave-form shows in a decisive manner that the currents occur together. (There is only one total current trace and there are no violent secondary waves superposed on the mean current.) Nevertheless this does not mean that the six currents start precisely together. As the voltage signal examined $(r + L \cdot d/dt)i$ falls very rapidly at the beginning because

of the $L \cdot di/dt$ term, if the six traces start one after the other, the signal will actually rise at the beginning, as successive currents step in, and start at about a sixth of the expected value. The successive entries of currents blunt the initial peak, (observed results). This would not be the case, of course, if the i 's obeyed the differential equation for all times, as in that case the sum would also obey the equation; but in this case each i is zero until conduction starts in the particular branch and (di/dt) suffers a definite step. This rather alters the aspect ratio of the trace seen on the oscilloscope even if all the currents start during the first microsecond.

Bearing these two points in mind it is nevertheless possible to get an approximate result for the resistance of the arc by measuring the time from the first zero of the curve to its minimum and from the first to the second (and often ultimate) zero as outlined in section 6.3 and in fact neglecting the first few microseconds for this particular purpose.

It would be rash to give the current explicitly as a function of time, especially as the dominant term in the trace is the term $L \cdot (di/dt)$.

This does not prevent the estimation of the maximum current, as the main factor is τ and, for nearly critically damped oscillation, the dependence on the resistance is weak.

Under these conditions the circuit inductance is almost the same as without external resistance, (Table V). The resistance of plasma and circuit exclusive of the external resistances is consistent with what it is without external resistance, covering the same range of values. The peak current, however, is considerably reduced.

Changing to hydrogen, introduced by a valve, or to neon does not make much difference, nor does shorting all the anodes together at the same potential (strapped anodes). There might be a systematic variation in the various parameters, but the various ranges overlap each other so much that it is dangerous to draw a conclusion at this stage.

Summaries of current measurements

Results (Tables II to V)

II. Current with transformers using neon.

Period = 20.2 μ secs.
 Duration of discharge = $3\frac{1}{2}$ periods. (up to 72 μ s)
 τ = 3.18 μ sec.
 u = 0.19 to 0.11
 R = 0.18 to 0.10 R_c
 C = 1 μ f
 L = 10.1 μ h
 Inductance of transformers = 3 μ h
 Residual inductance = 7 μ h per branch.
 R = 1.14 Ω to 0.64 Ω .
 Condenser voltage = 4 kv.
 Peak current per branch = 1 kA.

III. After the removal of transformers using neon.

Period = 11 μ sec.
 τ = 1.75 μ sec.
 C = 1 μ f.
 L = 3.1 μ h.
 Condenser voltage = 4 kv
 Peak current per branch = 1.95 kA.

IV. After shortening the leads, using the hydrogen injector (ch. 4.1) or neon.

Period = 9.5 μ sec.
 τ = 1.5 μ sec.
 C = 1 μ f.
 L = 2.26 μ h
 L/C = 1.5
 R = 0.4 Ω
 Condenser voltage = 4kv
 Peak current per branch = 2.26 kA.

v. With resistance (Hydrogen)

Main current resistances

Measured 0.85 Ω D.C., 0.28 μh

6 condensers:-

$$\tau = 1.45 \mu sec$$

$$u = 0.52$$

$$C = 1 \mu f$$

$$L = 2.1 \mu h$$

Circuit & plasma

resistance excluding R = 0.4 to 0.7 Ω

Condenser voltage = 4.4 kv

Peak current per branch = 1.62 kA.

12 condensers:-

$$\tau = 2.15 \mu sec$$

$$u = 0.64$$

$$C = 2 \mu f$$

$$L = 2.3 \mu h$$

= 0.4 to 0.7 Ω

= 1.97 kA.

6.5 Discussion of measurements.

The current trace on the Tektronix was photographed with writing speeds of 2 $\mu s/cm$ on the one axis and 10 volts/cm on the other. On the films the time of the first zero of the trace, of the minimum and the second zero were measured. The difference between the two zeroes gives the half-period $T/2$ and we call t_m the time from the first zero to the minimum (see Fig. 6.3).

We also know the charging voltage V , measured by an electrostatic voltmeter and the circuit capacitance which is equal to the condenser capacitance and measured on a standard meter. Stray capacitances are very small compared to 1 μf .

Using the notation of section 6.2 we get u from

$$u = \cos \varphi \quad (12) \quad \text{and} \quad \varphi = 2\pi t_m/T \quad (23)$$

$$\text{and} \quad \tau = t_m(\sin \varphi)/\varphi \quad (21)$$

L is given by $LC = \tau^2$; R by, $R = uR_c = 2\sqrt{L/C} \cdot \cos \varphi$

Let R^x be the resistance of circuit and plasma excluding that of the external resistance.

Table VI

	u	τ μsec	L μh	R ohm	R^x ohm
6 cond. 12 μ Hg of H ₂	0.43	1.42	2.02	1.21	0.36
6 " 5.9 μ Hg " "	{0.53 0.61	{1.49 1.43	{2.22 2.06	{1.53 1.76	{0.68 0.91
12 " 12 μ Hg " "	0.29	[1.82]	[1.67]	[0.53]	- 0.32
12 " 15 μ Hg " "	0.66	2.38	2.83	1.56	0.71
12 " strapped anodes	0.68	2.24	2.50	1.53	0.68
12 Condensers with valve injection of H ₂	{0.65 0.64 0.63	{2.3 1.93 2.17	{2.64 1.86 2.35	{1.49 1.24 1.37	{0.69 0.39 0.52
12 " and strapped anodes	0.6	1.98	1.96	1.19	0.34
12 with no magnetic field	0.64	2.1	2.2	1.35	0.50
Average values:-					
with 6 condensers (C = 1 μf)	0.52	1.45	2.10		} 0.4 to 0.7
" 12 " (C = 2 μf)	0.64	2.15	2.30		

These were used for table V.

There are 6 branches and 1 μf condensers were used.

Finally we get the maximum current from

$$i_{\max} = (V/\omega_0 L) e^{-u t_m / \tau} \quad (11)$$

and $\omega_0 L = \tau / C$

with 6 condensers $i_{\max} = 1.62$ kiloamp per branch

" 12 " $i_{\max} = 1.97$ kiloamp " "

Conclusions.

The currents measured are of the order of 1 to 2 kilo-amp per branch and are little affected by varying the conditions within the range possible with the apparatus.

References

1. Grover, Inductance measurements.

CHAPTER 7

THE CIRCULATING CURRENT

7.1 Introduction

The first objective of the research was to obtain circulating currents and to observe them. Three lines of attack were tried to measure the circulating currents, but only one was finally successful. The three methods used were to measure the magnetic field of the discharge externally, to measure it internally, and to measure any effect on the field coil current. Only the internal measurement had any success. This was finally achieved both by improvements in the apparatus and in the measuring technique.

At first, because it was the easiest, the external field was measured outside the tube using a small search coil. As this had to be done against a rapidly varying field, the signal was electrically balanced against the signal obtained from the applied external field alone, measured away from the discharge, so that only the disturbance due to the discharge would be measured (section 7.3). In an attempt to obtain results, instead of using hydrogen already in the tube at the time of the onset of the magnetic field, hydrogen was released from a titanium hydride injector so as to obtain greater pressures (see section 4.1). Even then the method was unsuccessful probably because the change of applied flux was too great and masked any effect, or because of the possible absence of circulating current, but also because the disturbance outside the tube is much less than on the axis (section 8.1).

It was hoped that the discharge might behave, through circulating currents, as a secondary to the magnetic field coils and that its resistance or inductance would be reflected into the primary circuit - the field circuit - as for a transformer. This would have changed the characteristics of the field current. As the field coil contained several turns at the very least and the plasma only counts as one turn, the "transformer turns ratio" is against us and reduces the reflected values of the secondary into the primary. Furthermore the magnetic coupling is weak. These factors all contributed in making this approach useless.

A first attempt was then made to measure a circulating current using an internal search coil, both with and without the titanium hydride injector. Initially the search coil

was connected to the cathode. The cathode however carries a large current with a consequent large voltage drop in series with the search coil. This was sufficient to mask any desired effect. The search coil was then made into an entirely independent circuit (section 7.2). As it was shown that the injector would never produce good results, this idea was abandoned and it was decided that the hydrogen would be brought in by a valve instead (section 2.3). At the same time the field coil was redesigned so that it could be used either on the d.c. mains to give a steady field, or from condensers to give an a.c. field of 11.7 milliseconds period instead of 42 microseconds. In this way the electro-motive force induced in the coil by direct transformer action was reduced to 2 volts except at current reversals when there is a 5 volt peak. This electromotive force was now small enough not to interfere with the search coil signal. This was the primary motive for increasing the inductance of the magnetic coil to 0.88 henry. Whether this also improved the circulating current is not known. At this stage also the electrodes were waxed over except at their tips so as to remove spurious discharge areas. The net result of all these changes was the noticing of the first signals which could be ascribed to circulating currents. There remained to get clear results from these.

The magnetic field anywhere near the discharge is the sum of two parts: one the applied magnetic field due to the currents in the field coil the other the magnetic field due to currents in the discharge itself and the electrodes. As only the rate of change of flux can be measured with a search coil, there is more chance of detecting interesting flux changes due to the discharge currents when the applied field is as steady as possible. The flux on the axis of the discharge is given by:

$$H_z = 4\pi/10 \left\{ \begin{array}{l} \text{total circulating current per cm of discharge} \\ + \text{coil current per cm of coil} \end{array} \right\}$$

so long as the field can be considered as straight and parallel to the axis at that level. This result remains correct whatever the radial distribution of circulating current density. By comparison of the magnetic field with and without plasma, one obtains an exact measure of the total circulating current.

If one assumes that the total circulating current can be concentrated as a lumped current at radius r , then the field H_c need not be measured at the level of the electrodes but further down as H'_c and H'_c is related to H_c by $H'_c = k H_c$ where k is a purely geometric factor depending on r and the distance of the coil below the electrodes. The exact value

of k will thus depend on the accuracy of this assumed value for r , but a small error in this will not change the order of magnitude of the results.

7.2 The search coil.

Apart from the modifications to the apparatus, section 2.3, which were necessary to produce circulating currents, quite a lot of research went into the design of the search coil so that it would produce meaningful results.

To save the complication of having to take yet another two leads through the vacuum envelope, the two leads of the internal search coil were connected initially to the cathode and the point electrode inside the cathode. The cathode was a sufficient mechanical support for the coil. The point electrode was that originally used for triggering the discharge, but, no longer used for this purpose, it was convenient to take the signal of the search coil out of the tube. The cathode was made the earthed point of the system, so there was no difficulty in connecting the oscilloscope between it and the point electrode. However a voltage occurred between the point where the coil was connected to the cathode and the point of measurement (and of earthing), because of the impedance of the cathode to the discharge current. This was sufficiently large to obscure any genuine signal due to a change in axial flux. The impedance voltage was about 160 volts, whereas, had there been a circulating current, the signal could only have been about 10 volts. This is less than the fluctuation of the impedance voltage from discharge to discharge.

The leads for the search coil thus had to be taken through the glass envelope independently of the rest of the circuit. The coil had to be totally insulated from the plasma, both to obtain good measurements and to avoid interfering with the discharge. To do this, the lead were taken through a ceramic tube and the coil totally immersed in "Araldite". Use was made of the side tube now no longer needed for the titanium hydride injector.

7.2.1 Description of the search Coil.

The two coils which were finally used were very similar consisting of 14 turns of gauge 33 copper wire, Lumex covered, wound into a close coil of mean diameter of 18.7 mm. The coil was prevented from unwinding by three clips of the same wire but electrically independent. The two leads were taken

out through a ceramic tube so that they would emerge outside the vacuum. The coil was set in 'Araldite' at a depth of at least 1 mm. The end of the ceramic tube nearest the coil was completely set in 'Araldite', and the Araldite was allowed to fill the ceramic tube completely. The ceramic tube was taken out of the vacuum envelope through a small glass tube which was sealed with a gas-tight seal of black 'Apiezon' wax. The Araldite was set in a ring so that it would slide over the cathode without any machining. A replacement coil was made because the ceramic tube proved very brittle and could not be repaired adequately. The direction in which the turns had been wound was found experimentally by transformer action with the a.c. magnetic field. The connection to the oscilloscope was made through two twin coaxial cables with their return leads earthed to the oscilloscope to reduce interference from other sources.

7.2.2 Characteristics of coil no II.

Before insulation:-

resistance : 0.25 ohms
 inductance : $5.4 \mu\text{h}$
 inductance from slope of C v. $1/f^2$ curve = $5.43 \mu\text{h}$
 intercept on C v. $1/f^2$ curve, (self capacitance) = 48 pf.
 ringing frequency : 9.8 Mc/s
 with araldite this is less by $1/\sqrt{k}$ where k is dielectric constant of araldite
 no of turns : 14
 diameter (mean): 18.7 mm.
 area of one turn: 2.75 cm^2
 Total area: 38.5 cm^2
 1% more than previous one (no. I)
 Shorting resistance to damp self oscillation, 150 ohms in parallel.

Coil no I

no of turns : 15
 diameter (mean): 18 mm
 area per turn : 2.54 cm^2
 Total area: 38.2 cm^2

7.3 Measuring Technique.

In the early experiments performed with an external coil, it was noticed that the discharge was accompanied by a certain amount of high frequency signal in the measuring coil. The measuring coil itself was balanced by a similar compensating coil positioned away from the region of the discharge, but still within the main magnetic field so that the component of the signal due to the change in the applied magnetic field could be virtually removed by careful adjustment of the position of this compensating coil. The high frequency signal was shown to be always present with a discharge even with no magnetic field or when only one branch of the discharge was used. It cannot therefore be due to any possible circulating current, when it occurred in the signal from the internal coil it was therefore with complete confidence that it was shown to be an autooscillation of the measuring coil and the oscilloscope which was easily damped out by a small shunt resistance of 150 ohms.

As for the external search coil, there was a great deal of high frequency "noise" in the signal received from the internal search coil. Investigation on the $0.1 \mu\text{s/cm}$ range showed a marked 5 Mc/s component. Starting with an amplitude of -200 v. to + 200 v., it gradually dropped to 0 v. to +200 v. and then 0 v. to +100 v. As it was essential to show that the mean of this trace was definitely positive, a resistance of 150 ohms was put across the search coil to damp the high frequency component.

Considering the internal resistance and inductance of the search coil, it can be shown that a 150 ohms short will remove from the signal any component at 5 Mc/s or over which is entirely due to auto-oscillations at the natural frequency of search coil and oscilloscope, but it will only halve a genuine signal due to a change of flux at these frequencies. It will not have a noticeable effect on the base signal (100 Kc/s). Thus no valuable information will be lost. As a result of this the 5 Mc/s component is entirely wiped out, leaving a random fluctuation of dB/dt of very much smaller amplitude. (Maximum peak to peak 50 V) and at random frequencies up to at least 50 Mc/s. These are probably inherent to the discharge and denote a minor turbulence, in the current and gas movements.

Numerous troubles were experienced with the Cossor oscilloscope used, but when a Tektronix became available most of these solved themselves. To trigger the oscilloscope so as to get the signal right from the very beginning of the discharge, another signal was fed from the current

measuring resistance (ch. 6) to the triggering terminal of the oscilloscope. The triggering controls were set to trigger on a positive rising signal to get the very beginning of the signal. To get a decent photograph of the oscilloscope trace, the brightness was set so that the static spot was just visible and the green filter removed. The Tektronix has a built-in, 50 steps, delay line totaling $0.2 \mu\text{s}$, to delay the measured signal long enough to allow the beam to brighten to its full intensity and the scope to trigger. By opening the shutter of the camera just before the discharge, the static spot is only very small and pin-points the origin on the film.

To make quite sure that we were in fact photographing the traces right from the start of the discharge, the measured signal was delayed using a length of coaxial cable. This showed that nothing was being missed, although the resulting signal was so distorted, that it was not felt worth while to use this as a permanent technique.

7.4 Genuineness of the signal.

Because of the troubles which had occurred earlier on and which are discussed in section 7.2 and because of spurious voltages occurring in the coil, the first time a genuine signal occurred it was strongly suspect. It was feared more especially that the mutual capacitance between the search-coil and the discharge might be high and that the voltage measured would be capacitively induced - or worse that there was a direct short circuit or a low resistance path, between the coil and the discharge as had occurred on many occasions. In such a case it would be reasonable to expect the voltage to be the same at both ends of the coil. However one end is connected directly to the earth of the scope and only the other is amplified, so that this unsymmetric treatment could well produce a net deflection on the plates of the oscilloscope. If this were the case, the deflection would be the same way whichever way round the coil was connected to the leads, depending only on the amplifier. When this was tested, changing over the coil leads very definitely changed over the polarity of the signal.

As a double check the coil was connected to the scope through a very simple insulating transformer consisting of two windings of ten turns of fine wire each, wound on cylinders of "Tufnol" $1\frac{3}{4}$ " and $2\frac{1}{4}$ " outside diameter respectively. $\frac{1}{8}$ inch of Tufnol and $\frac{1}{8}$ inch of air should be

enough to insulate any direct leak or capacitively induced voltage. When treated thus, the signal appeared unchanged on the oscilloscope, so that one can safely rule out a capacitively induced voltage. The signal must be of magnetically induced nature being a true measure of the rate of change of flux in the coil.

Needless to say the signal was taken to the scope by a coaxial cable. To check that the signal was not induced in the leads or the amplifier, the leads were connected without a coil, just shorted, and placed in various positions, but no signals of that order of magnitude were noticed. It is true that near to the spark gap a signal was observed, but of a different nature. At the distance of the search coil this spurious signal had vanished.

Another crucial test was to test that the signal inverted with the magnetic field. Had the signal not inverted, it would have been very suspect. However the signal did invert. Not only did it invert but its polarity depended only on the direction of the magnetic field, that is on the direction of the current in the field coils and not on whether the inside layers of the coil were connected to 4 kV or on the contrary earthed. This was done by changing the direction of the field alternatively by inverting the leads of the coil and turning it physically upside down. This was prompted by the very recent discovery that the phenomenon of ignition was sometimes due to a magnetic field, sometimes only to the field coil being suddenly at a high voltage above earth (see section 5.1, electric induction).

A less successful test was comparing the signal to what happened without magnetic field. At the beginning there had been a great deal of high frequency signal but this was an auto-oscillation due to the coil. This was removed, section 7.3. But never was the signal consistently removed. Sometimes the trace was rigorously flat or with one or many very small peaks, but often there was quite a randomness in the signal. A randomness which can only be due to the discharge itself and which represents some form of turbulence in the gas movement. But in this case the signal was much smaller than with a field and anyhow the average signal was zero over periods of time when the signal with the field would not have been zero. The trace without the magnetic field was thus clearly distinguishable from that with the magnetic field.

The circulating current is thus diamagnetic with respect to the field and zero with no field.

One more cause of a possible spurious signal will be considered: an excentricity of the coil or a tilt of its axis. The coil was placed so as to eliminate as much as possible any flux due to the electrode leads. The coil was placed concentric with the cathode, so that the cathode current passes through it. Neither a tilt nor an excentricity of the coil will make it link any net flux, only a combination of both. The same applies to the anode currents as they are symmetrically placed and parallel to the axis of the coil. The main discharge (radial) current is in a plane parallel to the coil so the coil is well placed to eliminate this cause of flux. But in all these cases the flux linked would be proportional to the main current. The direction of the signal would depend on the direction of the main current (hence of the electric field) and not on that of the magnetic field and this is not the case. Changing the direction of the magnetic field inverts the signal, changing the direction of the electric field has no effect on the signal, so the signal is not due to stray fields from the main current and the radial currents.

Although it cannot be said that all the causes liable to produce a spurious result have been checked, at least a number of the most obvious ones have been checked or removed without affecting the signal.

7.5 Properties of the circulating current.

7.5.1 Qualitative properties.

Before going into the detail of the scope traces obtained (Fig. 7.1-7.6) it will be best to summarize some of their more general properties and also the conditions under which the circulating current was noticed.

1. The polarity of the signal is always changed when the direction of the magnetic field is changed.
2. The polarity of the signal is left unchanged by changing the polarity of the electric field. However the signal again changes with the direction of the magnetic field.

Thus in all cases the circulating current is diamagnetic, i.e. in the direction opposite to the external field current. This is a very important fact as it rules out several explanations (chapter 10) and agrees with the polarity obtained from a theory of channel dynamics.

3. Two anodes and one cathode are always sufficient for causing a circulating current, but just one anode and one cathode only occasionally produce a circulating current.
4. On the other hand, connecting the six outer electrodes together does not prevent the circulating current from occurring and this even in the less stable case of a central anode.
5. The pressure on the other hand has a very marked effect. Above 14μ Hg there is no circulating current, 14μ Hg being a threshold, with a very small circulating current. The best range is from 7 to 9μ Hg. At 4μ Hg the circulating current is the largest observed but at any lower pressure the discharge would not even strike because of insufficient length between electrodes. With Neon the threshold is at a higher pressure.
6. The signal, dB/dt , lasts a little longer than the main current but this depends on the scale at which the main current is viewed. The latter decays exponentially and does not reach zero in any definite time.
7. Raising the value of the damping resistance by a factor of two or three does not change the nature of the circulating currents but only diminishes them.
8. When no damping resistance is used, the discharge is oscillatory. There is still a signal from the inner search coil but it is involved and the subject has not been followed up.
9. The whole discharge is essentially very similar when hydrogen is injected with the valve. There are only very minor changes both in the main current and in the circulating current.
10. Using Neon instead of Hydrogen does not make much difference either.
11. Without magnetic field the discharge in hydrogen is pinkish but with a magnetic field, it becomes pale blue.

Fig 7.1 and 7.2 With the cathode central

Scales 1 unit = 1 cm = 100 volts

1 unit = 1 cm = 5 μ sec

Fig 7.3 and 7.4 With the anode central

Scales 1 unit = 1 cm = 100 volts

1 unit = 1 cm = 5 μ sec

Fig 7.1 - 7.4 Oscillograms of the voltage induced in the internal search coil, measuring the rate of change of axial flux. The tube is initially evacuated. The hydrogen is released by an electro-magnetic valve. Radial current per branch = 2 kA.

Fig 7.5 and 7.6 Oscillograms of the voltage induced in the internal search coil measuring the rate of change of axial flux.
The radial current per branch is 2 kA and the cathode is at the centre

Scales 1 unit = 1 cm = 50 volts
1 unit = 1 cm = 2 μ sec

Fig 7.7 and 7.8 Axial magnetic flux obtained by integration of the oscillograms of Fig 7.5 and 7.6

12. When the hydrogen is already present in the tube the discharge goes only a few inches up the tube, but with the hydrogen let into a vacuum by the valve, the driving pressure pushes the discharge right up the tube (about 58 cm high) as judged by the luminosity of the discharge. This will be examined in section 8.1.

7.5.2 Quantitative properties. The circulating current.

For two reasons, the number of results available for a quantitative estimate of circulating currents is very much smaller than that which is available for qualitative results. Because the estimate of the change of axial magnetic field, ΔB , involves integration with respect to time, only those results can be used where a clear picture of the oscilloscope's trace is available. Thus early results with a lot of 5 Mc/s parasitic auto-oscillations have been discarded, as well as results where two or three branches only had been used. In both cases a great deal of qualitative information was gathered, either on film traces not clear enough for integration or through visual results which were only noted down in laboratory notes. These results, which form the bulk of the information gathered, agree with the results quoted here and in section (7.5.1). The condensers had the regrettable tendency to break down. In qualitative or development work, these were not replaced but merely disconnected as two branches are sufficient to demonstrate most properties.

In figures 7.1-7.6 we have six oscillograms obtained with a Tektronix oscilloscope. All six traces are of discharges with twelve 1 μ f condensers, two in each branch, with a damping resistance and a peak current of 2 kA per branch. They represent the rate of change of axial magnetic field in three typical cases. The first four traces refer to discharges where the tube is initially evacuated and where the discharge occurs when hydrogen is admitted through an electromagnetic valve into a d.c. magnetic field. The type studied was with the cathode central, but enough discharges were obtained with the anode central to establish the existence of circulating currents in that case as well and of some qualitative properties.

The next two traces (Fig. 7.5, 7.6) show the rate of change of the axial magnetic field in a discharge occurring in hydrogen already present in the discharge tube. To hold the voltage, there is initially no magnetic field. The discharge strikes when the magnetic field reaches a certain

AT ELECTRODES
AT COLL

HA

2000

700

GAUSS

600

1500

500

Circulating Current

1000

300

500

100

3 5 10 15 20 μ Hg

3

5

10

15

20 μ HgStatic Discharge

- Hydrogen
- Neon

Dynamic Discharge

- x Anode Central
- + Cathode Central
- Δ Half Field

applied
magnetic
fieldnoise level
with $B=0$ threshold
for
hydrogenDynamic
Discharge

Fig 7.9 Circulating Current v. Gas Pressure

value section 5.2. The magnetic field is obtained by discharging a set of condensers through the coil via a spark gap.

Although the six traces all look different, there is no systematic difference between the types of discharges as far as the changes of flux are concerned. For the first five microseconds or so the rate of change of flux is negative giving a reduction in the axial field as the circulating current is set up. During the next five microseconds or so, the flux comes back to about its original value as the circulating current dies down, with sometimes a few more small oscillations of the field before total quiescence is regained.

As an illustration, the last two traces, for a discharge in hydrogen at 12 and 7 μ Hg respectively (Figs 7.5 and 7.6), have been integrated by hand to give the magnetic field at the level of the coil as a function of time and plotted in figures 7.7 and 7.8 respectively. This field opposes the applied field. At the level of the circulating currents, the field is 2.8 times as large as at the level of the coil. So the peak field becomes 800 and 810 gauss respectively, with an applied field of 470 gauss. So the reduction in field is 1.7 times the applied field, i.e. reverse fields are set up.

In Fig. 7.7, the field does not come back to zero in the observed time. This is typical of most traces. The remaining field is anything up to a half of the peak field. It is obvious that finally the field must come back to its original value and that the circulating current must vanish, but before one concludes from Fig. 7.7 that there is still a circulating current after 14 μ sec, one must remember that by then the voltage observed is small, Fig. 7.5, and that its exact value depends on the response of the oscilloscope and the accurate setting of its grid axis. The same applies to the current measurement, so one cannot tell whether the circulating current does outlive the radial current or not.

It is worth while to note that there is often a zero signal between that positive and negative peaks of the voltage showing that during this period the axial magnetic field is constant and therefore also the circulating current, (that is assuming the circulating current remains at the same level in the tube and at the same radius).

The peak field of all the other oscillograms photographed are plotted in Fig. 7.9. They are also for a

current of 2 kiloamperes per branch of the discharge. The term dynamic hydrogen refers to hydrogen admitted by the electromagnetic valve. Static hydrogen refers to hydrogen which is already in the tube at a given pressure before the discharge starts.

The change of average magnetic field at the level of the coil is given by:

$$\begin{aligned}\Delta B &= \int \frac{dB}{dt} dt = \frac{1}{nA} 10^8 \int e dt \quad \text{gauss} \\ &= 2.6 \cdot 10^6 \int e dt \quad \text{gauss}\end{aligned}$$

where e is the voltage in volts induced in a coil of n turns of average area A and the integral is taken from time zero to the point where the trace crosses the axis. It was estimated that the coil was a distance 1.5 cm below the plane of the electrodes and hence of the circulating current and that the circulating current could be "lumped" as a filament at radius 1.5 cm so that the change of axial magnetic field at the level of the circulating current would be $2\sqrt{2}$ times as large and the circulating current given by:

$$\begin{aligned}i &= \frac{10r}{2\pi} \cdot \frac{1}{\sin^3\theta} \Delta B \\ &= 6.75 \Delta B \quad \text{amp}\end{aligned}$$

where r = radius of filament = 1.5 cm, $\sin\theta = \frac{1}{\sqrt{2}}$,

θ = half angle subtended by filament at search coil

and ΔB is the change of magnetic field at the level of the search coil. This assumes that the search coil is small enough so that the magnetic field is approximately uniform across it. The circulating current however is not a filament and it is not possible to determine its distribution so that the value obtained is at best only an estimate. The values of the change of magnetic field both at coil level and electrode level are also given in Fig. 7.9.

In view of this, the values quoted for the circulating currents represent only an order of magnitude, but this definitely establishes the presence of circulating currents.

I think the trend of the circulating current to vary with pressure is a real one although it could be indirect, through the tendency of the gas to need a higher magnetic

field before it conducts when the pressure is lower.
(Section 5.2). But not enough evidence is available.

The applied magnetic field was 560 gauss at its centre point with the discharge in dynamic hydrogen and 480 gauss at its centre point with the discharge in static hydrogen.

The Neon pressure was measured without recalibrating the Penning gauge, but using the hydrogen calibration. This will introduce some error in those readings.

7.5.3 Quantitative Properties, Other properties.

The total duration of the circulating current is

Dynamic hydrogen: 10-12 μsec with 2 μf condensers per branch
Static hydrogen : 11-14 μsec with 2 μf condensers per branch
10 μsec with 1 μf condenser per branch

The rates of change of flux and circulating current have been measured from oscillograms.

	dB/dt at coil gauss/ μsec	rate of change of circulating current amp/ μsec
Static Hydrogen (mean)	130	875
Dynamic hydrogen (mean)	260	1750
(max.)	364	2460
Neon (mean)	130	875

With currents of 2 kiloamperes in each branch, there is a total radial current of 12 kiloamperes so that the ratio of circulating current to total radial current becomes:

for dynamic hydrogen (max.)	0.41
Static hydrogen cathode central (max.)	0.29
anode central (max.)	0.25

The ratio of maximum rate of rise of circulating current to the maximum rate of rise of total radial current was also calculated for the case of 364 gauss/ μsec quoted above and found to be 0.20. This is not significantly different from the ratio of the currents.

From what can be seen above, the apparatus is very inflexible when it comes to varying the circulating currents by varying the parameters. This is regrettable as it would have been interesting to note the changes in circulating currents. The only change which had a definite effect was changing the number of condensers. This increases both the current and the duration of the discharge and hence the duration of the circulating current (section 6.4). The magnetic field for instance could not be increased, because its value is determined by the time it takes the discharge to strike after the start of the magnetic field. This is a parameter which could not be influenced. When a valve was used, a d.c. field could be used, but this again was limited, this time by the supply available.

One more thing however must be pointed out. It was surprising that a discharge would occur with 6 cathodes simultaneously to one central anode as this involves 6 simultaneous cathode spots. Current traces showed a good degree of equalization but the whole discharge was less stable (i.e. more random fluctuations of dB/dt). Circulating currents did occur but not always. The range of pressure on the one hand was less and also the hydrogen became too dirty, and suppressed circulating currents very much more rapidly than with a central cathode. Despite this, circulating currents were very easily observed.

7.5.4 Summary.

We group here for completeness's sake the quantities involved in the case of the maximum circulating current measured.

Applied electric potential	=	4.4 kv
Radial current in each branch	=	2 kiloamps
Total radial current	=	12 kiloamp
Applied magnetic field (centre)	=	480 gauss
Magnetic field of circulating current	=	-730 gauss at search coil and -2100 gauss at plane of electrodes
Circulating Current	=	4900 amps
Ratio of circulating to radial current	=	$\frac{4900}{12000} = 0.41$

7.6 Conclusion.

In conclusion it must be pointed out that although the numerical value of the circulating current is liable to be out by a factor of two or three, because of various assumptions, the total change of flux at the coil is known to probably 10% apart from the errors arising on integration. It can certainly be said that circulating currents were observed and that their order of magnitude is known. It is unfortunate that so little information could be derived quantitatively on which factors affect the size of the circulating current.

CHAPTER 8

THE VELOCITY OF EXPULSION

8.1 Once the existence of the circulating currents had been established by means of measurements with an internal coil placed 3 cm below the level of the discharge, it was decided to investigate the speed of drift of these circulating currents upwards, ie. perpendicular to the plane of the discharge, for the case when hydrogen is let in by a valve into a vacuum. To do so internally would have been very difficult as the coils would have been inside the plasma and not merely just below it as in the first instance, and this would have been the cause of a major disturbance. 4 coils of 4 turns each were then wound externally to the tube at various levels.

To ensure comparability of results, especially to ensure that the scope trace would start at the beginning of the discharge even in the absence of a measurable signal, the scope was triggered directly from the current-measuring-resistance as had been done for measurements of the internal field. This ensures identical starts to all traces, (section 7.3). The signal from the coils were damped with 150 ohm in parallel, to damp overcritically any self oscillation as for the internal field.

Being external, of course, the coils are less sensitive as they enclose the whole tube. Thus they measure the change in total flux Φ across the tube. In fact in the case of a coil $1\frac{1}{2}$ cm below the level of the electrodes the change in total flux is only about ± 150 lines over 22 cm^2 cross-section whereas changes at the axis amount to - 2000 gauss. At that level the discharge occurs at nearly constant total magnetic flux.

At the other levels however there was a substantial signal. This was quite different from that obtained in the case of the internal coil. Measured at 8 cm or 5.6 cm above the level of the anodes the trace had three distinct parts, (Fig. 8.1).

- i. a flat bit close to the axis for about $2 \mu\text{s}$ (slightly diamagnetic)
- ii. a vigorous signal of several positive and negative peaks
- iii. the whole signal was back to zero after $6 \mu\text{s}$
whereas the circulating current was known to last $12 \mu\text{s}$.

(i) From the coil at 8 cm
Scale 1 unit = 1 cm = 50 volts

(ii) From the coil at 6.5 cm
Scale 1 unit = 1 cm = 100 volts

(iii) From the coil at 3 cm
Scale 1 unit = 1 cm = 100 volts

(iv) From the internal coil
Scale 1 unit = 1 cm = 100 volts

Fig 8.1 Oscillograms of the voltage induced in three external coils and the internal coil for a discharge with magnetic field and valve injection
Scale 1 unit = 1 cm = 2 μ s.

The noteworthy point is that the vigorous signal does not start at $t = 0$ whereas the internal coil signal definitely starts at $t = 0$. A check with a coil at the level of the anodes showed that at that level the vigorous signal starts at $t = 0$, so the delay is not due to a wall phenomenon.

Considering that the coil is 8 cm above the anodes and has a large radius and few turns, it is not unconceivable that circulating currents occurring at the anodes should have little effect on it. This would explain why there is little signal before $2 \mu\text{s}$ and after $6 \mu\text{s}$ whereas circulating currents are known to last $12 \mu\text{s}$. In fact the first $2 \mu\text{s}$ are a diamagnetic signal, as would be expected, and represent -800 lines in $2 \mu\text{s}$. This increases considerably (-2000 lines) (Table II) when the coil is placed nearer to the plane of the anodes, and so does the signal after $6 \mu\text{s}$.

Between 2 and $6 \mu\text{s}$ the signal is totally different to what happens at the internal coil and can thus only represent currents occurring locally, Fig. 8.1.

The signal from $2 \mu\text{s}$ to $6 \mu\text{s}$ has several peaks both positive and negative even in the absence of an external magnetic field, and thus in the absence of circulating currents. This signal is obviously due to the magnetic fields of the radial current which has been trapped in the gasses and which the turbulence of the gas is forcing out of the plane of the electrodes. It must be remembered that the hydrogen is let in by a valve and that it has a considerable velocity perpendicular to the plane of the electrodes. In the absence of circulating currents the changes of flux range up to 800 lines of flux at rates of 2000 lines/ μsec . In the presence of the magnetic field, however, the activity is much greater. The trace then, is definitely inverted when the magnetic field is inverted. The first peak is usually paramagnetic representing an increase of 1200 - 1700 lines. So we have two phenomena one of natural turbulence, the other of magnetically induced turbulence.

Because the succession of peaks is more rapid than that observed at the electrodes and because the first peak is paramagnetic and not diamagnetic, it is more probable that we are observing successive fronts of circulating currents rather than an event varying in time but stationary in space. After $6 \mu\text{sec}$., the trace is again nearly zero whereas there are still circulating currents in the plane of the electrodes, so the local currents have stopped, either because the streaming has ceased, or because the currents are damped down before they reach the measuring coil.

The exact shape of the trace varies so much from discharge to discharge and so much like the way it does without a magnetic field that the plasmoid must have lost most of its organized structure by the time it has reached these coils.

If the conditions of injection are kept as constant as possible between discharges, it is found that the average time elapsing before the signal starts at a particular coil varies somewhat from coil to coil. A systematic investigation of this property (Table III) showed that the difference in transit times of the first wave front is consistent with an upward velocity of approximately 1.1×10^7 cm/sec.

Because in a previous experiment there had been drifts in the values of the initial delay before the appearance of a vigorous signal, the last film was taken changing several times from one coil to another to detect any systematic changes in the delay, but because a fresh hydrogen bottle was being used and injection conditions kept strictly constant, the values at each coil stayed sufficiently constant through the experiment for a mean to be taken.

The cause of this velocity has not been uniquely determined. Obviously as the signal from 2 μ s to 6 μ s occurs even in the absence of magnetic field, the velocity is not due uniquely to the magnetic field, but is mainly due to the pressure forces. Visually this is obvious: with the tube full of hydrogen, the discharge only goes up a few inches up the tube, whereas, with the tube previously evacuated and the hydrogen admitted by the valve, the discharge goes up to the top of the tube, a height of 23 inches, and almost fills the tube, Figures 8.2 and 8.3 show this clearly. These show the discharge photographed at night by its own light. The electrodes are at the level of the magnetic coil just visible at the bottom of the discharge tube. So the effect of the mass flow of the gas is convincing.

However, both in the case of the discharge in static hydrogen and with hydrogen admitted by the valve, the magnetic field makes a noticeable difference. The photographs were both taken under identical conditions, except that Fig. 8.2 was taken with a discharge without a magnetic field whereas for Fig. 8.3 there was a magnetic field. With a magnetic field the discharge fills the tube much more fully than without field, so the magnetic field certainly has an effect on the transport of the luminous part of the discharge. Logically, the magnetic field must have affected

FIG 8.2 LARGE DISCHARGE TUBE
WITHOUT MAGNETIC FIELD

FIG 8.3 LARGE DISCHARGE TUBE
WITH MAGNETIC FIELD

the velocity of the plasma perpendicular to the plane of the electrodes.

So the velocity is mainly caused by the pressure of hydrogen and the vacuum ahead, but also partly by the presence of a magnetic field. This is as it should be; the circulating current is in a direction opposite to the field current and so the plasma will be repelled by the field coil as soon as the plasma is out of the plane of symmetry of the coil.

Note:- The photographs were exposed for the whole duration of the discharge.

8.2 Summary.

The signal observed by external search coils several centimeters above the plane of the electrodes have been shown to be caused by local currents. These occur about 2 microseconds after the start of circulating currents, so that substantial currents drift upwards. From the delays to each successive coils, an upward velocity of $1.1 \cdot 10^7$ cm/sec has been deduced.

Table IExternal coils used in doing transit time measurements.

Coil	Cms above plane of electrodes	Diameter of coil	Area of coil	4 turns each
1	8 cm	5.7 cm	25.6 cm ²	
2	5.6 cm	5.9 cm	27.5 cm ²	
3	3 cm	5.3 cm	22.0 cm ²	
4	-1 cm	5.3 cm	22.0 cm ²	
(internal search coil:-		Total area 38.5 cm ²)		

Table IIDiamagnetic Signal. 0-2 μ sec.

at 3 cm	80 v.	$\approx$ 2000 lines/ μ s	- in 2 μ s	2000 lines
5.1 cm	25 v.	$\approx$ 600 " "		1200 "
8 cm	15 v.	$\approx$ 400 " "		800 "

at $-1\frac{1}{2}$ cm very little signal.Table IIIDetail of the last film: Transit time measurements at three coils
[to ensure that any drift in the properties would be noticed]

at	photos	actual values of transit time in μ s.
8 cm	1,2,3	2.5, 2.5, 2.4
5.6 cm	4,5,6,	2.2, 2.5, 2.3,
3 cm	7,8,(9),10,11,12,	2.1, 2.1, x, 2.1, 2.0, 2.0,
5.6 cm	13,14,15,16	2.6, 2.0, 2.3, 2.3,
8 cm	17,18,19,20,	2.7, 2.5, 2.5, 2.4,
5.6 cm	21,	2.2
3 cm	22,23,24	1.8, 2.4, 2.0
averages	8 cm	2.5 μ s
	5.6 cm	2.3 μ s
	3.0 cm	2.06 μ s
		velocity:-
		$1.2 \cdot 10^7$ cm/sec
		$1.1 \cdot 10^7$ cm/sec

CHAPTER 9

THE THEORY: INTRODUCTION

The main object of the research was to create a current closing back on to itself inside the plasma. A radial electric field and an axial magnetic field were set up and a change of the axial magnetic field was measured under suitable conditions. There is thus a component of the current in the tangential direction. The change of magnetic field was always diamagnetic, i.e. tending to oppose the applied field.

Several theoretical explanations are attempted in chapter 10 along classical lines, assuming the plasma to be continuous and cylindrically symmetrical. These fail mainly through their failure to predict a change of magnetic field which is always diamagnetic independently of the direction of the electric field. It is quite easy to predict that with the cathode central diamagnetic changes of field will occur, but the same theory wrongly predicts a paramagnetic change when the anode is at the centre.

So in chapter 11 the discharge is assumed to take place in a well defined narrow channel. This approach predicts the right polarity, a diamagnetic change, in both cases and furthermore the values of the change predicted is of the right order of magnitude.

Both treatments were done for a fairly typical case (section 7.5.4):-

Applied Voltage between the electrodes	
(initially)	= 4.4 kv
Applied magnetic field (centre)	= 480 gauss
Radial current (per branch)	= 2000 amps

With these conditions, observations gave

Maximum change of magnetic field at	
the plane of the electrodes	= -2100 gauss
Circulating current	= 4900 amps

Results obtained using channel dynamics (section 11.13.5):-

With anode central:

Change of magnetic field at the plane of the electrodes	= -2100 gauss
Net circulating current	= 4760 amps

With cathode central

Change of magnetic field at the plane of the electrodes	= -560 gauss
Net circulating current	= 1280 amps

CHAPTER 10

CIRCULATING CURRENTS: GENERAL THEORIES

In this chapter the discharge is treated as a continuous plasma with no sharp breaks. A model was chosen with cylindrical symmetry so that all variation lengthwise (along the z -axis) were ignored as well as any variation with θ . The only variations allowed were with the radius. Several approaches on those lines proved without success, usually because of their inability to predict the right sign of the change of magnetic field, in all cases.

First we look at a general physical picture of the type of causes which can establish a current perpendicular to an applied electric field. Secondly a detail microscopic analysis is done of the discharge. This neglects the fact that a plasma is essentially neutral. Thirdly, starting from Boltzmann's equation a macroscopic analysis is done for a neutral plasma, but neglecting the quadratic terms of the equation. These were found to be important so they were put in at the expense of neutrality. The neutrality of the plasma was finally reconsidered together with the quadratic terms. In general it was found that the collision terms were negligible except when the electrons were trapped by the magnetic field at one particular radius.

10.1 Introduction.

Basically a plasma in a magnetic field can be understood in two different ways both leading to the same result. In the one case the motion of individual particles can be calculated and hence the motion of the whole deduced, or the plasma can be treated as a continuous fluid and the macroscopic properties obtained directly starting from Boltzmann's equation. These are the microscopic and macroscopic approaches respectively (Spitzer², p.25).

The plasma, by definition, contains a high concentration of electrons and ions, so that, to a close approximation, the concentration of ions and electrons must be equal, otherwise the energy needed to separate ions from electrons is too high.

In a magnetic field B the notion^{1.2.3} of both ions and electrons parallel to the field, is totally unaffected by the presence of the field, whereas in the plane perpendicular to the field, both move in circles but in opposite directions.

The radius of curvature is given by

$$\rho = \frac{c}{eB^2} \underline{p} \times \underline{B} \quad (1)$$

where $\underline{p}$ is the momentum, and the angular frequency by

$$\omega = \frac{2eB}{mc} \quad (\text{Larmor frequency}) \quad (2)$$

An electric field (or a force) parallel to the magnetic field produces the expected acceleration, but an electric field or a force perpendicular to the magnetic field distorts the circular motion into a cycloid, such that the added motion is a drift perpendicular to both magnetic field and electric field (or force $\underline{F}$) and proportional to both.

$$\text{drift} = \frac{c}{e} \frac{\underline{F} \times \underline{B}}{B^2} \quad \text{cm/sec} \quad (3)$$

With overall neutrality of the plasma the drift may or may not produce a macroscopic current, and the circular motion, for a plasma in thermodynamic equilibrium does not lead to macroscopic currents as these are usually counter-balanced by currents due to wall forces, Alfvén, unless the plasma is actually contained by the external magnetic field.

10.2 Motion of a particle.

The equation of motion of a charged particle in a steady electric field, $\underline{E}$, and a steady magnetic field, $\underline{B}$, is

$$m \frac{d\underline{v}}{dt} = q \underline{v} \times \underline{B} + q \underline{E} \quad (4)$$

$$\text{this reduces to } \frac{d^2 \underline{v}}{dt^2} + \omega^2 \underline{v}_\perp = \left(\frac{q}{m}\right)^2 \underline{E} \times \underline{B} + \left(\frac{q}{m}\right) \underline{v} \cdot \nabla \underline{E} \quad (5)$$

where $\omega = qB/m$, m is the mass of the particle, q its charge in e.m.u.'s $\underline{v}_\perp$ the component of the velocity perpendicular to the magnetic field. If $\underline{v} \cdot \nabla \underline{E} = 0$, i.e. in an homogeneous electric field, we get

$$\underline{v} = \left[\underline{v}_{o\perp} - \left(\frac{\underline{E} \times \underline{B}}{B^2} \right) \right] \cos \omega t + \left[\underline{v}_{o\perp} \times \underline{k} + \frac{\underline{E}}{B} \right] \sin \omega t \\ + \frac{\underline{E} \times \underline{B}}{B^2} + \frac{\underline{E}_{\parallel o} t}{m} + \underline{v}_{\parallel o} \quad (6)$$

where suffix o indicates initial values at time zero.

The first two terms represent the circular motion of the particle. These are usually balanced in a macroscopic plasma by wall forces. Furthermore they would produce currents at the ion, (or electron) frequencies, two well known phenomena neither of which is relevant to the present phenomenon which is much slower. With $B = 470$ gauss, $\omega_+ = 4.5 \cdot 10^6$, $\omega_- = 8.25 \cdot 10^9$, whereas this phenomenon happens in several microseconds. The last two terms represent motion parallel to B , and independent of it. The third term is the drift velocity as in equation (3).

A force $\underline{F}$, has equal and opposite effects on ion and electron velocity, and so will cause a current in a plasma where $n_+ = n_-$. The electric field, $\underline{E}$, however, produces equal and opposite forces on ions and electrons and therefore equal drifts in the same direction, so that the ion current cancels the electron current. This is special of a neutral plasma that an electric field produces perpendicular to itself a mass velocity but no current. To produce a transverse current (in our case a circulating current as $\underline{E}$ is applied radially), the forces acting on ions and electrons must either be in the same direction (e.g. centrifugal force, gravity force) or in opposite direction but unequal (Hall current due to friction forces). These are the types of forces which we must investigate to try and explain the circulating currents obtained.

Chandrasekhar⁴ p.65 gives a convenient list of drifts, which I quote here for the sake of completeness.

$$\text{Electric drift} \quad \underline{V}_E = \underline{E} \times \underline{B} / B^2 \quad (7)$$

$$\text{Polarization drift} \quad \underline{V}_P = \frac{mc}{eB^2} \frac{\partial \underline{E}}{\partial t} \quad (8)$$

$$\text{Curvature drift} \quad \underline{V}_R = \frac{2 \frac{cW_{\parallel}}{eB^2}}{eB^2} \underline{E} \times (\underline{b} \cdot \nabla) \underline{b} \quad (9)$$

$$\text{Gradient drift} \quad \underline{V}_G = \frac{cW_{\perp}}{eB^2} \underline{b} \times \nabla \underline{B} \quad (10)$$

$$\text{External force drift } \underline{V}_F = \frac{c}{e} \frac{\underline{F} \times \underline{B}}{B^2} \quad (11)$$

and correspondingly a list of current densities in a neutral plasma.

$$\text{Magnetization current } \underline{J}_m = \nabla \times \underline{M} ; \underline{M} = -(W_\perp/B)\underline{n} \quad (12)$$

$$\text{Polarization current } \underline{J}_P = \frac{1}{4\pi} \frac{e}{c} \frac{\partial \underline{E}}{\partial t} ; \epsilon = \frac{4\pi\rho c^2}{B^2} + 1 \quad (13)$$

$$\text{Curvature current } \underline{J}_R = -2W_\parallel \underline{e}_2 \times \underline{B} / B^2 R_1 \quad (14)$$

$$\text{Gradient current } \underline{J}_G = W \underline{b} \times \nabla \underline{B} / B^2 \quad (15)$$

$$\text{External force current } \underline{J}_F = n \frac{\underline{F} \times \underline{B}}{B^2} \quad (16)$$

to this we add that caused by the cylindrical (r, θ) symmetry of the electric field

$$\text{"Centrifugal" force current } \underline{J}_c = n(\dot{M}_+^2 + m\dot{e}_-^2) \frac{\underline{r} \times \underline{B}}{B^2} \quad (17)$$

$$\text{and the Hall current } \underline{J}_H = \left(\frac{\omega_+^2}{\omega_+^2 + \nu_+^2} - \frac{\omega_-^2}{\omega_-^2 + \nu_-^2} \right) \frac{\underline{E} \times \underline{B}}{B^2} \quad (18)$$

where $\underline{b}$ is a unit vector parallel to the magnetic field,

$$w_\perp = 1/2 m v_\perp^2 = \text{energy due to motion perpendicular to the magnetic field,}$$

$$w_\parallel = 1/2 m v_\parallel^2 = \text{energy due to motion parallel to the magnetic field,}$$

$$W_\perp = \sum_{+-} n w_\perp, W_\parallel = \sum_{+-} n w_\parallel, R_1 = \text{radius of curvature of the lines of forces,}$$

$$\underline{e}_2 \text{ is a unit vector along the radius of curvature of the lines of forces}$$

$$n = \text{particle density, } \nu = \text{collision frequency.}$$

As pointed out by Bohr and van Leeuwen (Spitzer², p.27) the magnetization current for an isothermal plasma of uniform density is exactly counter-balanced by a current due to the wall force. So the magnetization current could only exist for a state of non-equilibrium, as for instance a plasma expanding radially. As the present plasma is contained by walls, it is unlikely that this would produce first order effects.

The polarization current is in the radial direction and so does not produce a circulating current whilst the curvature current with the magnetic field of a short coil results in an increase of the magnetic field in all circumstances, and would thus predict an effect in the wrong direction. This leaves us with J_G , J_F , J_C which is a special case of J_F , and J_H as possible causes for the circulating current. J_C will be considered in section (10.4) and J_H in section (10.3).

10.3 The Hall current.

Let us first of all examine the possibility of the circulating current being due to the Hall effect; i.e. to the friction which occurs due to the collision between the various kinds of particles in a plasma.

The movement of a free electron in a magnetic and an electric field is discussed by Delcroix³ (p.78) where he treats the conductivity as a tensor.

$$\underline{\sigma} = \sigma_o' \begin{vmatrix} \frac{\nu_1^2}{\nu_1^2 + \omega_b^2} & \frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} & 0 \\ -\frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} & \frac{\nu_1^2}{\nu_1^2 + \omega_b^2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad (19)$$

where ω has been put equal to zero as the electric field is steady, ν_1 is the momentum exchange collision frequency, $\omega_b = eB/mc$, the cyclotron frequency and

$$\sigma_o' = \frac{ne^2}{mc^2 \nu_1} \quad (20)$$

Considering only the plane perpendicular to B:

$$\underline{V} = \frac{\underline{j}}{ne/c} = \frac{\underline{E} \cdot \underline{\sigma}}{ne/c} = \frac{\underline{E}e}{mc \nu_1} \cdot \begin{vmatrix} \frac{\nu_1^2}{\nu_1^2 + \omega_b^2} & \frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} \\ -\frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} & \frac{\nu_1^2}{\nu_1^2 + \omega_b^2} \end{vmatrix} = \frac{\underline{E}e}{mc} \cdot \frac{1}{\omega_b} \cdot \begin{vmatrix} \frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} & \frac{\omega_b^2}{\nu_1^2 + \omega_b^2} \\ -\frac{\omega_b^2}{\nu_1^2 + \omega_b^2} & \frac{\nu_1 \omega_b}{\nu_1^2 + \omega_b^2} \end{vmatrix} \quad (21)$$

so that the velocity parallel to E, the radial velocity in our case, is

$$v_r = \frac{v_1 \omega_b}{v_1^2 + \omega_b^2} \cdot \frac{E}{B} \quad (22)$$

and perpendicular to E, the drift velocity equals

$$v_e = \frac{\omega_b^2}{v_1^2 + \omega_b^2} \cdot \frac{E}{B} \quad (23)$$

In a plasma where the ion density n_+ equals the electron density n_- ($= n$) the circulating currents and radial currents are given respectively by

$$i_c = |e|n(v_e^+ - v_e^-) \quad (24)$$

$$i_r = |e|n(v_r^+ - v_r^-)$$

and the ratio⁷

$$\frac{\text{circulating current}}{\text{radial current}} = i_c/i_r = \frac{\frac{\omega_+^2}{\omega_+^2 + v_+^2} - \frac{\omega_-^2}{\omega_-^2 + v_-^2}}{\frac{v_+ \omega_+}{\omega_+^2 + v_+^2} + \frac{v_- \omega_-}{\omega_-^2 + v_-^2}} \quad (25)$$

$$\text{where } \omega_+ = \left| \frac{eB}{Mc} \right| ; \quad \omega_- = \left| \frac{eB}{mc} \right|$$

1. If it can be shown that the electron currents prevail and that the ion current can be neglected equation (25) reduces to

$$i_c/i_r = - \frac{\omega_-}{v_-} \quad (26)$$

This would be the case for instance with weakly ionised dense plasmas.

2. If the plasma is fully ionised, or at least if the collisions with molecules can be neglected compared with the collision between charged particles then

$$v_+ = \frac{m}{M} v_- \quad (27)$$

[Delcroix³ p.135 gives values for the slowing down time, t_s :

$$t_{s-} = 1/v_1 ; \quad t_{s+} = 0.73(M/m)^{1/2} / v_1]$$

so

$$\frac{\omega_+}{v_+} = \frac{\omega_-}{v_-} \quad (28)$$

So for a fully ionised plasma we get no circulating current

$$i_c / i_r = 0 \quad (29)$$

Also, the ratio

$$\frac{\text{electronic radial current}}{\text{ionic radial current}} = \frac{\omega_- v_-}{\omega_+ v_+} \cdot \frac{\omega_+^2 + v_+^2}{\omega_-^2 + v_-^2} = 1 \quad (30)$$

even in the absence of a magnetic field. This applies as long as the motion of the ions and electrons are dominated by mutual collisions. Strictly speaking, when ions and electrons collide only with one another and not with stationary molecules eq. (21) is no longer exact and both circulating and radial current vanish Spitzer, p.34².

3. In the particular case of our experiments, it will be shown, section 10.5, that the electron - ion collisions far predominate over the collisions with molecules. So that the plasma can validly be considered as 'fully' ionised rather than weakly ionised, even though ionisation is not 100%. Equation (29) shows that under those circumstances collisions on their own cannot produce a circulating current.

Delcroix³, p.79, however, puts various limitations on this formula. Basically this approach neglects inertia terms by supposing that there are a sufficient number of collisions for a balance to be reached between the exchange of momentum due to collision and the electric force and that a steady state is reached. It will be shown in section 10.5.1 that, at least for ions, the electric field much exceeds the critical field beyond which this balance is no longer possible. Even if collisions cannot be altogether neglected, the regime is one where the ions are more or less in free flight.

To be more precise equation (25) is obtained for an infinite medium where collisions are numerous, even if the absolute value of v is not essential. It is applicable either if collisions are frequent (v/ω large) or, failing this, if both ions and electrons can spiral freely between collisions. But in this experiment the magnetic field is not strong enough compared to the electric field to prevent the ions reaching the cathode in the early part of the cycloid. So we cannot apply equation (21) to the ions.

10.4 The effect of cylindrical symmetry.

For concentric cylindrical electrodes, at first sight the most likely cause of a circulating current which is always diamagnetic, i.e. causing a reduction of the magnetic field, is the effect of the so-called "centrifugal" force;

the
$$\frac{MV_D^2}{e} \frac{c}{B^2 R^2} \frac{R \times B}{\text{drift}}$$

This is best tackled along the lines of the report on Ixion^{8,9}. If one considers an ion circling the centre electrode at a constant radius R in a magnetic field, $\underline{B}$, acting upwards and an electric field, $\underline{E}$, acting radially outwards, then the radial equation of motion is:

$$\frac{MV_D^2}{R} + \frac{eE}{c} + \frac{e}{c} B V_D = 0 \quad (31)$$

where V_D is the anticlockwise velocity of the ion tangential to its path (Fig. 1). With $V_L = \frac{eBR}{Mc}$, the Lamour velocity, and $V_E = \frac{E}{B}$ the electron drift velocity,

we get:
$$V_D^2 + V_L V_E + V_L V_D = 0 \quad (32)$$

and for electrons $(m/M) V_e^2 - V_L V_E - V_L V_e = 0 \quad (33)$

in a neutral plasma $n_+ = n_- = n$ so we get a circulating current density

$$j_e = \frac{ne}{c} (V_D - V_e) = - \frac{nM}{BR} (V_D^2 + \frac{m}{M} V_e^2) \quad (34)$$

However this analysis assumes implicitly that V_E/V_L is small. For if one solves equation (32) for V_D one gets

$$V_D = 1/2 V_L (-1 \pm \sqrt{1 - 4(V_E/V_L)}) \quad (35)$$

$$\text{So that for } V_D \text{ to be real } V_E/V_L \leq 1/4 \quad (36)$$

In our case $V_E = 4.25 \cdot 10^8$ cm/sec. $V_L = 4.5 \cdot 10^6$ cm/sec so $V_E/V_L = 95$ and V_D is complex.

Physically, however, equation (31) is only valid for a very special case when the ions and electrons move at exactly constant radius. This means that they have been projected at exactly the right speed to satisfy (31) from the start. Normally the charged particles will follow a cycloid instead of a circle at a mean radius R . The validity of equation (34) can nevertheless be extended to this case as well.

If we go back to the equation of motion which we write in polar coordinates, we get for a radial electric field E_r .

$$\text{and } \underline{B} = B_z \underline{k}$$

$$M\ddot{r} - Mr\dot{\theta}^2 = (e/c)(E_r + V_\theta B_z) \quad (37)$$

$$Mr\ddot{\theta} + 2M\dot{r}\dot{\theta} = -(e/c)V_r B_z \quad (38)$$

Neglecting half the corioli force, this is exactly as if a radial force $Mr\dot{\theta}^2$ had been added and the motion treated in rectangular geometry. This, together with the electric field, would cause a drift (eq. 10.7, 10.11)

$$\underline{V}_D = \frac{\underline{E} \times \underline{B}}{B^2} + \frac{c}{e} \frac{MV_D^2 \underline{R} \times \underline{B}}{B^2 R^2} \quad (39)$$

$$\underline{V}_e = \frac{\underline{E} \times \underline{B}}{B^2} - \frac{c}{e} m V_e^2 \frac{\underline{R} \times \underline{B}}{B^2 R^2} \quad (40)$$

so that the circulating current would still be given by equation

$$\underline{j}_\theta = nM(V_D^2 + \frac{m}{M}V_e^2) \frac{\underline{R} \times \underline{B}}{B^2 R^2} \quad (34a)$$

even if only approximately, where R is some average value of r .

But equation (34a) is still limited by the inequality (36). For the drift concept to be valid, the ions and electrons must be free to do complete cycloids without hitting electrodes. In our case, this is not possible because of the relatively weak magnetic field. An exact solution of equations (37) and (38) is

$$v_e = \frac{1}{2} \frac{eB}{Mc} \cdot \frac{1}{r} (r_o^2 - r^2) \quad (41)$$

and with the present values, the maximum v_e is $4.5 \cdot 10^6$ cm/sec compared to a fully developed drift of about $4.25 \cdot 10^8$ cm/sec. Thus the ion is only at the very beginning of its cycloid and the concept that the current is the difference of two drift velocities given by eqs. (39) (40) cannot be applied. The effects of the discharge can thus be treated neither as purely a Hall effect nor purely as a homopolar machine.

In the next section, we shall therefore investigate the motion of ions and electrons in far more detail, including the effect of collisions which have hitherto been ignored.

10.5 Detail Microscopic Analysis without a Magnetic Field.

Without magnetic field, there are only two forces to be considered, the force due to the electric field and that due to collisions. Four kinds of collisions are possible, that of electrons colliding against ions, $e - P$, electrons against molecules, $e - P$, ions against electrons $P - e$ and ions against molecules, $P - P$. Collisions against similar particles do not lead to any change in the average momentum of the group. The references used for the collision cross-sections are listed in section 10.8, and the resulting collision frequencies (ν) plotted against energy in Fig. 10.1, for $2 \cdot 10^{14}$ mols/cc, corresponding to the lowest pressure used.

10.5.1 Electrons.

Although the mean free path, at $300^\circ K$, is 10^{-3} cm and thus very much smaller than the size of the apparatus, the electric field is high and before the probability of collision has reached unity the energy has reached values at which collision is very improbable, because of the rapid decrease of the collision frequency with energy. One can define a critical field at which the collisions still balance the force of the electric field

$$E_c = \frac{mc}{2e} \cdot 10^{-8} w \nu$$

where w is the electron velocity. Above this the mean momentum of the electron increases, unchecked by collisions. In our case $E_c = 0.6$ kv/cm so that the electric field is well above this level. The probability of one collision is not zero but the probability of several collisions is low. However the collisions can still be treated as a drag force per unit mass

$$\text{drag} = w \nu \quad \text{cm/sec}^2$$

This has been plotted in Fig. 10.2. As can be seen, with 6% ionisation it is the collisions with ions which are important at low energies and the collisions with molecules at high energy.

10.5.2 Ions.

With ions the effect of collisions is even less except at high energies where ion molecule collisions come into play. So ions reach virtually the full velocity for the available voltage. The collisions with molecules are mainly charge exchange ones. Ionising collisions are about 100 times less frequent.

10.5.3 Drag force.

Measured as a force per unit mass the drag force is, on average, $w \nu$ cm/sec² where ν is $\bar{\nu}_1$ of Delcroix (p.135) [Spitzer], and is the frequency of collision for momentum exchange or relaxation frequency for first order anisotropy and is the reciprocal of Spitzer's slowing down time. The product $w \nu$ is plotted in Fig. 10.2, as well as the electric force on ions and electrons. From this it can be seen that with 6% ionisation the drag force on electrons is 3% of the electric force at low energies, 1% at high energies and less in between, for experiments at 5.6 μ Hg and 3% in both cases for experiments at 17 μ Hg. The drag on ions is negligible at both pressures as it only acts once the velocity is fully developed, where it reaches 100% of the electric force at 5.6 μ Hg and 3 times the electric force at 17 μ Hg.

Thus electrons and ions (except at their terminal velocity) can be considered as in free flight and collisions neglected.

Fig 10.1 Collision Table

Fig 10.2 Drag Table

10.5.4 Ionisation.

If it is assumed that electrons are formed anywhere in the discharge, the mean average velocity of electrons is about $1/2\sqrt{2}$ times the velocity of the full voltage and is about $1.3 \cdot 10^9$ cm/sec, whilst that of ions is $3.2 \cdot 10^7$ cm/sec. The ionic current is thus only $2\frac{1}{2}\%$ of the electronic current. Assuming that 2000 amps have to pass in a cross-section of about 1 cm^2 , this requires 6% ionisation at 5.6μ Hg and 2% at 17μ Hg. This must be increased somewhat as the peak current does not occur until the voltage has dropped considerably. There is thus not the slightest difficulty in explaining conduction in the absence of a magnetic field.

10.6 Detail Microscopic Analysis with a Magnetic Field.

10.6.0 With a magnetic field, $B = 470$ gauss, the most important quantities are the electron and ion cyclotron frequencies [Spitzer², p.2]

$$\omega_- = \frac{eB}{mc} = 8.25 \cdot 10^9 \text{ sec}^{-1} \quad (42)$$

$$\omega_+ = \frac{eB}{Mc} = 4.5 \cdot 10^6 \text{ sec}^{-1} \quad (43)$$

Figs 10.1 and 10.2

and the collision-free drift velocity:

$$\frac{E}{B} = 4.25 \cdot 10^8 \text{ cm/sec} \quad (44)$$

In the absence of collision, the depth of penetration into a magnetic field, B , of a charged particle due to the force of an electric field, E , is given by:

$$y_{\max} = \frac{2 \frac{eE}{mc}}{\omega^2} \quad (45)$$

where $y_{\max}$ is the maximum distance away from the electrode of the resulting cycloidal path. Using this expression we get for electrons and ions respectively:

$$y_{\max-} = 0.1 \text{ cm} \quad (46)$$

$$y_{\max+} = 183 \text{ cm} \quad (47)$$

In other words, with the outer electrode at a radius of 2 cm, the magnetic field has only a small effect on the ions, but so great an effect on the electrons that in the absence of collisions these cannot cross the gap between electrodes, but are forced to drift perpendicular to both electric and magnetic field. This being so the path of ions will only be slightly different from that at no magnetic field and collisions will only affect the ions near their terminal velocity, whereas for electrons, the effect of collisions must be included as only this effect enables them to cross the electrode gap. This effect depends critically on the ratio v/ω (section 10.3).

10.6.1 Electrons.

From Fig. 10.1, it is evident that the only time when collisions are important for electrons is at very low energies. Whereas, without the magnetic field, the electric force promptly accelerated the electrons to energies at which they were collision-free, the presence of the magnetic field constantly bends the path of the electrons back to regions of low energy. Electrons are thus periodically slowed down to energies where the collision frequency with ions is not negligible and where it matters over several cycles. Furthermore, the magnetic field hinders conduction between the electrodes so a higher ionisation is needed. This in turn increases the influence of collisions.

We have to calculate an effective collision frequency, bearing in mind that the steady state collision frequency varies by a factor of 10^4 from one part of the cycle to another and that all the time we are dealing with probabilities of collision.

Although the mean free path, λ , at room temperature is small, 10^{-4} cm, it increases so rapidly with velocity that the mean free path is always greater than the distance already travelled. We must deal with fractions of mean free paths. If one assumes, as in the calculation of the mean free path, that the probability of collision in a distance ds is ds/λ , then the total probability of collision is $\int \frac{dx}{\lambda}$. As $\lambda = w/v$, we find that the probability of collision is

$$\rho = \int \frac{ds}{\lambda} = \int \frac{ds}{w} v = \int v dt \quad (48)$$

This total probability reaches unity even if λ is always greater than the distance travelled.

The total probability was calculated numerically for total ionisation at $2 \cdot 10^{14}$ mols/cc using the electron-ion collision frequency which alone matters. Using the parametric form of the cycloid $[a(\theta - \sin\theta), a(1 - \cos\theta)]$, it was found that $\int_0^t v dt = 1.1$ after 10^0 and that the rest of the half cycloid can be neglected. The rate of collision is, therefore, about once per half cycloid or twice in the time of one cyclotron rotation; so

$$v = 2.2 \frac{\omega}{2\pi} = 1.1 \frac{\omega}{\pi} = 3 \cdot 10^9 \quad (49)$$

so
$$\frac{v}{\omega} = 0.363 \quad (50)$$

From section 10.3, equations (23) and (24) for full ionisation.

$$V_{r-} = \frac{v\omega}{v^2 + \omega^2} \cdot \frac{E}{B} = \frac{v/\omega}{1 + (v/\omega)^2} \cdot \frac{E}{B} \text{ cm/sec} \quad (51)$$

$$V_{\theta-} = \frac{\omega^2}{v^2 + \omega^2} \cdot \frac{E}{B} = \frac{1}{1 + (v/\omega)^2} \cdot \frac{E}{B} \text{ cm/sec} \quad (52)$$

10.6.2 Ions.

As was said in section 10.6.0, the magnetic field hardly disturbs the path of the ions and so collision processes are negligible as in the case without a magnetic field (except at high energies). An integration similar to that done for electrons shows that the probability of one collision is $1.7 \cdot 10^{-2}$ - quite negligible.

The drift velocity at the outer electrode can be obtained from momentum considerations and is:

$$\begin{array}{ll} 9 \cdot 10^6 \text{ cm/sec} & \text{in a rectangular axes system or} \\ 4.5 \cdot 10^6 \text{ cm/sec} & \text{in a cylindrical model with the anode} \\ & \text{central and} \\ 18 \cdot 10^6 \text{ cm/sec} & \text{in a cylindrical model with the cathode} \\ & \text{central.} \end{array}$$

The mean drift velocity is about a third of this or approximately

$$\begin{array}{ll} V_{\theta+} = 1.5 \cdot 10^6 \text{ cm/sec} & (\text{anode central}) \quad (53) \\ V_{\theta+} = 6 \cdot 10^6 \text{ cm/sec} & (\text{cathode central}) \end{array}$$

This accounts for only 1% of the final energy and will be neglected. With 4.4 kv available energy, the terminal velocity is $9.2 \cdot 10^7$ cm/sec, so that the average radial velocity is

$$V_{r+} = 4.6 \cdot 10^7 \text{ cm/sec} \quad (54)$$

10.6.3 Current.

The total radial current density is given by:

$$j_r = \frac{ne}{c} (V_{r+} - V_{r-})$$

so $j_r = \frac{ne}{c} \left\{ \frac{\nu/\omega}{1 + (\nu/\omega)^2} \cdot 4.25 \cdot 10^8 + 4.6 \cdot 10^7 \right\} \text{ amps/cm}^2 \quad (55)$

If the degree of ionisation is α and N the number of molecules initially, $n = \alpha N$ and $\nu = \alpha \nu_0$, where ν_0 is the collision frequency for full ionisation. The current carried by each of the six circuits is 2 kiloampères. If the average cross-section for conduction in the plasma is taken as 1 cm^2 per outer electrode, the radial current density is

$$2 \cdot 10^3 \text{ amp/cm}^2$$

Equating both values for the radial current density we get

$$\alpha N \frac{e}{c} \left\{ \frac{\alpha(\nu_0/\omega)}{1 + \alpha^2(\nu_0/\omega)^2} \cdot 4.25 \cdot 10^8 + 4.6 \cdot 10^7 \right\} = 2 \cdot 10^3 \quad (56)$$

substituting $\nu_0/\omega = 0.363$ and $N = 2 \cdot 10^{14}$ mols/cc we get

$$\alpha \left\{ \frac{0.363\alpha}{1 + 0.363^2\alpha^2} \cdot 4.25 \cdot 10^8 + 4.6 \cdot 10^7 \right\} = 0.625 \cdot 10^8 \quad (57)$$

This is a cubic for α and has one positive real root, $\alpha = 0.5$

i.e. 50% ionisation.

This gives 10^{14} electrons per cubic centimeter, and an electron radial velocity of

$$V_{r-} = 7.5 \cdot 10^7 \text{ cm/sec} \quad (58)$$

With different initial densities we still have 10^{14} electrons/cc as both the drag and the current depend only on the number of electrons (or ions) per cc and not on the initial pressure. The relative ionisation, however, drops to 17% for 17μ Hg pressure.

To summarize; equation (10.55) is not symmetric for ions and electrons for the following two reasons: firstly, because electrons move along full cycloids whereas ions only reach a fraction of the cycloid. Secondly because the collision integral is much less for low energy ions than for low energy electrons.

10.6.4 The circulating current.

With 10^{14} electrons per cc the collision frequency is $1.5 \cdot 10^9 \text{ sec}^{-1}$ so

$$\nu/\omega = 0.182$$

and

$$v_{e-} = \frac{1}{1 + (\nu/\omega)^2} \cdot \frac{E}{B} = 4.1 \cdot 10^8 \text{ cm/sec} \quad (59)$$

The drift velocity for the ions was found to be (10.6.2)

$$\begin{aligned} v_{e+} &= 1.5 \cdot 10^4 \text{ cm/sec} && \text{with the anode central} \\ v_{e+} &= 6 \cdot 10^6 \text{ cm/sec} && \text{with the cathode central.} \end{aligned} \quad (60)$$

Thus it can be seen that the ions only contribute a small, 1%, fraction of the total circulating current. The direction of the circulating current is thus governed by the electrons only. With the cathode central the electrons move outwards and therefore weaken the magnetic field, in agreement with the experimental results. With the anode central the electrons move inwards and therefore strengthen the magnetic field. This is contrary to the experimental results. This explanation therefore fails on the question of sign.

The circulating current density, however, is

$$\begin{aligned} j_e &= \frac{ne}{c} (v_{e+} - v_{e-}) \\ &\approx -\frac{ne}{c} v_{e-} \\ &= -6.8 \text{ kamps/cm}^2 \end{aligned}$$

Thus the ratio of

$$\frac{\text{circulating current density}}{\text{radial current density}} = 3.4 \quad (61)$$

This is of the right order of magnitude.

In deriving a value for ν it was assumed that all electrons started from thermal energy. In a collision only a few electrons lose all their energy. The rest are isotropically deflected. Only a very few of these will reach zero energy in their cycloid. The rest will always keep a "trapped" energy. As it is the collisions at the lowest energy which are the most important, the total probability of collision is thus diminished.

Furthermore the collisions which are important are the electron-ions ones. In all the accounts of collisions only collisions with stationary ions are considered. The fact that the ions themselves are moving radially has not been considered above. This will again reduce the frequency of collision. With a reduced frequency of collision the radial electron current is reduced, equation (51). The ionisation necessary is therefore increased and the drift becomes more nearly E/B or $4.25 \cdot 10^8$ cm/sec. So the drift velocity is hardly changed and hence this does not change the previous conclusion that the sign of the change in the magnetic field is incorrect when the anode is at the centre. The only change is a larger ionisation.

10.6.5 The Debye Length.

Although up to now the plasma has always been assumed to contain equal densities of ions and electrons, no estimate has been made as to how closely this equality is maintained. This is normally assessed by calculating the Debye length. Actually the distance beyond which the coulomb force is screened, it also represents the dimension of the volume from which all charges of one kind can be expelled using the thermal energy. Hence it represents the order of magnitude of charge separation. It is given by: (Spitzer² p.17)

$$h = \sqrt{\frac{kT}{4\pi n e^2}} = 6.90 \left(\frac{T}{n}\right)^{\frac{1}{2}} \text{ cm.} \quad (62)$$

If we assume that the temperature is due to the randomization of the energy acquired in the electric field, then if 10 v are randomized $h = 2.3 \cdot 10^{-4}$ cm and if 1 kv is randomized $h = 2.3 \cdot 10^{-3}$ cm. So whatever the actual amount of randomized energy the Debye length is much smaller than any physical dimension of the apparatus. This suggests that the plasma ought to be treated from a macroscopic point of view; as a continuous fluid.

10.7 Macroscopic Analysis.

10.7.1 Boltzmann Equation.

When studying a plasma from the macroscopic point of view, it is usual to treat each type of particle, ions, electrons and molecules as a continuous fluid in phase space. At each point in phase space, the density of the component studied is given by the distribution function $f(\underline{r}, \underline{w}, t)$, where $\underline{r}$ = position vector, $\underline{w}$ = velocity vector, t = time.

The various quantities of interest are then averaged over the velocity space to obtain the normal laws of hydrodynamics in terms of functions of space and time only.

The basic equation for f , discussed in detail by Chapman and Cowling⁶, is the Boltzmann equation, Delcroix³ p.89.

$$\frac{\partial f}{\partial t} + \underline{w} \cdot \frac{\partial f}{\partial \underline{r}} + \frac{\underline{F}}{m} \cdot \frac{\partial f}{\partial \underline{w}} = \left(\frac{\partial f}{\partial t} \right)_{\text{collision}} \quad (63)$$

The average value of a quantity A at position $\underline{r}$ and time t is then defined as

$$\overline{A(\underline{r}, t)} = \frac{1}{n} \int A(\underline{r}, \underline{w}, t) \cdot f \cdot d\underline{w} \quad (64)$$

By multiplying equation (10.63) by A and integrating through the whole of velocity space we get after several lines of algebra

with $A = 1$, the equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underline{v}) = 0 \quad (65)$$

with $A = m \underline{w}$, the equation of momentum

$$nm(\frac{\partial}{\partial t} + \underline{v} \cdot \nabla) \underline{v} = n\underline{F} - \nabla \cdot \underline{\Psi} + \underline{P} \quad (66)$$

$$\text{where } \underline{\Psi} = nm(\underline{w} - \underline{v})(\underline{w} - \underline{v}) \quad (67)$$

$$\underline{P} = \int m \underline{w}(\frac{\partial f}{\partial t})_{\text{coll}} d\underline{w} \quad (68)$$

$\underline{v} = \underline{\bar{w}}$, the average velocity at $(\underline{r}, t)$; $\underline{\Psi}$ is the kinetic pressure tensor and with $A = m(\underline{w} - \underline{v})(\underline{w} - \underline{v})$, an equation for kinetic pressure and heat flow which we shall not use.

10.7.2 Linearized Equation of Motion

For a plasma in a combined electric and magnetic field, we must examine simultaneously the equations for ions and electrons. These are coupled by the equation $n_+ \approx n_- = n$ as soon as the Debye length is small compared to the dimensions of the apparatus. The molecules only affect the ions and electrons through the collision terms P_{em} , P_{im} which we have shown in section (10.5) to be only secondary effects and therefore negligible.

For the ions equation (66) becomes

$$n_+ M(\frac{\partial}{\partial t} + \underline{v}_+ \cdot \nabla) \underline{v}_+ = n_+ \frac{e}{c}(\underline{E} + \underline{v}_+ \times \underline{B}) + n_+ \underline{F}_+ - \nabla \cdot \underline{\Psi}_+ + \underline{P}_{ie} + \underline{P}_{im} \quad (69)$$

and for electrons

$$n_- m(\frac{\partial}{\partial t} + \underline{v}_- \cdot \nabla) \underline{v}_- = -n_- \frac{e}{c}(\underline{E} + \underline{v}_- \times \underline{B}) + n_- \underline{F}_- - \nabla \cdot \underline{\Psi}_- + \underline{P}_{ei} + \underline{P}_{em} \quad (70)$$

by Newton's 3rd Law $\underline{P}_{ei} = -\underline{P}_{ie}$

Let $\underline{\Psi} = \underline{\Psi}_+ + \underline{\Psi}_-$

$$\underline{\Psi} = -(e/mc) \underline{\Psi}_- + (e/Mc) \underline{\Psi}_+$$

$$\rho \underline{v} = n_- m \underline{v}_- + n_+ M \underline{v}_+$$

$$\rho = n_- m + n_+ M$$

$$\sigma = (n_+ - n_-)(e/c)$$

$$\underline{j} = (e/c)(n_+ \underline{v}_+ - n_- \underline{v}_-)$$

We get - Delcroix³, p.158

$$\rho \frac{\partial \underline{V}}{\partial t} + \underline{V} \frac{\partial \rho}{\partial t} + \underline{V} \cdot (\underline{n}_- \underline{m} \underline{v}_- \underline{v}_-) + \underline{\nabla} \cdot (\underline{n}_+ \underline{m} \underline{v}_+ \underline{v}_+) \\ = \sigma \underline{E} + \underline{j} \times \underline{B} - \underline{\nabla} \cdot \underline{\Psi} + n_+ F_+ + n_- F_- \quad (71)$$

$$\frac{\partial \underline{j}}{\partial t} + (\underline{\nabla} \cdot \underline{n}_+ \underline{v}_+ \underline{v}_+ - \underline{\nabla} \cdot \underline{n}_- \underline{v}_- \underline{v}_-) (e/c) \\ = \frac{n_- e^2}{mc^2} \left[1 + \frac{m}{M} \left(\frac{\sigma}{n_- e/c} + 1 \right) \right] \underline{E} \\ + \frac{n_- e^2}{mc^2} \left\{ \underline{V} - \frac{\underline{j}}{n_- e/c} + \frac{\sigma}{n_- e/c} \underline{V}_+ + \frac{m}{M} \left[\frac{n_- M}{\rho} + \left(\frac{\sigma}{n_- e/c} + 1 \right) \right] \underline{V}_+ \right. \\ \left. - \frac{m}{M} \frac{n_- M}{\rho} \underline{V}_- \right\} \times \underline{B} \\ - \underline{\nabla} \cdot \underline{\Psi} - \frac{n_- e}{cm} \underline{F}_- + \frac{m}{M} \frac{n_+ e}{mc} \underline{F}_+ - \left(\frac{e}{mc} + \frac{e}{Mc} \right) P_{ei} \quad (72)$$

In the first instance we make the usual assumptions:

- (1) We neglect quadratic terms ($\underline{\nabla} \cdot \underline{n} \underline{m} \underline{v} \underline{v}$ etc. $\underline{V} \partial \rho / \partial t$). This is equivalent to neglecting $\underline{v} \cdot \underline{\nabla} \underline{v}$ in equation (66).
- (2) We assume exact macroscopic neutrality $\sigma = 0$
- (3) We neglect terms in (m/M)
- (4) We assume the distribution of random velocity to be isotropic so $\underline{\nabla} \cdot \underline{\Psi} = \underline{\nabla} p$

Although in a later section (10.7.3) the quadratic terms are included again in the analysis.

$$\text{So } \rho \frac{\partial \underline{V}}{\partial t} = \underline{j} \times \underline{B} + n_+ F_+ + n_- F_- - \underline{\nabla} p \quad (73)$$

$$\text{and } \frac{mc^2}{ne^2} \cdot \frac{\partial \underline{j}}{\partial t} = \underline{E} + \underline{V} \times \underline{B} - \frac{c}{en} \underline{j} \times \underline{B} - \frac{c}{en} P_{ei} - \frac{c}{e} \underline{F}_- + \frac{c}{en} \underline{\nabla} p_- \quad (74)$$

$$\text{therefore, } \frac{\partial^2 \underline{j}}{\partial t^2} + \omega_+ \omega_- \underline{j} + \frac{\partial \underline{j}}{\partial t} \times \underline{\omega}_- + \nu_1 \frac{\partial \underline{j}}{\partial t} \\ + \frac{\omega_+ \omega_-}{B^2} \underline{B} \times (n_+ F_+ + n_- F_-) - \frac{\omega_+ \omega_-}{B^2} \underline{B} \times \underline{\nabla} p = 0 \quad (75)$$

where $\omega_+ = \frac{eB}{Mc}$, $\omega_- = \frac{eB}{mc}$, $\nu_1 = \frac{e}{mc} \frac{Pei}{j}$

and ν is the electron - ion collision frequency of section 10.3. This has for solution:

$$\underline{j} = j_A e^{-\nu_1 t} \hat{\underline{a}} + j_B e^{-\nu_1 [\omega_- \omega_+ / (\omega_-^2 + \nu_1^2)] t} \hat{\underline{b}} + \frac{(n_+ \underline{F}_+ + n_- \underline{F}_-) \times \underline{B}}{B^2} - \frac{\nabla p \times \underline{B}}{B^2} \quad (76)$$

where $\hat{\underline{a}}$ is a unit vector rotating anticlockwise at frequency ω_-
 $\hat{\underline{b}}$ " " " " " clockwise at frequency $\frac{\omega_-^2}{\omega_-^2 + \nu_-^2} \omega_+$

note that if m/M is not neglected, a slight coupling between ω_- and ω_+ influences $\hat{\underline{a}}$.

The first four terms of equation (75) determine the form of the complementary function. This is the sum of two vectors rotating in the plane perpendicular to $\underline{B}$ and in opposite directions and with their magnitude decreasing exponentially. They rotate at or near ion and electron cyclotron frequency (period 1.2 microsec and $6 \cdot 10^{-4}$ microsec. respectively), so neither phenomenon corresponds to the one in question. The remaining terms lead to the particular integral, which is the macroscopic counter part of the drifts. This shows that the electric field alone is insufficient to cause a 'drift' current; and that collision processes, (if ion-molecule and electron-molecule collisions are negligible), cannot produce a drift current either. This was shown in section (10.3).

If in equation (76) we introduce the 'centrifugal' effect, discussed in section (10.4), by putting

$$\underline{F}_+ = \frac{M v_{\theta+}^2}{R} \hat{\underline{r}} \quad \text{and} \quad \underline{F}_- = \frac{m v_{\theta-}^2}{R} \hat{\underline{r}}$$

we get for the complementary function for the current

$$\underline{j} = n(M v_{\theta+}^2 + m v_{\theta-}^2) \frac{\underline{R} \times \underline{B}}{B^2} \quad (77)$$

As, however, equation (76) has been derived for an infinite plasma and it was shown in section (10.4) that the ions cannot perform a complete trajectory, equation (77) fails to apply for the same reason as (34) or (34a).

The last term of equation (76) causes the plasma to reduce the applied magnetic field if the pressure on the inside of the plasma is larger than on the outside. To do this the plasma must expand and take the lines of force with itself. (Frozen - in lines of forces - Cowling¹⁰ p.5). However, the plasma is initially in equilibrium and $\nabla p = 0$. The pressure is a measure of the random motion of the particles, and this will be small compared to the directed motion because of the small collision frequency (relaxation frequency). It is therefore improbable that ∇p should be the cause of the observed circulating currents.

10.7.3 Equation of Motion Retaining Quadratic Terms.

To go back to equations (71) and (72), these are the orders of magnitude of the more important terms.

Equation (72) [multiplied by c/en so that the quantities are in cm/sec^2] with $n = 10^{14}$ and $\Delta j/\Delta t = 2$ kiloamp. in 5 microseconds.

$$\begin{aligned}\frac{c}{ne} \frac{\partial j}{\partial t} &= 2.5 \cdot 10^{13} \\ \nabla \cdot \underline{\underline{V}} \underline{\underline{V}} &= 2 - 7 \cdot 10^{18} \\ \frac{eE}{mc} &= 3 \cdot 10^{18} \\ \frac{eB}{mc} \times \underline{\underline{V}} &= 10^{17} - 10^{19} \\ \frac{j \times B}{mn} &= 10^{18} \\ \underline{\underline{v}} \underline{\underline{v}} &= 3 \cdot 10^{18}\end{aligned}$$

Equation (71)

$\rho \partial v / \partial t$	$= 6 \cdot 10^3$	dynes/cc
$\nabla \cdot n m \underline{\underline{v}} \underline{\underline{v}}$	$= 7 \cdot 10^5$	"
$\nabla \cdot n M \underline{\underline{v}}_+ \underline{\underline{v}}_+$	$= 4 \cdot 10^5$	"
$j \times B$	$= 10^5$	"

It is apparent that

- (1) we should have retained the quadratic terms $\nabla \cdot \underline{v}_+ \underline{v}_+$ etc.
- (2) as we are not interested in the particular integral of equation (75) both $\partial j / \partial t$ and $\partial v / \partial t$ can be neglected.
- (3) we can certainly neglect any effect due to $(n_+ - n_-)$ because of the smallness of the Debye length.⁺
- (4) centrifugal terms can be neglected if necessary.
- (5) we also neglect P_{ei} as this cancels out.

Under these conditions it is easier to go back to equations (69, 70) which we write

$$n \underline{Mv}_+ \cdot \nabla \underline{v}_+ = n \frac{e}{c} (\underline{E} + \underline{v}_+ \times \underline{B}) \quad (78)$$

$$n m \underline{v}_- \cdot \nabla \underline{v}_- = -n \frac{e}{c} (\underline{E} + \underline{v}_- \times \underline{B}) \quad (79)$$

If we assume cylindrical symmetry: $\partial / \partial \theta = 0$, $\partial / \partial z = 0$, equation (78) becomes

$$v_r \frac{dv_r}{dr} - \frac{v_\theta^2}{r} = \frac{e}{Mc} (E_r + v_\theta B) \quad (80)$$

$$v_r \frac{dv_\theta}{dr} + \frac{v_\theta v_r}{r} = -\frac{e}{Mc} v_r B \quad (81)$$

and the solution is

$$v_\theta = \frac{eB}{Mc} \frac{1}{r} \frac{r_0^2 - r^2}{2} \quad (82)$$

$$v_r^2 = -\frac{2e}{Mc} (V - V_0) - v_\theta^2 \quad (83)$$

For electrons the Larmor radius is so small under these conditions that the electrons would never be able to cross the gap between the electrodes without the collision term (see section 10.6.0). So we include P_{ei} in e.g. (79) and get

$$v_r \frac{dv_r}{dr} - \frac{v_\theta^2}{r} = -\frac{e}{mc} E - \nu v_r - \frac{eB}{mc} v_\theta \quad (84)$$

$$v_r \frac{dv_\theta}{dr} + \frac{v_\theta v_r}{r} = \frac{eB}{mc} v_r - \nu v_\theta \quad (85)$$

substituting $y = v_\theta r$, $x = v_r r$, we can neglect x^2 and y^2 and the solution is:

$$v_\theta = \frac{-\omega \frac{eE}{mc}}{\omega^2 + v^2} \quad v_r = \frac{-v \frac{eE}{mc}}{\omega^2 + v^2} \quad (86), (87)$$

It is unfortunate that to find a mathematical solution one should be forced to ignore either the centrifugal force or the friction, however in this case they are not both important simultaneously.

The solutions to equations (78), (79) are, however, identical to those already obtained with free trajectories in section 10.6. There is thus no advantage in using a macroscopic model rather than a microscopic one. The conclusions drawn then are valid here too and this model fails to explain the observed fact that the magnetic field is reduced in both cases. We have assumed above that both ions and electrons start at the electrodes. This simplification will not change the general conclusions.

Furthermore equations (82), (83) and (86), (87) cannot represent a neutral plasma. The equation of continuity of particles is

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underline{v}) = 0 \quad (88)$$

$$\text{or} \quad \frac{\partial n}{\partial t} + \underline{v} \cdot \nabla n + n \operatorname{div} \underline{v} = 0 \quad (89)$$

If we consider the electrons $v_r \propto E \propto 1/r$ so with cylindrical symmetry

$$\operatorname{div} \underline{v} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) = 0 \quad (90)$$

$$\text{so} \quad \underline{v} \cdot \nabla n = - \frac{\partial n}{\partial t} \quad (91)$$

and a stationary state exists for a uniform concentration of electrons. If we consider the positive charges however $\operatorname{div} \underline{v}_+ \neq 0$ so a stationary state exists only for a non uniform concentration.

$$\underline{v} \cdot \nabla n = -n \operatorname{div} \underline{v} \neq 0 \quad (92)$$

So the plasma cannot be neutral everywhere at once. As we have shown in section 10.6.5 that the plasma must be closely neutral, this invalidates the present approach.

We thus see that we must start the argument by making $n_+ = n_-$ right from the very beginning. This is only possible if one also assumes induced electric fields which will keep n_+ equal to n_- . These fields are caused by the very small difference between n_+ and n_- and are unknown to start with. With cylindrical symmetry and $\partial/\partial z = 0$, $\partial/\partial \theta = 0$, the equation of continuity becomes:

$$\frac{\partial n}{\partial t} + v_r \frac{\partial n}{\partial r} + \frac{n}{r} \frac{\partial}{\partial r}(r v_r) = 0 \quad (93)$$

of which the rather obvious solution is:

$$r n v_r = \text{const.} \quad (94)$$

So that for neutrality of the plasma

$$v_{r+} = -C v_{r-} \quad (95)$$

where C is a (positive) constant. However this leads to very improbable assumptions, so that finally it was decided to investigate the problem from a totally different aspect and to assume that the discharge forms six distinct channels. This is investigated in the next chapter and has proved much more successful.

10.8 Cross-section Data.

In obtaining the curves of figures 10.1 and 10.2, the following data was used.

Electron-ion collisions.	Delcroix ³ , p.152
Electron-molecule collisions	Delcroix ³ , p.23, 152
	Massey Burhop ¹¹ , p.177
Ion-molecule collisions	Massey Burhop ¹¹ , p.480, 526
	Cobine ¹² , p.32
Ion-electron collisions	Delcroix ³ , p.152,135.

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CHAPTER 11

CHANNEL DYNAMICS

11.0 Summary.

In this chapter, the discharge is considered to take place in well defined channels, so that an expression for the curvature of this channel can be obtained from the laws of motion. A first integration leads to a cut-off condition for this form of discharge. A second integration leads to the actual equation of the channel in polar coordinates, to the value of the change in axial magnetic field at the centre of the tube, and to a value for the ratio of circulating current to radial current.

The sign of the change of magnetic field is predicted correctly both for the anode and for the cathode at the centre. The value of the change of magnetic field with a central anode is of the right order of magnitude; with the cathode at the centre it is too low by a factor of about 2 to 3. Although the degree of ionisation required cannot be computed directly, an estimate shows it to be reasonable (10-70%). The effect of the self field has not been included; this may lead to a kink instability.

11.1 Introduction.

In this chapter we shall assume that the discharge is confined to only a small part of the volume available, that in fact it follows a well defined channel. This does not prevent the existence of ions and electrons outside the channel, but these are assumed to be of negligible concentration as far as current carrying is concerned. The six arcs are taken to be independent of one another.

It is also assumed that the channel is relatively well defined and that the region of change from negligible ionisation to ionisation capable of carrying the current is sufficiently narrow to be neglected compared to the width of the channel.

Overall neutrality must be preserved. The average separation of charges in the boundary layer will be of the order of one Debye length, which, at a density in the region of 10^{14} ions/cc, is $7 \cdot 10^{-5}$ cm if the energy randomized is

equal to one volt and $7 \cdot 10^{-3}$ cm with a random energy of 10 kv. So throughout the possible range of energies the Debye length is small compared to the dimensions of the apparatus. Neither ions nor electrons can escape from the channel. Any movement must be of the channel as a whole, with ion flank and electron flank moving together, - within a Debye length of each other. This does not, however, prevent relative movement of electrons and ions along the channel, constituting the current. To maintain the charge separation small there must be an intense electric field perpendicular to the channel between ions and electrons, this field being a direct result of the charge on the surface of the channel.

In the absence of ions, the radius of the epicyclic movement of the electrons in the fields in question is about $\frac{1}{2}$ mm. This is large compared with the Debye length and such epicycloids would take the electrons outside the channel. This means that the electric field induced by filament separation must be large enough virtually to cancel the curvature of the electron path and to transfer it to the ion path. The electric field perpendicular to the channel is thus expected to be about $E' \approx v_{t-} \cdot B$. The channel will thus be curved and this will cause a change in the magnetic field on the axis.

11.2 An expression for the acceleration.

11.2.1 If we take our axes along the channel and along the normal to the channel respectively and use the symbols defined in section 11.2.2 (Fig. 11.1), we can state that:

$$\begin{aligned} N_+ &= N_- && \text{(densities)} \\ v_{n+} &= v_{n-} && \text{(velocities normal to the channel)} \\ \text{and } \frac{\partial v_{n+}}{\partial t} &= \frac{\partial v_{n-}}{\partial t} && \text{(time derivatives)} \end{aligned}$$

The last equation needs some explanation. It would be wrong to say that

$$\frac{dv_{n+}}{dt} = \frac{dv_{n-}}{dt}$$

using the value of the actual acceleration. The quantity which remains equal for electrons and ions is the velocity of the flank. Thus local (partial) derivatives of ion and electron velocities must be used.

These in general will not be the same as the total derivatives for two reasons. Firstly the ions and electrons at a point A will have different accelerations, yet the same velocity, because of the transport derivative $\underline{v} \cdot \nabla \underline{v}$. The ions and electrons come from opposite sides and at different velocities. Secondly the curvature of the channel will introduce unequal centrifugal terms.

So

$$\frac{dv_{n+}}{dt} \neq \frac{dv_{n-}}{dt}$$

but

$$\left(\frac{\partial v_{n+}}{\partial t} \right)_s = \left(\frac{\partial v_{n-}}{\partial t} \right)_s$$

and as $v_{n+} = v_{n-}$ all the way along the channel

$$\left(\frac{\partial v_{n+}}{\partial s} \right)_t = \left(\frac{\partial v_{n-}}{\partial s} \right)_t$$

11.2.2 With $\underline{B}$ perpendicular to $\underline{E}$, the problem is essentially a two dimensional problem in the plane perpendicular to $\underline{B}$. Motion parallel to $\underline{B}$ is independent of $\underline{B}$ (and therefore trivial).

Let the vector velocity of the particle be $\underline{v}$ (Fig. 11.1), let its component tangential to the channel be v_t and normal to the channel v_n . We define curvilinear axes along the plasma channel $(\underline{\hat{u}})$ and perpendicular to it and to $\underline{B}$, $(\underline{\hat{n}})$, at any particular instant, so that $\underline{\hat{n}}$ is anticlockwise from $\underline{\hat{u}}$. We define $\underline{\hat{b}} = \underline{\hat{u}} \times \underline{\hat{n}}$ and $\underline{\hat{u}}, \underline{\hat{n}}, \underline{\hat{b}}$ form a right handed system of unit vectors. A particle however can only be in the channel so two coordinates suffice to define any relevant point in the plasma; the distance s along the channel, and the time t which defines the position of the channel in space. Therefore $\underline{v} = \underline{v}(t, s)$

$$v_t = \underline{v} \cdot \underline{\hat{u}} \quad , \quad \text{the velocity tangential to the channel}$$

$$v_n = \underline{v} \cdot \underline{\hat{n}} \quad , \quad \text{the velocity normal to the channel}$$

and
$$\underline{v} = v_t \underline{\hat{u}} + v_n \underline{\hat{n}}$$

Let us find an explicit expression for the acceleration

$$\begin{aligned}\underline{a} &= \frac{d\underline{v}}{dt} = \frac{d}{dt}(v_t \underline{\hat{u}} + v_n \underline{\hat{n}}) \\ &= \frac{dv_t}{dt} \underline{\hat{u}} + v_t \frac{d\underline{\hat{u}}}{dt} + \frac{dv_n}{dt} \underline{\hat{n}} + v_n \frac{d\underline{\hat{n}}}{dt}\end{aligned}\quad (1)$$

Consider first the changes of the axes. Being unit vectors, translation will have no effect, however rotation will. This can be due to two causes: (1) the rotation of the axes by virtue of movement along the channel, (2) the rotation of the channel itself at the point in question (Note, the channel is not a trajectory). If α denotes the angle which $\underline{\hat{u}}$ makes with a fixed axes $\underline{\hat{i}}$, then

$$d\alpha = \left(\frac{\partial \alpha}{\partial t}\right)_s dt + \left(\frac{\partial \alpha}{\partial s}\right)_t ds$$

Therefore

$$\begin{aligned}\frac{d\alpha}{dt} &= \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial s} \frac{ds}{dt} \\ &= \frac{\partial \alpha}{\partial t} + f v_t\end{aligned}$$

where f is the curvature (cm^{-1}) so $f = \partial \alpha / \partial s$ by definition and $ds/dt = v_t$. But $\underline{\hat{n}}$ is a unit vector, so $d\underline{\hat{n}}/dt$ is perpendicular to it and has magnitude da/dt . If α increases anticlockwise

$$\frac{d\underline{\hat{u}}}{dt} = \frac{d\alpha}{dt} \underline{\hat{n}}; \quad \frac{d\underline{\hat{n}}}{dt} = - \frac{d\alpha}{dt} \underline{\hat{u}}$$

So

$$\frac{d\underline{\hat{u}}}{dt} = \underline{\hat{n}} \left(\frac{\partial \alpha}{\partial t} + f v_t \right); \quad \frac{d\underline{\hat{n}}}{dt} = - \underline{\hat{u}} \left(\frac{\partial \alpha}{\partial t} + f v_t \right)$$

The change in v_t can be due to only two factors: movement along the channel and movement of the channel perpendicular to itself, as the particle moves at constant s , where s is the distance along the channel.

$$dv_t = \left(\frac{\partial v_t}{\partial t}\right)_s dt + \left(\frac{\partial v_t}{\partial s}\right)_t ds$$

Therefore

$$\frac{dv_t}{dt} = \frac{\partial v_t}{\partial t} + v_t \left(\frac{\partial v_t}{\partial s}\right)$$

and

$$\frac{dv_n}{dt} = \frac{\partial v_n}{\partial t} + v_t \left(\frac{\partial v_n}{\partial s}\right)$$

Collecting terms

$$\begin{aligned} \underline{a} = & \frac{\partial v_t}{\partial t} \underline{\hat{u}} + \frac{\partial v_n}{\partial t} \underline{\hat{n}} + v_t \left(\frac{\partial v_n}{\partial s} \underline{\hat{n}} + \frac{\partial v_t}{\partial s} \underline{\hat{u}} \right) \\ & + f \cdot (v_t^2 \underline{\hat{n}} - v_t v_n \underline{\hat{u}}) + \frac{\partial a}{\partial t} (v_t \underline{\hat{n}} - v_n \underline{\hat{u}}) \quad (2) \end{aligned}$$

where f is the curvature of the channel.

11.3 General conditions.

It will now be established that the solution we require is a steady state one if $\underline{E}$ and $\underline{B}$ are constant.

With crossed electric and magnetic fields, it is well known that the velocity which an ion (or electron) acquires perpendicular to both $\underline{E}$ and $\underline{B}$ is proportional to the distance which the ion has moved in the $\underline{E}$ direction from its initial position, by the law of conservation of total momentum. Or more simply:

Let the fields be uniform and steady with the anode on $x = 0$, and cathode on $x = X$, so

$$\underline{E} = E_x \underline{i}$$

$$\underline{B} = B \underline{k} \quad (\text{Fig. 11.2a})$$

$$\begin{aligned} a_y = \frac{dv_y}{dt} &= \frac{e}{Mc} (E_y + [\underline{v} \times \underline{B}]_y) \\ &= - \frac{eB}{Mc} \cdot v_x = - \frac{eB}{Mc} \frac{dx}{dt} \quad (E_y = 0) \end{aligned}$$

Therefore
$$v_y = - \frac{eB}{Mc} \int_0^x \frac{dx}{dt} \cdot dt = - \frac{eB}{Mc} x$$

so long as we neglect kT compared to eV and assume the ion starts from rest. By conservation of energy:

$$v_y^2 + v_x^2 = \frac{2eEx}{Mc}$$

so that both v_y and v_x are functions only of x and all trajectories have the same shape and are oriented in the same way, so long as the ion starts on the y -axis. If furthermore all ions are only emitted from a narrow area, successive trajectories will superpose into a stationary path.

If we now consider a channel between planes $x = 0$, $x = X$ (Fig. 11.2b) we must include the electric field caused by charge separation (Hallfield). If we call it $\underline{E}'$ we have

$$\begin{aligned} v_{y+} &= \frac{e}{Mc} \int_0^x -v_x B \, dt + \frac{e}{Mc} \int_0^x E'_y \, dt \\ &= -\frac{eB}{Mc} x + \frac{e}{Mc} \int_0^x \frac{E'_y}{v_{x+}} dx \end{aligned} \quad (3)$$

$$\text{and} \quad v_y^2 + v_x^2 = \frac{2eEx}{Mc} + \frac{2e}{Mc} \int_0^x E'_x \, dx \quad (3a)$$

If again ions and electron both start from rest from small areas on anode and cathode respectively, there is no reason why E'_y or E'_x should vary with time or with y , except at the edges of the channel. So the channel is still a succession of superposed trajectories and is thus stationary. Should the emissive region be extended, there is the possibility of the channel sliding along the electrode as on rails. In this apparatus however the outer electrode is only 1 mm in diameter so this should lock the channel.

With cylindrical symmetry the argument is essentially the same, equation (3) being replaced by one derived from the conservation of total angular momentum. The outer electrode in the apparatus is formed by 6 independent electrodes which should lock the discharges into 6 steady channels.

So $V_{n+} = V_{n-} = 0$ and $\frac{\partial}{\partial t} \equiv 0$ and therefore,

$$\frac{\partial v_{n+}}{\partial t} = 0, \quad \frac{\partial v_{n-}}{\partial t} = 0, \quad \frac{\partial v_{n+}}{\partial s} = 0, \quad \frac{\partial v_{n-}}{\partial s} = 0$$

11.4 Equation of Motion.

For both ions and electrons equation (2) reduces to

$$\underline{a} = v_t \frac{\partial v_t}{\partial s} \underline{\hat{u}} + v_t^2 \underline{\hat{n}}$$

$$\text{or} \quad mfv_{t-}^2 = -\frac{e}{c}(\underline{E} \cdot \underline{\hat{n}} + \underline{E}' \cdot \underline{\hat{n}} - v_{t-} B) \quad (4)$$

$$Mfv_{t+}^2 = \frac{e}{c}(\underline{E} \cdot \underline{\hat{n}} + \underline{E}' \cdot \underline{\hat{n}} - v_{t+} B) \quad (5)$$

$$\text{and} \quad mv_{t-} \frac{\partial v_{t-}}{\partial s} = -\frac{e}{c}(\underline{E} \cdot \underline{\hat{u}} + \underline{E}' \cdot \underline{\hat{u}}) \quad \text{as} \quad v_n = 0 \quad (6)$$

$$M v_{t+} \frac{\partial v_{t+}}{\partial s} = \frac{e}{c}(\underline{E} \cdot \underline{\hat{u}} + \underline{E}' \cdot \underline{\hat{u}}) \quad (7)$$

where $\underline{E}$ is the applied electric field and $\underline{E}'$ the electric field due to charge separation. Integrating (6)

$$\begin{aligned} \frac{1}{2} mv_{t-}^2 &= -\frac{e}{c} \int (\underline{E} + \underline{E}') \cdot \underline{\hat{u}} \, ds \\ &= -\frac{e}{c} \int_0^s (\underline{E} + \underline{E}') \cdot \underline{ds} \\ &= \frac{e}{c} V \quad \text{as both } \text{curl } \underline{E} \text{ and } \text{curl } \underline{E}' \text{ are zero } \left(\frac{\partial B}{\partial t} = 0 \right) \end{aligned}$$

$$mv_{t-}^2 = \frac{2e}{c} V$$

$$\text{similarly} \quad M v_{t+}^2 = \frac{2e}{c} (V_0 - V)$$

where V_0 is the voltage between the electrodes

and $V = V_0$ at the anodes

and $V = 0$ at the cathode.

Adding (4) and (5) to eliminate $\underline{E} \cdot \underline{\hat{n}}$ and $\underline{E}' \cdot \underline{\hat{n}}$ and substituting the above expressions we get

$$f.2 \frac{e}{c} V_0 = \frac{e}{c} (v_{t-} - v_{t+}) B \quad (8)$$

which can be written in the following forms

$$f = \frac{B}{2V_0} (v_{t-} - v_{t+}) \quad (9)$$

$$f = - \frac{icB}{2eNV_0} \quad \begin{array}{l} \text{where } i \text{ is the current} \\ \text{density} \\ N \text{ the ion density} \end{array}$$

$$\text{or} \quad i = - \frac{Ne}{c} \cdot \frac{2V_0}{B} \cdot f$$

Note:- Throughout this argument, the self-field of the current has been neglected. It could be the cause of a kink instability. However equation (9) is independent of current, so this is in fact a low current approximation.

if the anode is at $s = 0$

and the cathode at $s = s_0$,

$$v_{t+} = + \sqrt{\frac{2e(V_0 - V)}{Mc}}$$

and

$$v_{t-} = - \sqrt{\frac{2eV}{mc}}$$

so writing (9) explicitly,

$$f = - \frac{B}{2V_0} \left\{ \sqrt{\frac{2eV}{mc}} + \sqrt{\frac{2e(V_0 - V)}{Mc}} \right\} \quad (10)$$

where the minus sign merely indicates that the curvature is the opposite way to the way it is defined in Fig. 11.1.

11.5 Channel in a Plane Field.

Provided we assume some law for the voltage distribution, equation (10) gives us a differential equation for the curve of the channel. In the first instance we shall simplify and assume a rectangular geometry, with two infinite parallel plane electrodes, distance a apart (Fig. 11.3).

If we neglect space charge effects

E is constant and

$$V = \frac{a - x}{a} V_0$$

equation (10) becomes

$$\begin{aligned}
 f &= -\frac{B}{2V_0} \left\{ \sqrt{\frac{2e(a-x)V_0}{mca}} + \sqrt{\frac{2eV_0x}{Mc a}} \right\} \\
 &= -B \sqrt{\frac{e}{2V_0 mca}} \left\{ \sqrt{a-x} + \sqrt{\frac{m}{M}} \sqrt{x} \right\} \\
 &= -Q \frac{1}{\sqrt{a}} \left\{ \sqrt{a-x} + \sqrt{\frac{m}{M}} \sqrt{x} \right\} \quad (11)
 \end{aligned}$$

$$\text{where } Q = B \sqrt{\frac{e}{2V_0 mc}} \quad \text{cm}^{-1}$$

and is the Larmor curvature for electrons with the terminal velocity.

So we have

$$\frac{\frac{d^2y}{dx^2}}{\left(\left(\frac{dy}{dx}\right)^2 + 1\right)^{3/2}} = -\frac{Q}{\sqrt{a}} \left\{ \sqrt{a-x} + \sqrt{\frac{m}{M}} \sqrt{x} \right\} \quad (12)$$

$$\text{put } p = \frac{dy}{dx} ; \quad \frac{dp}{dx} = \frac{d^2y}{dx^2}$$

$$\text{and } \frac{p}{\sqrt{1+p^2}} = \frac{2}{3} \frac{Q}{\sqrt{a}} \left\{ (a-x)^{3/2} - \sqrt{\frac{m}{M}} x^{3/2} + C \right\}$$

We shall assume that the channel leaves the anode at right angle to it. This assumption will be discussed in detail in section 11.7 and a proof given. So that

$$\text{at } x = 0, \frac{dy}{dx} = 0 \quad \text{therefore } p = 0 \quad \text{and } C = -a^{3/2}$$

$$\text{so } \frac{dy}{dx} = p = \frac{\frac{2}{3} \frac{Q}{\sqrt{a}} \left\{ (a-x)^{3/2} - \sqrt{\frac{m}{M}} x^{3/2} - a^{3/2} \right\}}{\sqrt{1 - \frac{4}{9} \frac{Q^2}{a} \left\{ (a-x)^{3/2} - \sqrt{\frac{m}{M}} x^{3/2} - a^{3/2} \right\}^2}} \quad (13)$$

which can be solved by numerical integration.

However if we put

$$\frac{dy}{dx} = \tan \theta \quad (\text{Fig. 11.3})$$

$$dx = ds \cos \theta$$

$$\text{and } f = d\theta/ds \quad (\text{by definition})$$

$$\cos \theta \, d\theta = -\frac{Q}{\sqrt{a}} (\sqrt{a-x} + \sqrt{\frac{m}{M}} \sqrt{x}) dx$$

$$\text{Therefore, } \sin \theta = \frac{2}{3} \frac{Q}{\sqrt{a}} \left\{ (a-x)^{3/2} - \sqrt{\frac{m}{M}} x^{3/2} - a^{3/2} \right\} \quad (14)$$

obviously, if the theory is to be applicable

$$-1 \leq \sin \theta \leq 1$$

for the range

$$0 \leq x \leq a$$

so conduction will only take place if

$$\frac{2}{3} Qa \left(1 + \sqrt{\frac{m}{M}} \right) \leq 1 \quad (15)$$

This is very similar to the cut-off value of magnetron theory.

With $B = 470$ gauss

$$V_0 = 4 \text{ kv}$$

and $a = 2 \text{ cm},$

$$Q = 2.2 \text{ cm}^{-1}$$

$$\text{and } \frac{2}{3} Qa = 2.93 \quad (\text{neglecting the term in } \sqrt{\frac{m}{M}})$$

Thus in a plane field with the data experimentally used we should not get conduction.

It can easily be seen that changing the law obeyed by the voltages will only change the numerical constant in front of Qa , so there is some hope that the above values will conduct in a cylindrical model.

11.6 Channel in a Cylindrical Field.

11.6.1 Anode Central

We now use the voltage distribution which exists between two continuous concentric cylindrical electrodes when space charge is neglected:

$$V = V_0 \frac{\ln(r/r_2)}{\ln(r_1/r_2)} \quad (16)$$

where r_1 is the radius of the (common) central electrode (Fig. 11.4)

r_2 is the radius of the cylinder representing the six outer electrodes

$$V = V_0 \text{ at } r = r_1 \text{ (anode)}$$

$$V = 0 \text{ at } r = r_2 \text{ (cathode)}$$

Substituting this in equation (10) we get:

$$\begin{aligned} f &= -\frac{B}{2V_0} \sqrt{\frac{2eV_0}{mc}} \left\{ \sqrt{\frac{\ln(r/r_2)}{\ln(r_1/r_2)}} + \sqrt{\frac{m}{M}} \sqrt{\frac{\ln(r_1/r)}{\ln(r_1/r_2)}} \right\} \\ &= -Q \left\{ \sqrt{\frac{\ln(r/r_2)}{\ln(r_1/r_2)}} + \sqrt{\frac{m}{M}} \sqrt{\frac{\ln(r_1/r)}{\ln(r_1/r_2)}} \right\} \end{aligned} \quad (17)$$

$$\text{but } f = \frac{u^3(u + \ddot{u})}{(u^2 + \dot{u}^2)^{3/2}} \quad \text{where } u = 1/r, \dot{u} = \frac{du}{d\theta}; \ddot{u} = \frac{d^2u}{d\theta^2}$$

$$\text{so } \frac{u^3(u + \ddot{u})}{(u^2 + \dot{u}^2)^{3/2}} = -Q \left\{ \sqrt{\frac{\ln(u/u_2)}{\ln(u_1/u_2)}} + \sqrt{\frac{m}{M}} \sqrt{\frac{\ln(u_1/u)}{\ln(u_1/u_2)}} \right\} \quad (18)$$

This does not contain θ explicitly and solves to:

$$\begin{aligned} \frac{1}{\sqrt{u^2 + \left(\frac{du}{d\theta}\right)^2}} &\approx \frac{Q}{\sqrt{\ell n(u_1/u_2)}} \int \frac{\sqrt{\ell n(u/u_2)}}{u^3} du \\ &= -\frac{Q}{\sqrt{\ell n(r_2/r_1)}} \int r \sqrt{\ell n(r_2/r)} dr \quad (19) \end{aligned}$$

where the term $\sqrt{m/M}$ is neglected as it is not important (section 11.13.1).

If we use tangential polar coordinates, Fig. 11.4, where r is the distance from a fixed point, in this case the centre and q (usually called p) is the perpendicular distance from the origin to the tangent at r , the curvature is given by

$$f = \frac{1}{r} \frac{dq}{dr}$$

$$\begin{aligned} \text{so } q_r - q_1 &= + \int r f dr \\ &= -\frac{Q}{\sqrt{\ell n(r_2/r_1)}} \int_{r_1}^r r \sqrt{\ell n(r_2/r)} dr \quad (20) \end{aligned}$$

substitute $z = \sqrt{2\ell n(r_2/r)}$

giving $r = r_2 e^{-z^2/2}$

and integrating by parts we get

$$q_r - q_1 = \frac{-Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_2^2}{\sqrt{2}} \int_{\sqrt{2\ell n(r_2/r_1)}}^{\sqrt{2\ell n(r_2/r)}} -e^{-z^2} \cdot z^2 \cdot dz \quad (21)$$

$$\begin{aligned} &= \frac{-Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_2^2}{4\sqrt{2}} \left\{ 2\sqrt{2\ell n(r_2/r)} \left(\frac{r}{r_2}\right)^2 - 2\sqrt{2\ell n(r_2/r_1)} \left(\frac{r_1}{r_2}\right)^2 \right. \\ &\quad \left. + \sqrt{\pi} \operatorname{erf}(\sqrt{2\ell n(r_2/r_1)}) \right. \\ &\quad \left. - \sqrt{\pi} \operatorname{erf}(\sqrt{2\ell n(r_2/r)}) \right\} \quad (22) \end{aligned}$$

11.6.2 Cathode Central.

If the central electrode is the cathode

$$V = 0 \quad \text{at } r = r_1$$

$$V = V_0 \quad \text{at } r = r_2$$

$$\text{so } V = V_0 \frac{\ell n(r/r_1)}{\ell n(r_2/r_1)}$$

$$\text{and } f = Q \left\{ \sqrt{\frac{\ell n(r/r_1)}{\ell n(r_2/r_1)}} + \sqrt{\frac{m}{M}} \sqrt{\frac{\ell n(r_2/r)}{\ell n(r_2/r_1)}} \right\} \quad (23)$$

$$\text{so } q_r - q_1 = \frac{Q}{\sqrt{\ell n(r_2/r_1)}} \int_{r_1}^r r \sqrt{\ell n(r/r_1)} \, dr \quad (24)$$

$$\text{substitute } z = \sqrt{2 \ell n(r/r_1)} ; \quad r = r_1 e^{+z^2/2}$$

we get

$$q_r - q_1 = \frac{Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_1^2}{\sqrt{2}} \int_{\sqrt{2 \ell n(r_1/r_1)}}^{\sqrt{2 \ell n(r/r_1)}} e^{z^2} \cdot z^2 \cdot dz \quad (25)$$

$$= \frac{Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_1^2}{4\sqrt{2}} \left\{ 2 \sqrt{2 \ell n(r/r_1)} \left(\frac{r}{r_1} \right)^2 + i \sqrt{\pi} \operatorname{erf}(i \sqrt{2 \ell n(r/r_1)}) \right\} \quad (26)$$

Fig 11.5

Fig 11.6

Fig 11.7 Anode Central

Fig 11.8 Cathode Central

11.7 End Conditions.

Consider the anode and let the channel leave the surface of the anode at an angle β to the normal. (Fig. 11.5) If we neglect any space charge effects $\underline{E}$ will be perpendicular to the anode surface and the induced field $\underline{E}'$ is perpendicular to the channel.

$$V = V_0$$

Therefore $v_{t+} = 0$, $v_{t-} = -\sqrt{\frac{2eV_0}{mc}}$

$$\text{so } f = -\frac{B}{2v_0} \sqrt{\frac{2eV_0}{mc}}$$

Consider the ions

$$v_{t+} = 0 \quad \text{so} \quad f v_{t+}^2 = 0 \quad \text{and} \quad v_{t+} B = 0$$

$$\text{so the force in } x - \text{direction} = \frac{e}{c}(E - E' \sin \beta)$$

$$\text{" " } y - \text{direction} = \frac{e}{c}(E' \cos \beta)$$

also as $v_{t+} = 0$ the tangent to the path at the origin is in the same direction as the acceleration at the origin, so

$$\tan \beta = \frac{E' \cos \beta}{E - E' \sin \beta}$$

Consider the electrons: equation (4)

$$m f v_{t-}^2 = -\frac{e}{c}(\underline{E} \cdot \hat{n} + \underline{E}' \cdot \hat{n} - v_{t-} B)$$

substituting

$$-\frac{mB}{2v_0} \left(\frac{2eV_0}{mc}\right)^{3/2} = -\frac{e}{c} \left[\left(\frac{2eV_0}{mc}\right)^{1/2} \cdot B - E \sin \beta + E' \right]$$

$$\text{so } E' = E \sin \beta$$

$$\text{and } \tan \beta = \frac{E \sin \beta \cdot \cos \beta}{E - E \sin^2 \beta} = \tan \beta$$

so β can have any value.

But resolving the electric vectors parallel to the anode surface we get:

$$E' \cos \beta = E \sin \beta \cos \beta = \frac{1}{2} E \sin 2\beta$$

and the anode being a conductor this must be zero. So $\beta = 0$. This applies equally well at the cathode, so the channel must leave both electrodes perpendicular to the surface.

Without doubt the detail process at the electrodes is very much more complicated than this simple model, but there does not seem to be any good reason why the channel should not leave the electrodes at right angle.

The direction of the normal to the channel is always on the same side of the channel, as it is in the direction of the $\underline{j} \times \underline{B}$ force and neither vector changes sign. This means that such a channel cannot leave both electrodes at right angle if the electrodes are concentric cylinders. This would only be possible in the case of no magnetic field. Fortunately in our experiment outer electrode is not just one continuous surface, but six electrodes, symmetrically placed on a circle, radius r_2 , concentric with the inner electrode. This means that on the whole the voltage distribution will resemble that of two concentric cylinders, and yet the channel, leaving the central cylinder along a radius, will still be able to approach one of the outer electrodes from any direction and meet it perpendicular to its surface. Admittedly this will restrict the path to 6 discreet ones, but this is in keeping with what was found earlier (see 11.3).

The central electrode is continuous so that the arc, in leaving its surface at right angle, must do so on a radius of the apparatus

$$\text{hence } \underline{q_1} = 0 \quad (27)$$

whether this happens to be the anode or the cathode. If we now substitute this value for q_1 into equations (22) and (26) respectively, we get for q_2 :

$$q_2 = - \frac{Q}{\sqrt{2\ell n(r_2/r_1)}} \frac{r_2^2}{2\sqrt{2}} \left\{ - \sqrt{2\ell n(r_2/r_1)} \left(\frac{r_1}{r_2} \right)^2 + \frac{\sqrt{\pi}}{2} \operatorname{erf}(\sqrt{2\ell n(r_2/r_1)}) \right\} \quad (28)$$

when the anode is at the centre

$$\text{and } q_2 = + \frac{Q}{\sqrt{\ell \ln(r_2/r_1)}} \frac{r_1^2}{2\sqrt{2}} \left\{ + \sqrt{2\ell \ln(r_2/r_1)} \left(\frac{r_2}{r_1} \right)^2 + \frac{\sqrt{\pi}}{2} i \operatorname{erf}(i \sqrt{2\ell \ln(r_2/r_1)}) \right\} \quad (29)$$

when the cathode is at the centre.

11.8 Cut-off conditions.

Now $\sin \alpha = q/r$ (Fig. 11.4) so as before, (11.5), this limits the range of applicability of the theory. If α is to remain real

$$\left| \frac{q}{r} \right| \leq 1$$

for all values of r between r_1 and r_2 , otherwise the calculated channel curls back and fails to span the electrodes. In particular:

$$\left| \frac{q}{r} \right|_{r_2} \leq 1 \quad (30)$$

This again gives us a cut-off condition as for a plane field. With anode central

$$Q \left\{ \frac{\sqrt{\pi}}{4} \cdot \frac{r_2}{\sqrt{2\ell \ln(r_2/r_1)}} \operatorname{erf}(\sqrt{2\ell \ln(r_2/r_1)}) - \frac{1}{2} \frac{r_1^2}{r_2} \right\} \leq 1 \quad (31)$$

With cathode central

$$Q \left\{ \frac{\sqrt{\pi}}{4} \frac{r_1^2/r_2}{\sqrt{2\ell \ln(r_2/r_1)}} i \operatorname{erf}(i \sqrt{2\ell \ln(r_2/r_1)}) + \frac{1}{2} r_2 \right\} \leq 1 \quad (32)$$

In fact this is not strictly true in the first case (anode central), because $|q/r|$ reaches a maximum just before r_2 so that it is this value which must be used. The maximum is very flat and the error is about 2% and will be neglected (section 11.13.2).

If we use the same values as before, $Q = 2.2 \text{ cm}^{-1}$, $r_1 = 0.5 \text{ cm}$, $r_2 = 2 \text{ cm}$, we get with the anode central

$$q_2 = -Q \left\{ r_2^2 \frac{\sqrt{\pi}}{4} \frac{\text{erf}(\sqrt{2\ell n(r_2/r_1)})}{\sqrt{2\ell n(r_2/r_1)}} - \frac{1}{2} r_1^2 \right\}$$

$$= -2.2 (1.052 - 0.125) = -2.04$$

so $\left| \frac{q}{r} \right|_{r_2} = 1.02 \quad (33)$

and with cathode central

$$q_2 = +Q \left\{ r_1^2 \frac{\sqrt{\pi}}{4} \frac{i \text{erf}(i\sqrt{2\ell n(r_2/r_1)})}{\sqrt{2\ell n(r_2/r_1)}} + \frac{1}{2} r_2^2 \right\}$$

$$= 2.2 (-0.453 + 2) = +3.4$$

so $\left| \frac{q}{r} \right|_{r_2} = 1.7 \quad (34)$

11.9 Discussion of assumptions.

With the anode central we can assume that we have conduction, if we remember that none of the quantities involved were measured with an accuracy greater than 5%, but the theory fails to explain conduction with a central cathode, so we shall examine the assumptions.

The following are the main assumptions involved; apart from those normally assumed in this type of work

- (a) The channel is stationary for constant $\underline{E}$ and $\underline{B}$.
- (b) The channel leaves both electrodes perpendicular to the surface.
- (c) Space charge is ignored.
- (d) The outer electrode is assumed continuous in deriving the voltage distribution.

The argument underlying the assumption that the channel is stationary, seems reasonable, but it is not rigorous. This assumption lies at the very root of this approach and so cannot be changed, without altering the method completely. So despite the fact that the contrary is quite conceivable, this assumption will be kept unchanged.

The effect of assuming a different angle of incidence of the channel onto the electrodes is easily established. It changes the value of q_1 and relation (30) becomes

$$\left| \frac{q - q_1}{r} \right|_{r_2} \leq 1$$

This gives us a little bit more latitude as q_1 can have any value up to r_1 , but then another end condition must be found if there is to be only one channel per anode and not a multitude all with different α 's and it is difficult to see how there can be a steady component of the electric field parallel to a conducting surface at that surface.

Furthermore the assumption $q_1 = 0$ gives the particular direction to the channel and is in fact directly responsible for the channel's magnetic field opposing the external field. For this reason alone, it is worth while keeping it unchanged.

The space charge comes in directly in the derivation of the voltage distribution. If there is a space charge, i.e. $n \neq n_-$, then the first effect will be to change the form of $V(r)$. This in turn changes the numerical coefficient γ in the general cut off relationship

$$\gamma Qa \leq 1$$

However it has not been ascertained whether this effect would help or hinder conduction.

Assuming that the outer electrodes are separate also changes the voltage distribution. Using a very rough model, it is shown in the next section that $|q/r|$ at $r_2 = 1.3$, so this is a change in the right direction, so that altogether this approach leads to results of the right order of magnitude.

11.10 Detail calculation.

We shall now investigate the effect of assuming 6 separate anodes instead of the one cylinder, radius r_2 . The space between electrodes is divided into two regions, one around the cathode with the same voltage distribution as before, the other around the anode with a voltage distribution symmetrical around the anode (Fig. 11.6).

Assumptions.

region 1. voltage distribution unchanged from $r_1 = 0.5$ cm to $r_3 = 1$ cm

region 2. the voltage distribution has cylindrical symmetry around the anode; total voltage from cathode to anode unchanged.
 inner radius $r_5 = 0.05$ cm (radius of anode)
 outer radius $r_4 = 1$ cm ($= r_2 - r_3$)

region 1.

$$V = V_0 \frac{\ln(r/r_1)}{\ln(r_2/r_1)} \quad r_1 \leq r \leq r_3$$

$$\text{so} \quad f = Q \left\{ \sqrt{\frac{\ln(r/r_1)}{\ln(r_2/r_1)}} \right\}; \quad dq = f r dr$$

$$\text{so} \quad q_3 - q_1 = \int_1^3 r f dr \quad (\text{as for cathode central})$$

$$= \frac{Q}{\sqrt{\ln(r_2/r_1)}} \frac{r_1^2}{2\sqrt{2}} \left\{ \sqrt{2\ln(r_3/r_1)} \left(\frac{r_3}{r_1} \right)^2 + \frac{i\sqrt{\pi}}{2} \operatorname{erf}(i\sqrt{2\ln(r_3/r_1)}) \right\}$$

$$= 2.2 \left\{ \frac{1}{2} \sqrt{\frac{2\ln 2}{2\ln 4}} \cdot 1^2 + \frac{i\sqrt{\pi}}{4\sqrt{2\ln 4}} \operatorname{erf}(i\sqrt{2\ln 2}) \right\}$$

$$= 2.2 (0.354 - 0.154) = 0.44$$

region 2.

$$\text{at } r_3, \quad V = V_0 \frac{\ln(r_3/r_1)}{\ln(r_2/r_1)} = V_0 \frac{\ln 2}{\ln 4} = \frac{1}{2} V_0$$

$$\text{so } V_4 = \frac{1}{2} V_0, \quad V_5 = V_0$$

$$\text{and } V = \frac{V_0}{2} + \frac{V_0}{2} \frac{\ln(r_4/x)}{\ln(r_4/r_5)} = \frac{V_0}{2} \frac{\ln(r_4^2/r_5 x)}{\ln(r_4/r_5)}$$

$$\begin{aligned} f &= \frac{B}{2V_0} v_{t-} = \frac{B}{2V_0} \sqrt{\frac{2Ve}{mc}} \\ &= \frac{Q}{\sqrt{2}} \sqrt{\frac{\ln(r_4^2/r_5 x)}{\ln(r_4/r_5)}} \simeq \frac{Q}{\sqrt{2}} \sqrt{\frac{\ln[r_4^2/(r_2 - r)]}{\ln(r_4/r_5)}} \end{aligned}$$

This approximation involves a minor change in the spatial distribution of V

$$\text{so } q_2 - q_3 = \int_3^2 \frac{Q}{\sqrt{2}} \frac{1}{\sqrt{\ln(r_4/r_5)}} \cdot r \sqrt{\ln\left(\frac{r_4^2}{r_5} / (r_2 - r)\right)} dr$$

substituting $z = \sqrt{2 \ln[r_4^2/r_5(r_2 - r)]}$ and $y = z/\sqrt{2}$

$$\begin{aligned} q_2 - q_3 &= \frac{Q}{\sqrt{2 \ln(r_4/r_5)}} \left\{ -\frac{1}{\sqrt{2}} \left(\frac{r_4^2}{r_5}\right)^2 \int_3^2 z^2 e^{-z^2} dz + \frac{2r_2 r_4^2}{r_5} \int_3^2 y^2 e^{-y^2} dy \right\} \\ &= \frac{Q}{\sqrt{2 \ln(r_4/r_5)}} \left\{ \frac{1}{\sqrt{2}} \left(\frac{r_4^2}{r_5}\right)^2 \left[\left[\frac{z}{2} e^{-z^2} \right]_3^2 - \left[\frac{1}{2} \frac{\sqrt{\pi}}{2} \operatorname{erf} z \right]_3^2 \right] \right. \\ &\quad \left. - \frac{2r_2 r_4^2}{r_5} \left[\left[\frac{y}{2} e^{-y^2} \right]_3^2 - \left[\frac{1}{2} \frac{\sqrt{\pi}}{2} \operatorname{erf} y \right]_3^2 \right] \right\} \end{aligned}$$

because the error function of a large number tends to one, all the erf's cancel, and

$$\begin{aligned}
q_2 - q_3 &= \frac{Q}{\sqrt{2\ell n(r_4/r_5)}} \left\{ -\frac{\sqrt{2\ell n(r_4/r_5)}}{2} \frac{r_4^2}{\sqrt{2}} + \frac{1}{2} \sqrt{4\ell n(r_4/r_5)} \frac{r_5^2}{\sqrt{2}} \right. \\
&\quad \left. + \frac{1}{\sqrt{2}} \sqrt{2\ell n(r_4/r_5)} \cdot r_2 r_4 - \frac{1}{\sqrt{2}} \sqrt{4\ell n(r_4/r_5)} r_5 r_2 \right\} \\
&= 2.2(-0.354 + 0.00125 + 1.414 - 0.1) = 2.2(0.96) = 2.11
\end{aligned}$$

$$\text{so } q_2 - q_1 = 2.55 = 2.6$$

$$\text{so } \left| \frac{q}{r} \right|_2 = 1.3 \quad (35)$$

11.11 The polarity of the magnetic field change.

In deciding the polarity of the change of the magnetic field along the axis of the discharge tube (i.e. along the central electrode), it must be remembered that the discharge has neither rigidity nor tensile strength. It is the single trajectory common to both ions and electrons. The centre of curvature is therefore on the same side as the $\underline{j} \times \underline{B}$ force (Fig. 11.7 and 11.8). [The $\underline{j} \times \underline{B}$ force is the only force perpendicular to the channel as the electric force drops out of equation (8).] It is true that the electrodes impose an end condition on the discharge, but only by fixing the initial direction of flight.

With the anode at the centre (Fig. 11.7), the current is outward, the $\underline{j} \times \underline{B}$ force clockwise and hence the curvature clockwise. With $\underline{B}$ as before and the cathode central, (Fig. 11.8), the current is reversed and so the curvature is reversed. This agrees with q_2 being negative when the anode is in the centre and positive with the cathode in the centre. But in the first case the current is outward and in the second case inward, so the circulating component of the current is clockwise in both cases. Therefore in either case the field on the axis due to the circulating component of the current opposes the original field (Table 11.1).

This is the great advantage of this approach: it predicts correctly the polarity of the resulting change of flux in both cases, with a reduction of the magnetic field whatever the direction of $\underline{E}$.

Table 11.1

	<u>Anode Central</u> <u>(Fig. 7)</u>	<u>Cathode Central</u> <u>(Fig. 8)</u>
External magnetic field	upwards	upwards
Electric field	outwards	inwards
Hence Current	outwards	inwards
and curvature	negative	positive
and $q_1 = 0$ and	$q_2 = -2.04$	$q_2 = +3.4$
and $\theta_2 - \theta_1$	negative	positive
and the circulating component of current	clockwise	clockwise
so field of this current at $r = 0$	downwards	downwards

So that in both cases the change of magnetic field
opposes the external field at $r = 0$.

11.12.1 The second integration.

We can take the integration process a step further and find θ in terms of r . With the notation of Fig. 11.4

$$q = r \sin \alpha \quad (36)$$

and $\tan(\alpha + \delta\alpha) = r \frac{\delta\theta}{\delta r}$

so in the limit $\tan \alpha = r \frac{d\theta}{dr}$

Therefore,

$$d\theta = \frac{1}{r} \tan \alpha \cdot dr = \frac{1}{r} \cdot \frac{q}{\sqrt{r^2 - q^2}} dr$$

and,

$$\theta_2 - \theta_1 = \int_1^2 \frac{1}{r} \frac{q}{\sqrt{r^2 - q^2}} dr = \int_1^2 \frac{\tan \alpha}{r} dr \quad (37)$$

This has been calculated numerically in section (11.13.5) and plotted in (Fig. 11.9).

11.12.2 The Axial Magnetic Field.

At the centre, $r = 0$, the magnetic field due to the arc current is given by the Biot Savart law.

$$\begin{aligned}
 dB &= \frac{I ds \sin \alpha}{r^2} & I \text{ in e.m.u.} \\
 &= I \frac{rd\theta}{\sin \alpha} \cdot \frac{\sin \alpha}{r^2} = I \frac{d\theta}{r} \\
 B &= I \int_1^2 \frac{1}{r^2} \frac{a}{r^2 - a^2} \cdot dr \\
 &= I \int_1^2 \frac{\tan \alpha}{r^2} dr & (38)
 \end{aligned}$$

This also has been calculated numerically in section (11.13.5).

11.12.3 The ion concentration.

This is given by

$$j = \frac{Ne}{c} (v_+ - v_-) \quad \begin{array}{l} \text{where } j = \text{current} \\ \text{density} \\ N = \text{ion con-} \\ \text{centration} \end{array}$$

and will be examined with the numerical values.

11.13 Numerical Results.

11.13.1 Estimate of error involved in neglecting $\sqrt{m/M}$ terms.

$$f = -Q \left\{ \sqrt{\frac{\ln(r/r_2)}{\ln(r_1/r_2)}} + \sqrt{\frac{m}{M}} \sqrt{\frac{\ln(r_1/r)}{\ln(r_1/r_2)}} \right\} \quad \text{eq. (17)}$$

$$\text{so } q = \frac{-Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_2^2}{4\sqrt{2}} \left\{ 2\sqrt{2\ell n(r_2/r)} \left(\frac{r}{r_2}\right)^2 - 2\sqrt{2\ell n(r_2/r_1)} \left(\frac{r_1}{r_2}\right)^2 \right. \\ \left. + \sqrt{\pi} \operatorname{erf}(\sqrt{2\ell n(r_2/r_1)}) - \sqrt{\pi} \operatorname{erf}(\sqrt{2\ell n(r_2/r)}) \right\} \\ - Q\sqrt{\frac{m}{M}} \frac{1}{\sqrt{\ell n(r_2/r_1)}} \frac{r_1^2}{4\sqrt{2}} \left\{ 2\sqrt{2\ell n(r/r_1)} \left(\frac{r}{r_1}\right)^2 + i\sqrt{\pi} \operatorname{erf}(i\sqrt{2\ell n(r/r_1)}) \right\}$$

$$\text{for } r = r_2 \quad q_2 = -Q(0.927 + \sqrt{m/M} \ 1.547)$$

$$\text{so error} = \sqrt{\frac{m}{M}} \frac{1.547}{0.927} = 3.9\%$$

for $r = r_1$ both contributions are zero, but their ratio is

$$\frac{dq_+/dr}{dq_-/dr} = \frac{rf_+}{rf_-} = \frac{f_+}{f_-} = \frac{0}{1}$$

$$\text{so error} = 0\%$$

with the cathode central, for $r = r_2$

$$\text{the error is } \sqrt{\frac{m}{M}} \frac{0.927}{1.547} = 1.4\%$$

11.13.2 Cut off condition.

In section (11.8) it was mentioned that the cut off value was not given by $|q/r|_{r_2}$ but by $|q/r|_{\max}$. This introduces only a 2% correction^{r2} in the cut-off value, which was, therefore, neglected.

In the case with the anode central, $|q/r|$ has a maximum when

$$rdq - qdr = 0$$

but

$$dq = frdr$$

so the maximum occurs when $q = fr^2$

$$\begin{aligned} \text{so } -\frac{Q}{\sqrt{\ell n(r_2/r_1)}} \frac{r_2^2}{2} & \left\{ \sqrt{\ell n(r_2/r)} \left(\frac{r}{r_2}\right)^2 - \sqrt{\ell n(r_2/r_1)} \left(\frac{r_1}{r_2}\right)^2 \right. \\ & \left. + \frac{\sqrt{\pi}}{2\sqrt{2}} \operatorname{erf}(\sqrt{2\ell n(r_2/r_1)}) - \frac{\sqrt{\pi}}{2\sqrt{2}} \operatorname{erf}(\sqrt{2\ell n(r_2/r)}) \right\} \\ & = -r^2 Q \frac{\sqrt{\ell n(r_2/r)}}{\sqrt{\ell n(r_2/r_1)}} \end{aligned} \quad (39)$$

with $r_1 = 0.5$ $r_2 = 2$ put $x = r_2/r$

and equation (39) reduces to:

$$\frac{\sqrt{\ell n x^2}}{x^2} + \frac{\sqrt{\pi}}{2} \operatorname{erf}(\sqrt{\ell n x^2}) = 0.766 \quad 1 \leq x \leq 4 \quad (40)$$

solving by successive approximations, using a series for erf:

$$x = 1.096$$

$$r = 1.825$$

$$\text{and } q = fr^2 = -0.861 Q$$

$$\text{so } \left| \frac{q}{r} \right|_{\max} = 0.472 Q$$

$$\text{and } Q_{\max} = 2.12 \text{ for cut-off}$$

$$\text{whereas before, } Q_{r_2} = 2.16$$

an error of 2%

In the numerical integration this error is actually 3%.
With the cathode central we have

$$x^2 \sqrt{\ell n x^2} - \frac{i\sqrt{\pi}}{2} \operatorname{erf}(i\sqrt{\ell n x^2}) = 0 \quad \text{where } x = r/r_1$$

and this is always positive except for $x = 1$, $r = r_1$,
 $|q/r| = 0$ so there is no maximum for $|q/r|$,

and the cut-off is still obtained at $r = r_2$.

11.13.3 Error functions.

The error function with an imaginary argument can either be calculated from the series

$$\begin{aligned} \frac{\sqrt{\pi}}{2} \operatorname{erf}(iy) &= \int_0^{iy} e^{-n^2} dn \\ &= i \left(y + \frac{y^3}{3 \cdot 1!} + \frac{y^5}{5 \cdot 2!} + \frac{y^7}{7 \cdot 3!} + \dots \right) \end{aligned} \quad (41)$$

which is absolutely convergent but does so only slowly, or from tables of

$$\frac{\sqrt{\pi}}{2} \operatorname{erf}(iy) = i \int_0^y e^{-n^2} dn$$

11.13.4 Transverse Electric Field.

The electric field induced by charge separation at right angles to the channel is found as follows

$$E = \frac{B}{2V_0} (v_{t-} - v_{t+}) \quad (\text{equation 9})$$

so substituting in equation (4), we get

$$\frac{B}{V_0} \left[\sqrt{\frac{2eV}{mc}} (V - V_0) + \sqrt{\frac{2e(V_0 - V)}{Mc}} V \right] = \underline{E} \cdot \underline{\hat{n}} + \underline{E}' \cdot \underline{\hat{n}} \quad (42)$$

$$\text{or } \underline{E} \cdot \underline{\hat{n}} + \underline{E}' \cdot \underline{\hat{n}} = v_{t+} v_{t-} B \frac{(Mv_{t+} + mv_{t-})}{Mv_{t+}^2 + mv_{t-}^2}$$

This is zero for v_{t+} or $v_{t-} = 0$ (electrodes) and for

$$Mv_{t+} + mv_{t-} = 0$$

$$\text{or } V = \frac{M}{M+m} V_0$$

$$\text{As } \underline{E}' \cdot \underline{\hat{n}} = -E \sin \alpha = -V_0 \frac{\sin \alpha}{r \ln(r_2/r_1)},$$

we get the following typical values for $\underline{E' \cdot \hat{n}}$, the induced field:-

$$\begin{array}{ll} \text{at } r = 1.0, 1.5 \text{ cm} & \underline{E' \cdot \hat{n}} \approx -8.5 \text{ kv/cm} \text{ in both cases} \\ \text{at electrodes (section 11.7)} & \underline{E' \cdot \hat{n}} = 0 \text{ kv/cm} \end{array}$$

11.13.5 Numerical Integration.

As outlined in sections 11.12.1 and 11.12.2, both Θ and B , the axial flux, can be obtained by numerical integration.

$$\Theta_r - \Theta_1 = \int_{r_1}^r \frac{\tan \alpha}{r} dr \quad (37)$$

$$\text{and} \quad B = I \int_1^2 \frac{\tan \alpha}{r^2} dr \quad (38)$$

I in e.m.u.'s

A table of $\frac{\tan \alpha}{r}$ and $\frac{\tan \alpha}{r^2}$ was calculated at intervals of 0.05 cm and of 0.01 cm² between 1.75 cm and 1.85 cm. The resulting curves were sufficiently smooth to be approximated by a succession of straight lines and the integrals obtained by summation. With the anode central, the calculation was performed with 4 figures on a desk machine, so each point should be accurate to about 1%. With the cathode central the accuracy of the tables was not so good, and the calculation was done with a slide rule.

The results are plotted in Fig. 11.9 and as polar diagrams in figures 11.10 and 11.11.

	anode central	cathode central
$\Theta_2 - \Theta_1$	143°	38.5°
equivalent no. of turns with 2kA per channel	2.38	0.64
circulating current	<u>4.76 kA</u>	<u>1.3 kA</u>
$\frac{\text{circulating current}}{\text{total radial current}}$	0.40	0.11
magnetic flux on axis	1.74 I	0.46 I
	= <u>2100 gauss</u>	<u>560 gauss</u>

Notes

Anode Central:- Because $Q = 2.2 \text{ cm}^{-1}$ would have been higher than the cut-off value, V was changed to 4.4 kV so that $Q = 2.1 \text{ cm}^{-1}$. This is justifiable as the accuracy of many measurements was not greater than a few per cent.
($B_{\text{external}} = 470 \text{ gauss}$)

Cathode Central:- With the cathode central, the discrepancy is even higher (section 11.8). To make the calculation possible Q was chosen to be 95% of its cut-off value (about 1.25 cm^{-1}). As a result of this the agreement with practical results is not as good as for the anode central.

11.13.6 Further Numerical Results.Table 11.2. Anode Central

Radius cm	0.5	1.0	1.5	2.0
ion velocity cm/sec	0	$6.5 \cdot 10^7$	$8.2 \cdot 10^7$	$9.2 \cdot 10^7$
electron " "	$3.9 \cdot 10^9$	$2.8 \cdot 10^9$	$1.8 \cdot 10^9$	0
f cm^{-1}	2.1	1.5	1.0	0.05
induced transverse field kV/cm	0	8.5	8.5	0
electrons/cc (1 mm^2)	$3.2 \cdot 10^{14}$	$4.3 \cdot 10^{14}$	$6.6 \cdot 10^{14}$	$14 \cdot 10^{14}$
(10 mm^2)	$3.2 \cdot 10^{13}$	$4.3 \cdot 10^{13}$	$6.6 \cdot 10^{13}$	$14 \cdot 10^{13}$

11.13.7 Ionisation.

At the lowest pressures at which the arc was obtained the initial density of molecules is $2 \cdot 10^{14}$ molecules per cm^3 . The number of ions (or electrons) needed has been calculated in table 11.2, assuming 2 kiloamp. per channel and 1 mm^2 and 10 mm^2 channel cross-section respectively. It is at once apparent that a 1 mm^2 cross-section is impossible but a 10 mm^2 cross-section requires 70% peak ionisation and about 30% elsewhere. A cross-section of 10 mm^2 or more thus leads to a reasonable ionisation. A larger cross-section or a higher pressure both make the degree of ionisation necessary even less.

Conclusions:-

The theoretical analysis just performed is summarized at the beginning of chapter 11. The main results obtained are compared with the practical results in the introduction to the theory, chapter 9. The agreement is good, for this type of work. The whole research and the theory are summarized and discussed in the introduction chapter 1, more especially in section 1.2 where conclusions are drawn, and in the abstract p.2.

Fig 11.9 Polar Coordinates of the Channels

Fig 11.10 Path of Channel

Anode Central

Fig 11-11 Path of Channel

Cathode Central