New Receiver for Joint Blind Equalization and Carrier Phase Recovery of QAM Signals

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Abstract—This paper introduces a new receiver for joint blind equalization and carrier phase recovery. The new receiver overcomes several disadvantages available in literature in this area by accomplishing modulus equalization and phase recovery in two independent operations. The proposed structure enables alternative and independent approaches to be taken for the design of the modulus and phase equalizer respectively, and in the context we propose two new algorithms for the modulus equalizer and the phase shifter. In particular, we propose a new constant modulus (CM) and modulus decision directed (MDD) hybrid algorithm for the equalizer. The CM and MDD both influence the adaptation of equalizer weight taps simultaneously. This enhances both the convergence rate and the steady state performance. The selection of step sizes for the modulus equalizer and the phase shifter is discussed. We compare the performance of our receiver with other previous receivers via computer simulations.

I. INTRODUCTION

Equalization of a communication channel without dependence on a training sequence for start-up is known as blind equalization. The constant modulus algorithm (CMA), introduced by Godard [1], is one of the most popular adaptive blind equalization algorithms. However CMA suffers from phase ambiguity, and to combat this problem several algorithms for joint blind equalization and carrier phase recovery have been proposed in [1], [2], [3], [4], [5] and [6]. These receivers can be divided into two broad classes. These are examined below in some detail. Nevertheless, both classes of receivers have their own disadvantages. In this paper, we introduce a new receiver with independent modulus equalization and phase recovery, which can overcome these disadvantages.

II. REVIEW OF PREVIOUS WORKS

The receivers for blind equalization and carrier phase recovery developed so far can be divided into the following two classes; (a) those receivers, typically represented in [1], [2], and [3], attach a separate phase tracking loop to the CMA equalizer. In order to obtain a faster convergence rate and avoid excessive adjustment for QAM signals, they can switch to the Decision Directed (DD) algorithm after using CMA to open the channel eye at start-up. However, there are two conditions which need to be fulfilled before this switch can be performed. The first is that the tracking phase must have converged prior to switching, as the DD algorithm directly uses the distance between receiver outputs and hard decisions in the adaptation process, a process known to be sensitive

to phase errors. The second condition that needs to be met is that the opening of the channel eye must reach a sufficient level, which is more stringent than fundamental requirement to suppress the decision error probability. Both conditions result in a delay in equalizer convergence. These aspects are not explicitly specified in [2] and [3].

(b) those receivers, typically represented in [4], [5], and [6], propose modified algorithms mostly based on CMA to accomplish simultaneous channel equalization and phase recovery. For example in [4], the authors introduce a modified constant modulus algorithm (MCMA), which decomposes CMA into real and imaginary parts. Thus their receiver does not need an additional phase tracking loop. However, we find that this algorithm can only provide good performance when the phase error is a random constant shift. This means that no carrier frequency offset is considered or tolerated. When such an offset is present, the phase distortion is not a constant random shift but a rotation with a fixed angular velocity, which is a non-stationary process. Thus, the split into real and imaginary axes causes both parts to become non-stationary. In practice, for the case of a non-stationary environment, a lower bound is imposed on the equalizer step size. This suppresses the tracking gap and ensures a correct decision, however, it leads to a degradation in the steady state performance. Furthermore, when there is fast phase change and as a result of a restriction on step size, the receiver performance will suffer severe degradation and might even lose the ability to reconstruct the transmitted signal. This disadvantage is present not only in [4], but also in other receivers (see [5] and [6]), in which the cost functions are directly matched to signal constellations. In a similar manner, because the DD algorithm is also sensitive to phase errors, the receivers based on CM-DD switch algorithms share the same problem.

III. PROPOSED RECEIVER

A. $T/2$ Fractionally Spaced System Model

The model representing the communication system and receiver to be used in this paper is shown in Fig. 1. Thus we adopt the $T/2$ fractionally spaced multi-channel model. The inherent advantages of this model for channel equalization have been carefully reviewed in the survey article [7].

Given a LTI channel and pulse-shape discrete impulse response $c(k)$ of finite length $2M$, the channel coefficients

of the even and odd sub-channels are given by

$$\begin{aligned} c_n^e &= c(2n) \\ c_n^o &= c(2n+1) . \end{aligned}$$

Because the fractionally spaced CMA (FS-CMA) is robust with respect to channel disparity [8], we do not assume that there are no common zeros between the even and odd sub-channels. Similarly, the carrier phase error of the even and odd sub-channels due to carrier frequency offset Δf are given by

$$\begin{aligned} \varphi_n^e &= 2\pi\Delta f \frac{2nT}{2} \\ \varphi_n^o &= 2\pi\Delta f \frac{(2n+1)T}{2} , \end{aligned}$$

where T denotes the baud interval. Let $\mathbf{w}_n^o$ and $\mathbf{w}_n^e$ denote the column weight tap vectors for the odd and even sub-equalizers of length N , and collected in one column vector $\mathbf{w}_n = [(\mathbf{w}_n^o)^T, (\mathbf{w}_n^e)^T]^T$, where superscript T denotes transpose of vector. In this paper, upper and lower case bold letters denote matrices and vectors.

Further let r_n^e and r_n^o denote the input signal to the odd and even sub-equalizers respectively, and define column vector $\mathbf{r}_n$ as the equalizer input regressor vector

$$\mathbf{r}_n = [r_n^e, \dots, r_{n-N+1}^e, r_n^o, \dots, r_{n-N+1}^o]^T .$$

In a similar manner, we also collect the channel noise in one column vector as

$$\mathbf{v}_n = [v_n^e, \dots, v_{n-N+1}^e, v_n^o, \dots, v_{n-N+1}^o]^T ,$$

where v_n^e and v_n^o represent the additive white and complex Gaussian noise sources at the even and odd sub-channels, respectively.

Introduce the $(N+M-1) \times N$ Toeplitz matrix combining the channel impulse response and the phase error for the even sub-channel

$$\mathbf{C}_n^e \triangleq \begin{bmatrix} c_0^e e^{j\varphi_n^e} & & & & \\ c_1^e e^{j\varphi_n^e} & c_0^e e^{j\varphi_{n-1}^e} & & & \\ \vdots & c_1^e e^{j\varphi_{n-1}^e} & & & \\ c_{M-1}^e e^{j\varphi_n^e} & \vdots & \ddots & c_0^e e^{j\varphi_{n-N+1}^e} & \\ & c_{M-1}^e e^{j\varphi_{n-1}^e} & & c_1^e e^{j\varphi_{n-N+1}^e} & \\ & & & \vdots & \\ & & & c_{M-1}^e e^{j\varphi_{n-N+1}^e} & \end{bmatrix} ,$$

and similarly for $\mathbf{C}_n^o$. Then, defining the $(N+M-1) \times 2N$ matrix $\mathbf{C}_n \triangleq [\mathbf{C}_n^e, \mathbf{C}_n^o]$, we can write $\mathbf{r}_n = \mathbf{C}_n \mathbf{a}_n + \mathbf{v}_n$, where $\mathbf{a}_n = [a_n, a_{n-1}, \dots, a_{n-N-M+2}]^T$ is the source vector. In this paper, a_n is assumed to be circular symmetric, independent identical-distributed (i.i.d.) and sub-Gaussian.

From Fig. 1, the modulus equalizer output z_n and the phase shifter output y_n are given by

$$z_n = \mathbf{w}_n^H \mathbf{r}_n \quad (1)$$

$$y_n = z_n e^{j\theta_n} , \quad (2)$$

Fig. 1. Communication system and receiver model.

where θ_n is the phase tracking parameter, and superscript H denotes Hermitian of vector.

B. Proposed Receiver

The proposed receiver consists of two independent operations, namely the modulus equalizer and the phase shifter. The modulus equalizer uses a new constant modulus and modulus decision directed hybrid algorithm (CM-MDDA) to penalize the modulus deviation of the received signal, while the phase shifter recovers the phase ambiguity.

First, we introduce the adaptation of the modulus equalizer. Unlike previous CM-DD switch algorithms, the proposed algorithm allows both the CM and MDD cost functions to influence the adaptation of $\mathbf{w}_n$ jointly. At every iteration, the error function performs a balance between the following two cost functions

$$J_{CM} = \frac{1}{4} E\{(\gamma^2 - |z_n|^2)^2\} \quad (3)$$

$$J_{MDD} = \frac{1}{4} E\{(|s_n|^2 - |z_n|^2)^2\} , \quad (4)$$

where $\gamma^2 = E\{|a_n|^4\}/E\{|a_n|^2\}^2$ is the Godard dispersion constant, and s_n is the feedback hard decision shown in Fig. 1. The MDD algorithm was first proposed in [9] for QAM signals because it only penalizes the modulus deviation and is immune to phase errors of received signals. In order to improve its stability of convergence, an improved algorithm, known as CADAMA, was proposed in [10] by combining the CM and MDD algorithms. However, this algorithm is also based on switching from CM to MDD. Thus it clearly differs from our proposed approach, which aims to strike a balance at each iteration between the two objective functions.

From (1), (3) and (4), the stochastic updating equation of $\mathbf{w}_n$ is now given by

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_n^w (\alpha_n e_{CM} + (1 - \alpha_n) e_{MDD})^* \mathbf{r}_n \quad (5a)$$

$$e_{CM} = (\gamma^2 - |z_n|^2) z_n \quad (5b)$$

$$e_{MDD} = (|s_n|^2 - |z_n|^2) z_n , \quad (5c)$$

where μ_n^w is the step-size of the modulus equalizer at time instant n , e_{CM} and e_{MDD} denote the CM and MDD errors respectively, and α_n is a weighting factor that trades off these two errors, which is introduced below. The symbol $*$ denotes complex conjugation.

The phase shifter to provide the required phase equalization is a modification of the Decision Directed approach of [11]. The significant modification is to render this specific algorithm insensitive to the modulus of the signal. We suggest the objective function for this purpose as

$$J_{PH} = E\left\{\left|\frac{s_n}{|s_n|} - \frac{y_n}{|y_n|}\right|^2\right\} = E\left\{\left|\frac{s_n}{|s_n|} - \frac{z_n e^{j\theta_n}}{|z_n|}\right|^2\right\}. \quad (6)$$

Thus, from (2) and (6), the updating equation of θ_n is given by

$$\theta_{n+1} = \theta_n - \mu_\theta \frac{\Im(y_n s_n^*)}{|y_n s_n|}, \quad (7)$$

where μ_θ is the phase tracking step-size and $\Im$ denotes the imaginary projection operator.

C. Selection of Weight Factor α_n

Let us rewrite (5) as

$$\begin{aligned} \mathbf{w}_{n+1} &= \mathbf{w}_n + \mu_n^w (\gamma_{new}^2 - |z_n|^2) z_n^* \mathbf{r}_n \\ \gamma_{new}^2 &= \alpha_n \gamma^2 + (1 - \alpha_n) |s_n|^2. \end{aligned} \quad (8)$$

This equation indicates that at every iteration, the proposed algorithm introduces a new parameter γ_{new}^2 to penalize the modulus deviation of equalizer outputs. The parameter γ_{new}^2 combines the source dispersion constant and the hard decision modulus, and is controlled by the weight factor α_n . At start-up, in order to use CMA to open the channel eye, γ_{new}^2 is chosen close to the source dispersion constant γ^2 . This means that α_n is initialized as 1. However, since for non-constant modulus sources, the CM cost function does not go to zero at any of the signal points in the constellations, the adjustment exhibited by CMA can be excessive, [10]. Furthermore, the MDD cost function goes to zero at each ring of QAM constellations, [9]. Thus, using the hard decision modulus instead of the source dispersion constant can lead to a much smaller mean square error (MSE) at steady state. Note that for constant modulus sources, this new algorithm is identical to CMA.

Based on the above, the weight factor α_n is updated as

$$\alpha_{n+1} = \left[\lambda \alpha_n + (1 - \lambda) \frac{|s_n - y_n|^2}{R_{th}^2} \right]_{\alpha_-}^{\alpha_+}, \quad (9)$$

where $\lambda \in [0, 1)$ controls the length of the effective data window and R_{th} is a self-defined distance threshold. We discuss the selection of R_{th} later in this section. $[\cdot]_{\alpha_-}^{\alpha_+}$ denotes truncation to the limits of the range $[\alpha_-, \alpha_+]$, and α_- and α_+ are set as 0 and 1, respectively. First we consider the case without carrier frequency offset. The numerator in (9) is the squared error of the receiver output at time instant n . Provided that the effective data window is sufficiently long, α_n can converge to a value close to the ratio of MSE to R_{th} at steady state. Thus, if MSE is large at start-up, the algorithm relies more on CMA. During the steady state, if MSE has been reduced to small enough value, α_n converges to a small value very close to 0 and the algorithm relies more on the MDD algorithm. Then we take the influence of the carrier frequency offset into consideration. Because

of the presence of phase tracking gap, using MSE of the receiver outputs (combining the modulus equalizer and the phase shifter) to adjust α_n makes the modulus equalizer partially dependent on the performance of the phase shifter. However, such arrangement is reasonable. With the adoption of the “minimum Euclidean distance” in the decision device, large phase errors might lead to decision errors, especially for QAM signals. CMA uses the source dispersion constant in the adaptation process and is immune to these decision errors, while the MDD algorithm uses hard decision modulus. Therefore, relying more on CMA temporarily can alleviate the influence of such decision errors and increase the possibility of convergence.

In this part, we discuss the selection of R_{th} . According to [12], the open-eye condition can be expressed by

$$|y_n - a_{n-\delta}| < \frac{D}{2} \quad \text{for all } n,$$

where D is the minimum distance between the symbols in the constellations and δ is an unknown delay. In practice, $a_{n-\delta}$ is usually substituted by the feedback decision s_n . Thus, without taking phase rotation into account, the optimal selection of R_{th} is $D/2$. With phase rotation, the selection of R_{th} as $D/2$ is not optimal. However, if the phase tracking gap is controlled sufficiently small, such selection can also provide a good transient and steady state performance.

Finally, in our algorithm, the MDD algorithm is allowed to influence the adaptation process of $\mathbf{w}_n$ when $|s_n - y_n|$ is less than $D/2$, which means that the channel eye has not been clearly opened. This feature enhances the convergence rate of the algorithm. In addition, under strong noise environments, some CM-DD switch algorithms need to switch frequently between CM and DD algorithms even at steady state. However, our proposed algorithm can perform a balance between them, thereby allowing a more stable steady state performance.

D. Selection of Step Size μ_n^w and μ_θ

Owing to the independence between the modulus equalizer and the phase shifter, the step sizes μ_n^w and μ_θ can be selected as different values to provide better performance.

First, μ_θ is selected as a constant value in this algorithm. Because the carrier frequency offset normally introduces a phase rotation with a fixed angular velocity which is a non-stationary process, the selection of μ_θ actually can be regarded as a compromise between the phase variance and the tracking gap at steady state. Because of the adoption of the “minimum Euclidean distance” decision role, the tracking gap must be kept small enough to ensure correct decisions. It has been proved in [11] that the tracking gap plays a more significant role than the variance of θ_n in the total phase misadjustment. Furthermore, the tracking gap is inversely proportional to μ_θ^2 . Therefore, contrary to the conventional proposal of small selection in [13], μ_θ should be selected as a substantially large number to enhance the robustness of the phase tracking algorithm.

As to the modulus equalizer, the step size μ_n^w is adapted via the Ordinary Differential Equation (ODE) method in [14] to improve the transient behavior.

From (8), the updating equation of μ_n^w is given by

$$\mu_{n+1}^w = [\mu_n^w + \rho \Re(e_n \mathbf{r}_n^H \Psi_n)]_{\mu_-}^{\mu_+}, \quad (10)$$

where ρ is the learning rate of μ_n^w , $\Re$ denotes the real projection operator, and Ψ_n represents the derivative $\partial \mathbf{w}_n / \partial \mu_n^w$. The parameters μ_+ and μ_- decide the truncation range of μ_n^w . The combined error e_n of e_{CM} and e_{MDD} in (5b) and (5c) is given by

$$e_n = (\gamma_{new}^2 - |z_n|^2) z_n.$$

The updating equation of Ψ_n is given by

$$\Psi_{n+1} = [I + \mu_n^w (\gamma_{new}^2 - 2|z_n|^2) \mathbf{r}_n \mathbf{r}_n^H] \Psi_n + e_n^* \mathbf{r}_n, \quad (11)$$

where I is an identity matrix.

As to the lower and upper bounds of μ_n^w , we set μ_- to 0, while μ_+ is determined at every iteration according to the modulus equalizer performance, rather than as a pre-determined constant.

We propose the selection of μ_+ in our algorithm analogous to the idea of the normalized constant modulus algorithm (NCMA) in [15]. From the introduction of prior and posterior errors, by solving a quadratic inequality in μ_n^w , (see [15]), we can obtain that μ_n^w must fulfill the following inequality to prevent excessive adjustment in the adaptation process

$$\mu_n^w \leq \frac{1}{(|z_n|^2 + |z_n| \gamma_{new}) \|\mathbf{r}_n\|_2^2}, \quad (12)$$

where $\|\cdot\|_2$ denotes the 2-norm of vector. In [15], μ_n^w is suggested to be chosen directly as this value after multiplying a small constant β . However, we find that this can cause a degradation in the steady state performance, because the step size reaches its maximum value at every iteration. Furthermore, because decision errors can also influence the result of (12), we suggest that it be set as the upper bound of the step size in the ODE method. Therefore, the upper bound μ_+ is finally given by

$$\mu_+ = \frac{\beta}{(|z_n|^2 + |z_n| \gamma_{new}) \|\mathbf{r}_n\|_2^2}. \quad (13)$$

IV. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed receiver using computer simulations and compare it with previous designs. In all simulations, the channel information was obtained from Chan 10 of the Signal Processing Information Base (SPIB) microwave channel model. The input source signal a_n was unit power 16-QAM. Thus R_{th} was selected as $1/\sqrt{10}$. The sub-equalizer length N was selected as 16 and the sub-channel length M was selected as 17. We used the single-spike method to initialize the modulus equalizer weight tap vector $\mathbf{w}_n$, which means that only one component in $\mathbf{w}_n$ was set as 1 and all others were set as 0. The 1 was set at the same position for all algorithms to eliminate the initialization effect on receiver performances. To

evaluate the receiver performance, we define the output signal-to-interference-and-noise ratio (SINR) as

$$SINR = \frac{\max(|\mathbf{q}_n|^2)}{\|\mathbf{q}_n\|_2^2 - \max(|\mathbf{q}_n|^2) + \|\mathbf{w}_n\|_2^2 \sigma^2},$$

where $\mathbf{q}_n = \mathbf{C}_n \mathbf{w}_n$ is the combined channel and equalizer system response, $\max(\cdot)$ denotes the maximum element selector, and σ^2 is the variance of the additive noise. All learning curves shown were obtained by averaging over 10 individual trials.

First, we compared the performance of the proposed receiver with MCMA in [4] and a CM-DD switch algorithm (CM-DDSA) under the condition of no carrier frequency offset. The signal-to-noise ratio (SNR) of the receiver input was set as 30dB. In our algorithm, the phase tracking step size μ_θ was selected as 0.1. In the ODE method, the modulus equalizer step size learning rate ρ was selected as 10^{-7} . The upper bound controlling constant β was chosen as 0.1. The initialization of μ_n^w is not important [14] and can be selected as any small value, e.g. 10^{-5} , and Ψ_n was initialized as a zero vector. The data window length controlling parameter λ was selected as 0.99. We also used the ODE method to adapt the step size in MCMA and CM-DDSA with the same step size learning rate. Fig. 2 compares the output SINRs of CM-MDDA, CM-DDSA and MCMA. All three algorithms perform closely at steady state, while the proposed CM-MDDA has the fastest convergence rate. The convergence of CM-DDSA is delayed due to the wait period for switching. As to MCMA, because it splits CMA into two parts along the real and imaginary axes, the 16-QAM signal is also divided into two 4-PAM signals. Thus, MCMA actually consists of two real CMAs, each dealing with one 4-PAM signal. Because 4-PAM signal is non-constant modulus, the performance of each real CMA in MCMA is degraded. Therefore, the convergence of MCMA is slower than that of CM-MDDA and its SINR at steady state is also around 1dB smaller.

Fig. 2. Comparison of SINR between CM-MDDA, CM-DDSA and MCMA without carrier frequency offset.

Then, we took the carrier frequency offset into the comparison. We kept all parameters unchanged. The carrier frequency offset $\Delta f T$ was selected as same as in [4], i.e. 10^{-4} . Because the DD algorithm is not able to recover phase errors when phase rotation is tolerated, we only compared the performance

between CM-MDDA and MCMA. Fig. 3 compares the output SINRs of these algorithms. CM-MDDA is able to maintain a similar performance, but MCMA suffers severe performance degradation. The SINR of MCMA is not only over 10dB smaller than that of CM-MDDA but it fluctuates more severely. Fig. 4 shows that the step size of MCMA is much larger than that of CM-MDDA. The non-stationary property in both the real and imaginary parts of MCMA causes the ODE method to converge to the large step size, and this leads to the performance degradation. In addition, Fig. 5 compares 3000 points of receiver outputs at steady state between CM-MDDA and MCMA. The deviation of receiver outputs of CM-MDDA from the signal points in the constellation is much smaller than that of MCMA.

V. CONCLUSION

In this paper, a new receiver is proposed for joint blind equalization and carrier phase recovery. We construct the new receiver with independent modulus equalization and phase recovery. Based on the receiver structure, we propose a new CM-MDD hybrid algorithm for the modulus equalizer to enhance both the convergence rate and the steady state performance. Unlike other CM-DD switch algorithms, the new algorithm allows both the CM and MDD cost functions to influence the adaptation of the modulus equalizer weight tap vector $\mathbf{w}_n$ simultaneously and can automatically reach a balance at steady state. We also modify the cost function for phase recovery to be modulus independent. The step size for the phase shifter is chosen as a large number, while ODE method is used in adapting the step size of the modulus equalizer. In conclusion, the proposed receiver overcomes some disadvantages in previous designs and improves both the convergence rate and the steady state performance.

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Fig. 3. Comparison of SINR between CM-MDDA and MCMA with carrier frequency offset.

Fig. 4. Comparison of step size between CM-MDDA and MCMA.

Fig. 5. Comparison of output constellation between CM-MDDA and MCMA. (a) CM-MDDA. (b) MCMA.